Lecture 2: Population growth rates

- Arc of the course
- Intro
  - Biodiversity is important for ecosystem function
  - Many causes for the loss of biodiversity
- Module 1 – population growth
- Module 2 – habitat loss and protection
- Module 3 - climate

Declining Biodiversity

Declining global biodiversity can be explained by reference to the five *HIPCO* factors

H Habitat Loss & fragmentation

I Invasive Species

P Pollution

C Climate Change

O Overharvesting (direct killing)

(We're not addressing invasive species or pollution.)

What do we need to know to manage populations and ecosystems and restore biodiversity and ecosystem function?

1. What do we know about populations?

2. How can we manage local biodiversity and ecosystem functions?

3. How can we manage global cycles?

Population Growth

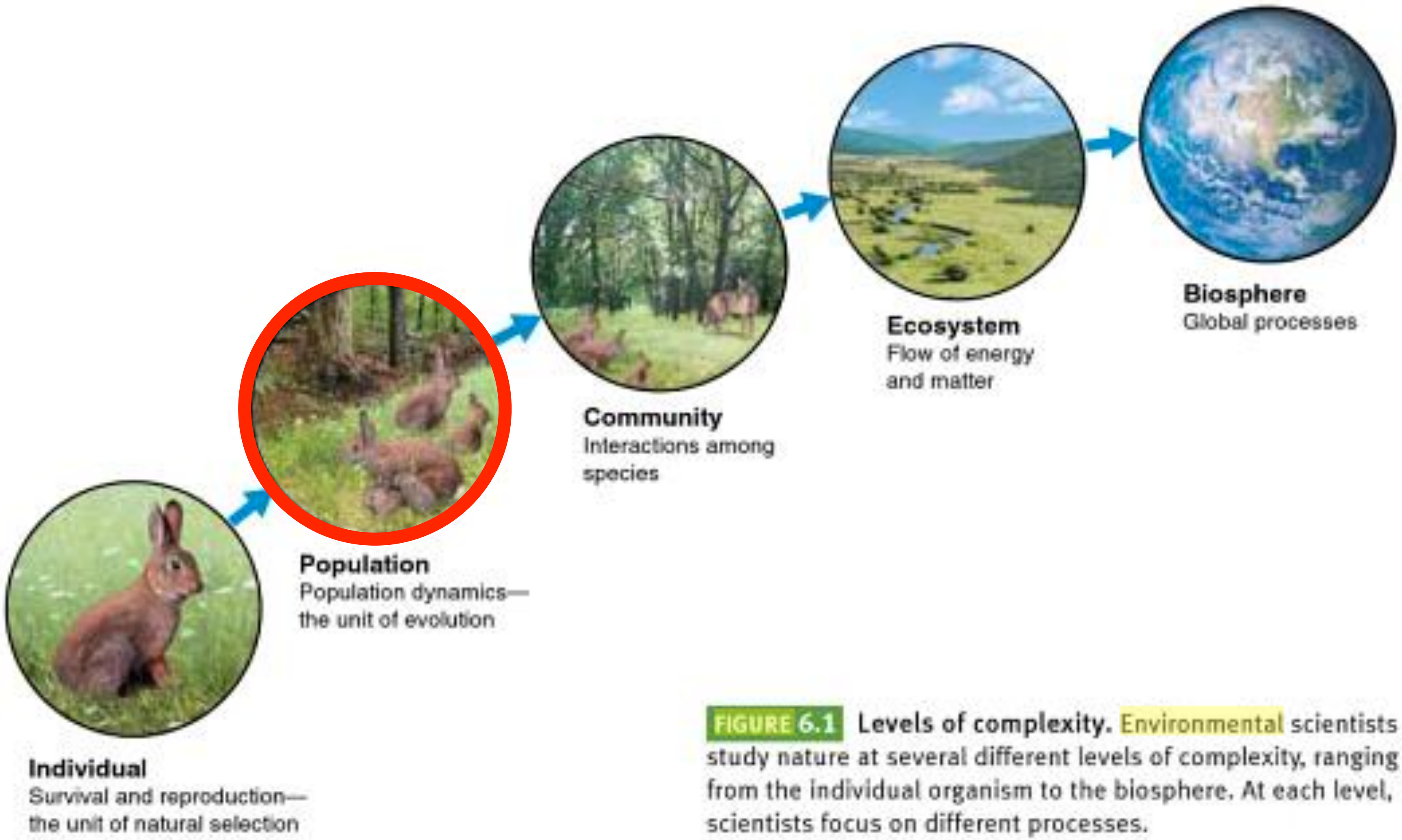

FIGURE 6.1 **Levels of complexity.** Environmental scientists study nature at several different levels of complexity, ranging from the individual organism to the biosphere. At each level, scientists focus on different processes.

Information about births and deaths is essential to predict future population size and inform management.

ECOLOGY 2e, Figure 9.3

How would you get this information?

Population dynamics: What primary factors affect population size?

$$\mathbf{N_{t+1} = N_t + B - D + I - E}$$

- N_t = initial population size at time t
- B = number of individuals born <u>during the time interval</u>
- D = number of individuals that die <u>during t</u>
- I = number that immigrate during t
- E = number that emigrate during t

Calculation 1. Complete the new population size for these two examples

$$N_{t+1} = N_t + B - D + I - E$$

Population of finches on an island over 1 yr

- $N_t = 463$
- B = 87
- D = 93
- I = 39
- E = 14
- $N_{t+1} =$

Population of Daphnia in a small pond over summer (3 months)

- $N_t = 14, 207$
- B = 8,212
- D = 4,945
- $N_{t+1} =$

1. Completed population size examples

$$N_{t+1} = N_t + B - D + I - E$$

