Portland State UNIVERSITY

Fariborz Maseeh Department of Mathematics and Statistics

MTH 256: Applied Ordinary Differential Equations

Updated Fall 2018

Course Description: Solution techniques in ordinary differential equations; applications.

Credits: 4

Prerequisites: Math 252 and Math 261

Course Objectives: The course introduces elementary solution techniques and methods of qualitative analysis of ordinary differential equations. It also discusses some applications of the theory of ordinary differential equations in engineering and physical sciences.

Student Learning Outcomes: Upon completion of this course the student should be able to:

- Solve some elementary ordinary differential equations.
- Solve systems of linear ordinary differential equations with constant coefficients.
- Be familiar with the concept of the Laplace transform and use it effectively.
- Be familiar with the concept of an equilibrium solution, its different types, and understand the relation between the type of an equilibrium solution and the long-term behavior of solutions.
- Identify the equilibrium solutions of some elementary ordinary differential equations and system of ordinary differential equations.
- Use effectively these solutions techniques in applications.

Topics:

1. *First Order Differential Equations:* Analytic techniques such as separation of variables and the method of the integrating factors Qualitative techniques: slope fields, equilibrium, phase line, bifurcation. Existence and uniqueness of solutions Numerical techniques: Euler's method.

2. *Systems of First-Order Equations:* The geometry of systems of equations. The damped harmonic oscillator. Modeling via systems: Lorenz equations, the SIR model of an epidemic.

3. *Systems of Linear First-Order Equations:* Properties of linear systems in two dimensions. Phase planes for linear systems: real eigenvalues, complex eigenvalues,

repeated and zero eigenvalues. Second-Order linear equations. Linear systems in three dimensions.

4. *Forcing and Resonance:* Forced harmonic oscillator. Sinusoidal forcing. Undamped forcing and resonance. The steady state.

5. *Laplace Transforms:* Laplace transforms. Second-order equations. Discontinuous functions, delta function and impulse forcing. Convolution.

6. *Nonlinear Systems:* Equilibrium point analysis. Hamiltonian systems.

Textbook:

Paul Blanchard, Robert L. Devaney and Glen R. Hall, *Differential Equations*, Brooks/Cole, 4th ed.

MTH 256 Textbook Mapping:

Paul Blanchard, Robert L. Devaney and Glen R. Hall, *Differential Equations*, 4th ed., Brooks/Cole.

Chapter 1. First-Order Differential Equations

1.1 Modeling via Differential Equations
1.2 Analytic Techniques: Separation of Variables
1.3 Qualitative Technique: Slope Fields
1.4 Numerical Technique: Euler's Method
1.5 Existence and Uniqueness of Solutions
1.6 Equilibrium and the Phase Line
1.7 Bifurcations
1.8 Linear Equations
1.9 Integrating Factors for Linear Equations

Chapter 2. First-Order Systems

2.1 Modeling via Systems
2.2 The Geometry of Systems
2.3 The Damped Harmonic Oscillator
2.4 Additional Analytic Methods for Special Systems
2.6 Existence and Uniqueness for Systems (optional)
2.7 The SIR Model of an Epidemic (optional)
2.8 The Lorenz Equations (optional)

Chapter 3. Linear Systems

3.1 Properties of Linear Systems and the Linearity Principle
3.2 Straight-Line Solutions
3.3 Phase Planes for the Linear Systems with Real Eigenvalues
3.4 Complex Eigenvalues
3.5 Special Cases: Repeated and Zero Eigenvalues
3.6 Second-Order Linear Equations
3.8 Linear Systems in Three Dimensions

Chapter 4. Forcing and Resonance

4.1 Forced Harmonic Oscillators
4.2 Sinusoidal Forcing

4.3 Undamped Forcing and Resonance
4.4 Amplitude and Phase of the Steady State

Chapter 5. Nonlinear Systems
5.1 Equilibrium Point Analysis
5.3 Hamiltonian Systems (optional)

Chapter 6. Laplace Transforms
6.1 Laplace Transforms
6.2 Discontinuous Functions
6.2 Second-Order Equations
6.4 Delta Functions and Impulse Forcing
6.5 Convolutions

Other sections may also be covered depending on the interests of students and instructors.