Population of Finches on an island during 1 year

N_t =	463	463
+ B =	87	87
- D =	93	+ 39
+ I =	39	589
- E =	14	-107
N_{t+1} =		482

Population of Daphnia in a small pond over 3 summer months

N_t:	14, 207
+ B:	8,212
	22,419
- D:	4,945

N_{t+1} = 17,474

Definitions

- What does ***per capita*** mean?
- What does **rate** mean?

Developing a growth rate equation to predict population change (1)

- For simple predictions, we assume no E or I. Why?
- Formula becomes: $N_{t+1} = N_t + B - D$
- To allow you to model population size across time periods, re-write $N_{t+1} - N_t$ as: $dN/dt = B-D$
  d is shorthand for 'the change in' (derivative of)
- The change in N (pop size) with respect to changing t (time) is written: dN/dt = population growth rate
- dN/dt is an operator (don't divide N by t)

Developing a growth rate equation to predict population change (2)

- Birth and death are combined into the term growth rate, number of births minus deaths in the time period: B – D
- Growth rate is typically expressed per capita, (because population size affects the population-level growth rate): B/N – D/N = r; b-d=r.
  b = per capita birth rate; d = per capita death rate
- Rate of change in N can be written: $\mathbf{dN/dt = rN_t}$

Exponential growth is described as $\mathbf{dN/dt = rN_t}$

The intrinsic growth rate (r)

- Definition: Under ideal conditions, with unlimited resources available, every population has a particular maximum growth potential for growth.
- Notated as r or r_{max}
- = b – d under ideal conditions

Calculation 2. Calculate growth rate

Population of Finches on a remote island during 1 yr

- $N_t = 463$
- $B = 87$
- $D = 93$

What is the growth rate, r?

$= b - d = B/N - D/N = (B-D)/N$

$= r$

2. Calculate growth rates

Population of Finches on a remote island during 1 year

- N_t = 463
- B = 87
- D = 93
- B/N-D/N = (87-93)/463 = r = -0.013

Population of Daphnia in a small pond over 3 months

- N_t = 14, 207
- B = 8,212
- D = 4,945

What is the growth rate, r?

2. Calculated growth rates for examples

Population of Finches on a remote island during 1 year

N_t = 463

- B = 87
- D = 93

- What is the growth rate?
  r= -0.013

Population of Daphnia in a small pond over 3 summer months

- N_t = 14,207
- B = 8,212
- D = 4,945
- What is the growth rate?
  B/N-D/N =
  (8,212-4,945)/14,207
-
  = r = 0.23

3. Check in on r

- Write a group definition of r
- What information do you need to calculate r?

Quick Review of log and e:

Logarithms are the inverse of exponentials

—The Relationship—

$y = b^x$ ....is equivalent to.............. $\log_b(y) = x$
(means the exact same thing as)

$100 = 10^2$; $\log_{10}(100)=2$; $\log_{10}(10^2)=2$; $\log(100)=2$

The base of the log can be 10 or e…

e, Euler's constant ≈ 2.718, is the base of the natural logarithm,

$\log_e = \ln(x)$

In Excel, e^x is written as exp(x);

using your calculator…

1. What is the natural log of 2 = $\log_e(2) = \ln(2)$ = _____? *type 2, press ln*
2. $e^{3.73767}$ = ___ ? *type 3.73767, then press e^x (2nd function)*
3. log(10)= ____? ln(e) = ____ ? $\ln(e^x)$= _____

(Natural log = area under the curve of y=1/x)

Quick Review of log and e:

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The base of the log can be 10 or e…

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$\log_e = \ln(x)$

In Excel, e^x is written as exp(x);

using your calculator…

1. What is the natural log of 2 = $\log_e(2) = \ln(2)$ = _0.69____?
2. $e^{3.73767}$ = _42___ ?
3. log(10)= _1____? ln(e) = _1____ ? $\ln(e^x)$= _x_____

(Natural log = area under the curve of y=1/x)

Definitions: Review

Population: individuals of a species that live in the same area at a given time

Species: a group of organisms that are morphologically, behaviorally, or biochemically distinct from other groups and that produce viable offspring with each other but can't with others

Community: suite of species that co-occur in time and space

Biodiversity: variety of life forms

Ecosystem: interacting biotic & abiotic components of an area

Biome: region & its biological community shaped by regional temperatures, amount of precipitation, soil, & disturbance (land) or in salinity, depth, and flow (aquatic; less used)

Biosphere: the combination of all living organisms & our ecosystems; where life resides

r = per capita growth rate and often intrinsic growth rate, the per capita growth rate of a population in ideal conditions

Exponential Growth

In many species, the population can reproduce *continuously*, and generations can overlap.

When populations increase by a <u>constant</u> proportion of the population size (yet by ever more individuals each time period), there is **exponential growth** (or if declining, exponential decay).

E.g., a population doubles in size every 10 yr

The ***exponential growth*** *model* describes a population with overlapping generations that grows **continuously** at a **fixed** rate.

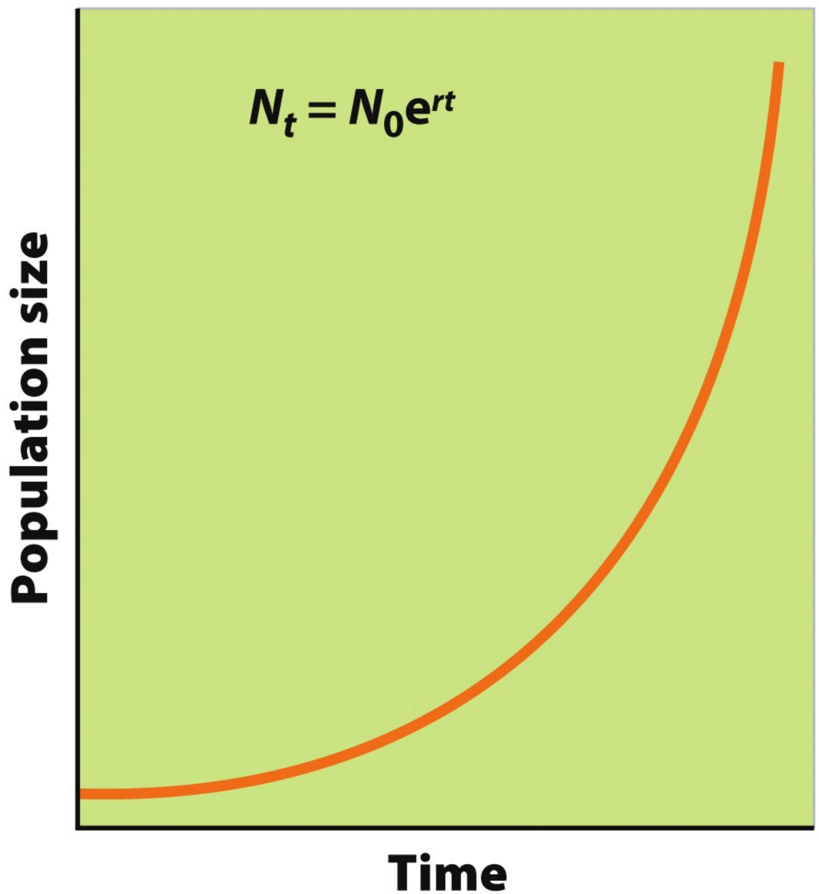
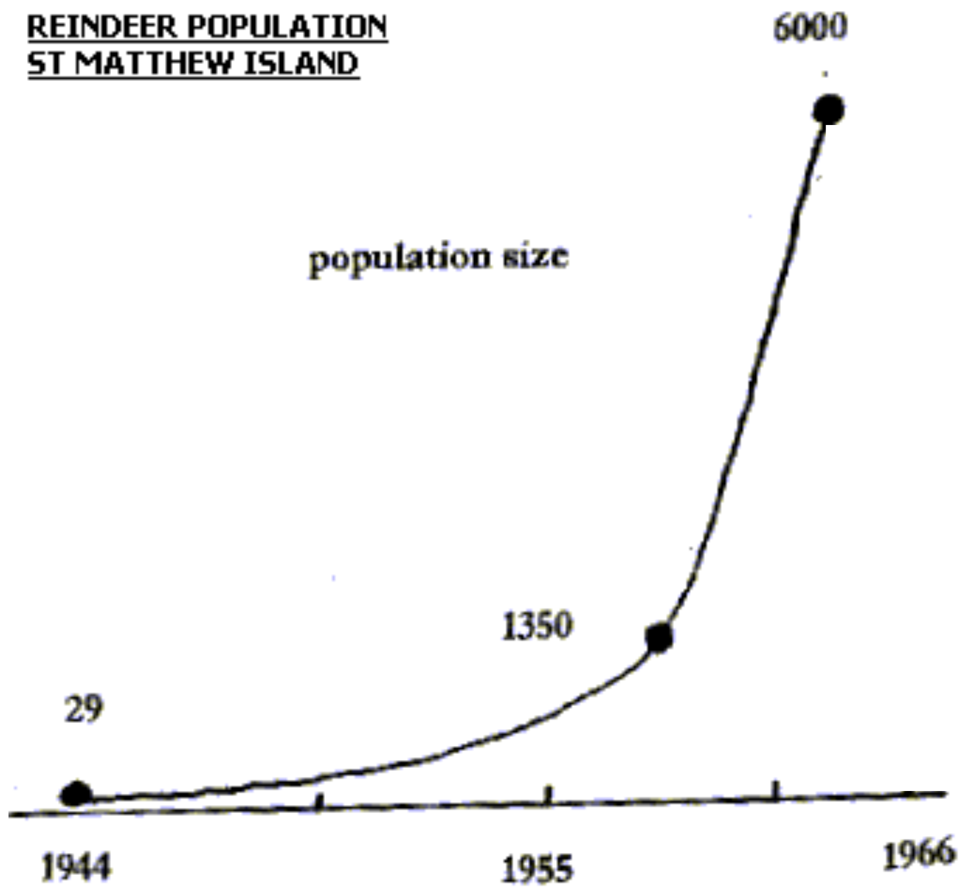
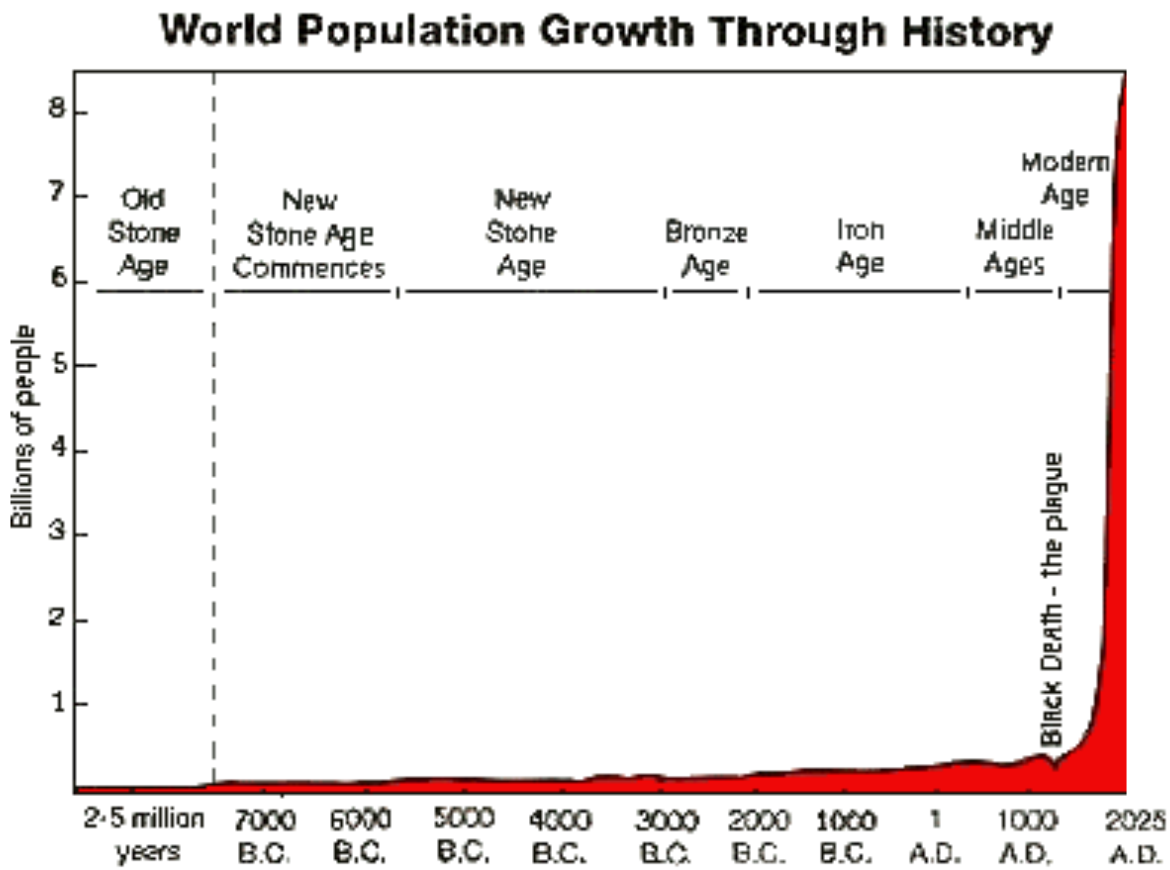


Figure 6.5
Environmental Science

Example Population Growth



Human Population Growth



Exponential growth formula, the base model for population growth

$$N_t = N_0 e^{rt}$$

- Use this formula to predict for an exponentially ‘growing’ population one of: population size, intrinsic growth rate, or elapsed time
- This formula assumes that conditions for survival and reproduction are ideal (= perfect)
- This formula has many applications
- (this equation is the integral of $dN/dt = rN$)

4. What do each of these variables mean?

- **N_t =**
- **N_0 =**
- **e =**
- **r =**
- **t =**

5. Calculate population size after 1, 2, and 3 months of growth for this example:

Fruit flies in a lab culture with ideal conditions,

- Initial population size (at time zero), $N_0 = 147$
- Time, $t = 1$ month
- Intrinsic growth rate, $r = 0.1$
- Formula: $N_t = N_0 e^{rt}$

- All together for the first time step
- Solving for N_t

6. Calculated size over three months

Fruit flies in a lab culture with ideal conditions,

– Initial population size (at time zero), $N_0 = 147$

– Time, $t = 1$ month

– Intrinsic growth rate, $r = 0.1$

– Formula: $N_t = N_0 e^{rt}$

1 mo: $N_t = 147 * e^{0.1*1mo} = 162$

2 mo:

3 mo:

6. Calculated size over three months

Fruit flies in a lab culture with ideal conditions,

- Initial population size (at time zero), $N_0 = 147$
- Time, $t = 1$ month
- Intrinsic growth rate, $r = 0.1$
- Formula: $N_t = N_0 e^{rt}$

1 mo: $N_t = 147 * e^{0.1*1\ mo} = 162$

2 mo: $N_t = 147 * e^{0.1*2\ mo} = 179$

3 mo: $N_t = 147 * e^{0.1*3\ mo} = 198$

(note: the population grows by a greater amount each time step because more individuals are reproducing)

Ended here on Wed

7. Challenge questions:

a. Draw N vs t for $r = 0$, $N_0 = 500$, for $t = 0, 10, 20$

b. Draw N vs t for $r = 1$, $N_0 = 500$, for $t=0, 2, 5, 10$

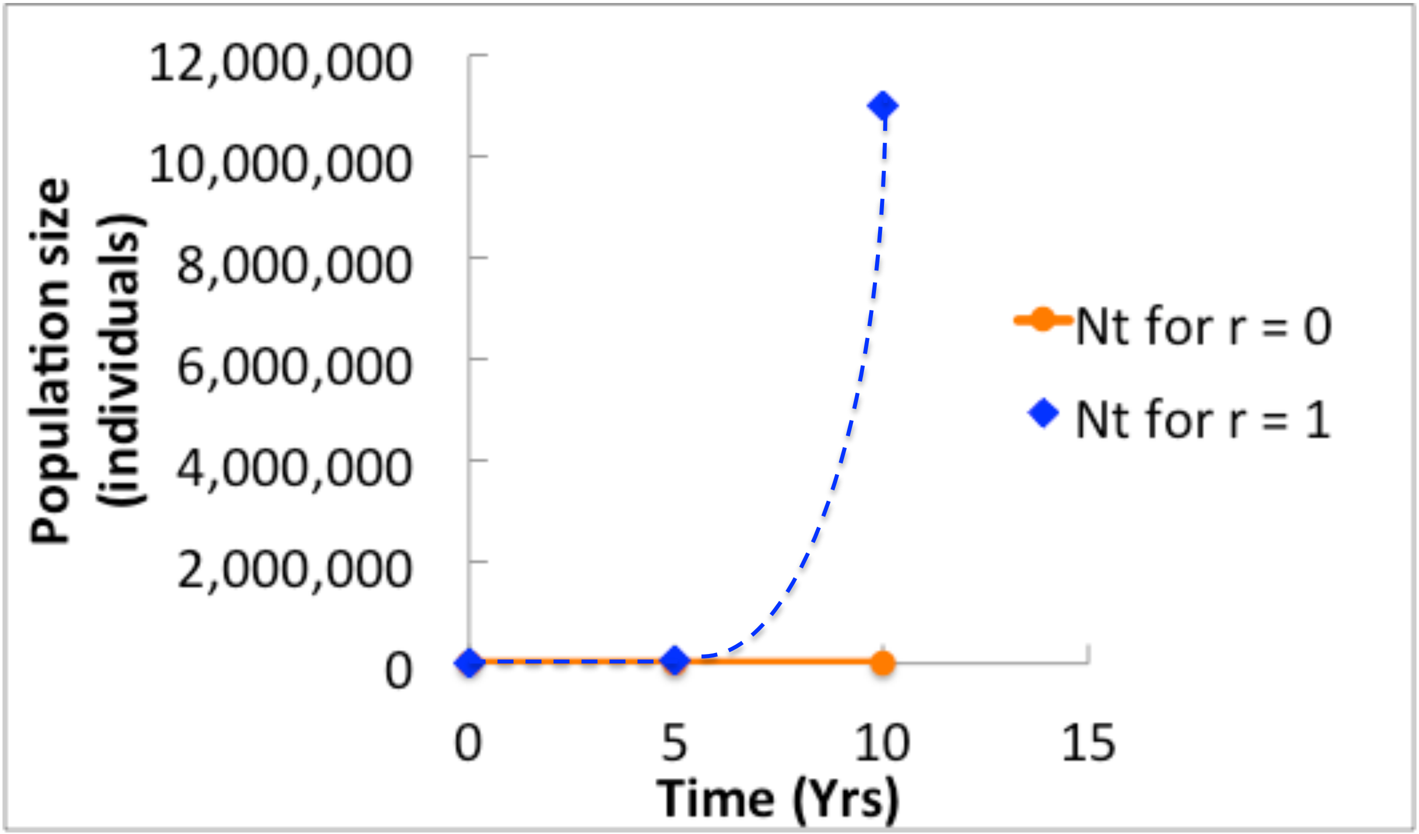
(put above 2 on the same graph)

c. Draw dN/dt on the Y axis versus N on the X axis. (What equation would you use?)

d. What would the slope be?

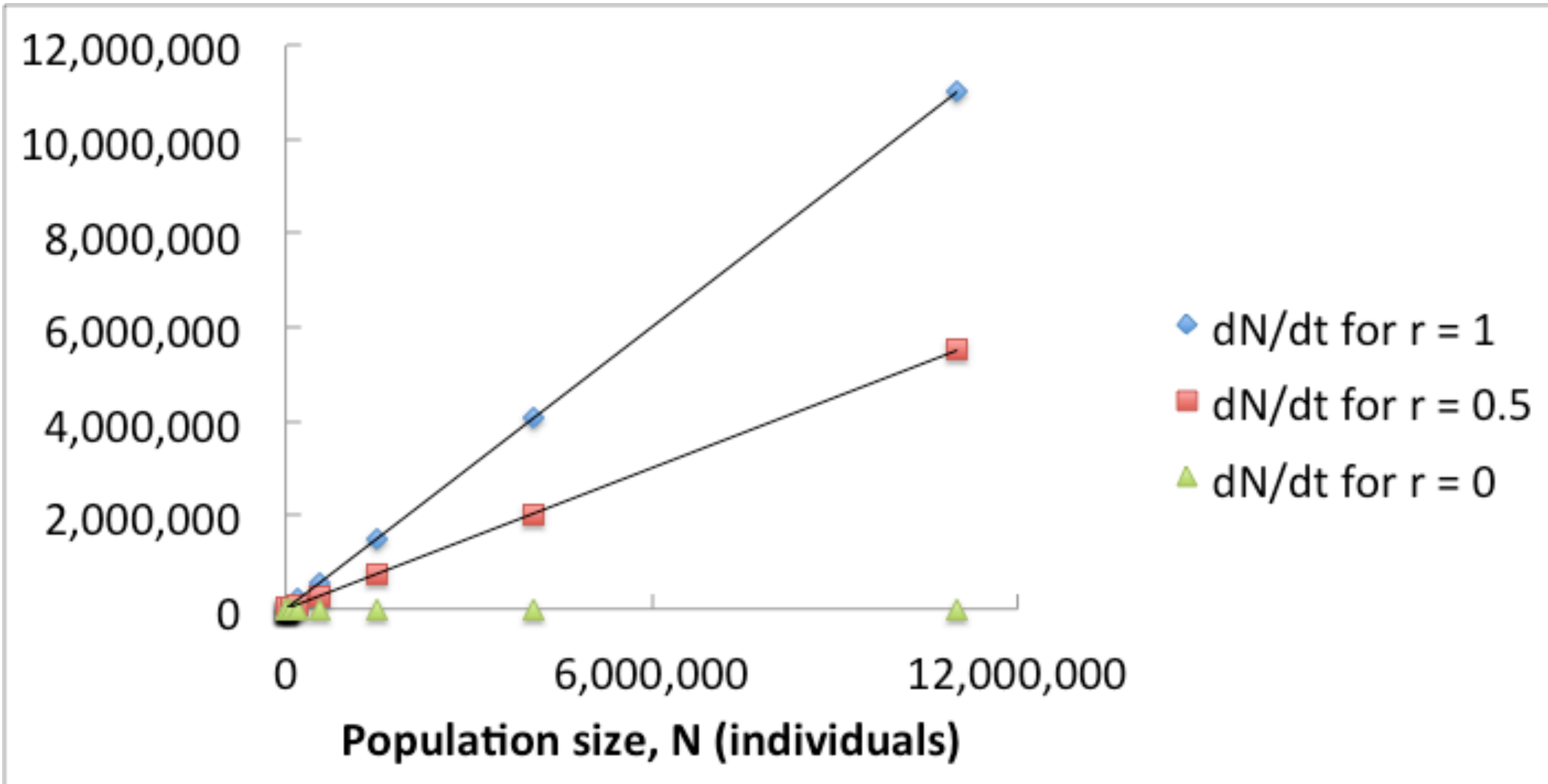
7. Challenge questions a, b:

N vs t for r = 0 & 1, N_0 = 500, for t=0, 2, 5, 10 yr



7. Challenge c & d: dN/dt vs N for N_0=500, r = 0, 0.5, 1

d. $dN/dt = rN$; slope = r



The **doubling time** (t_d) of a population is the number of years it will take the population to double in size.

$$t_d = \ln(2)/r$$

ln(2) is a constant ≈ 0.693. (~0.7)

The larger *r*, the less time it takes for the population to double.

Initial population size does not matter here

8. Doubling time problems,
Solve for t_d using $t_d = \ln(2)/r$:

a. A continuously growing population of bears has a population size of 300 and its intrinsic rate of increase is 0.07 per year. Assuming that this rate of increase remains the same, about how long should it take for the population to reach 600?
A. 5 years B. 10 years C. 14 years D. 20 years E. 28 years

b. In 2012 the human population hit 7 bil and growth rate is 0.0118. What is the doubling time for the human population? In what year do we expect our population to hit 14 billion?

8. Doubling time problems, using $t_d = \ln(2)/r$:

a. A continuously growing population of bears has a population size of 300 (N_0) and its intrinsic rate of increase is 0.07 per year (r). Assuming that this rate of increase remains the same, about how long should it take for the population to reach 600 (= 2x N_0)?
Ln (2)/r ~= 0.7/0.07 per yr = 10 yr

b. In Oct 2011 the human population hit 7 bil and growth rate, r, is 0.0118. What is the doubling time for the human population? In what year do we expect our population to hit 14 billion if this rate keeps constant?
ln (2)/r = .69/0.118 = 58.47 yr
2011.83+58.47 yr = 2070 (8 billion in 2024!)

9. Optional challenge: derive doubling time equation

$$t_d = \ln(2)/r$$

- Derive from $N_t = N_0 e^{rt}$
- To start, what is N_t in terms of N_0 given that we're doubling?

Exponential relationships between y and x appear linear when graphed on semi-log paper.
Why use a log scale for the y axis?