

Portland State University
General Physics Workshop
Problem Set 3

Electric Circuits and Capacitors

Equations and Relations:

Electrical power: $P = IV = I^2R = \frac{V^2}{R}$

Resistance:

$$R_{series} = R_1 + R_2 + R_3 + \dots = \sum_i R_i$$

$$\frac{1}{R_{parallel}} = \sum_i \frac{1}{R_i}$$

Electric Field: $E = \frac{F}{q_0}$

Electric Potential: $E = -\frac{\Delta V}{\Delta s}$

Capacitance: $C = \frac{Q}{V}$

Parallel-plate capacitor: $E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$

$$C = \frac{\epsilon_0 A}{d}$$

Capacitance with dielectrics: $C = \kappa C_0$

Energy in Capacitor:

$$U = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}$$

Electrical energy density: $u_E = \frac{1}{2}\epsilon_0 E^2$

Capacitance:

$$C_{parallel} = C_1 + C_2 + C_3 + \dots = \sum_i C_i$$

$$\frac{1}{C_{series}} = \sum_i \frac{1}{C_i}$$

Charging a capacitor:

$$q(t) = q_0(1 - e^{-t/\tau}) = CV(1 - e^{-t/\tau})$$

$$I(t) = I_0 e^{-t/\tau} = \frac{V}{R} e^{-t/\tau}$$

$$V(t) = V_0(1 - e^{-t/\tau})$$

Discharging a capacitor:

$$q(t) = q_0 e^{-t/\tau}$$

$$|I(t)| = I_0 e^{-t/\tau}$$

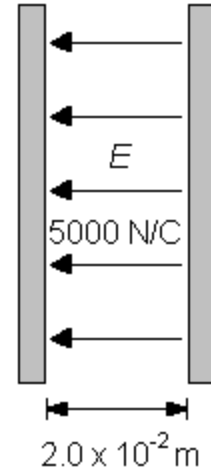
$$V(t) = V_0 e^{-\frac{t}{\tau}}$$

Time constant:

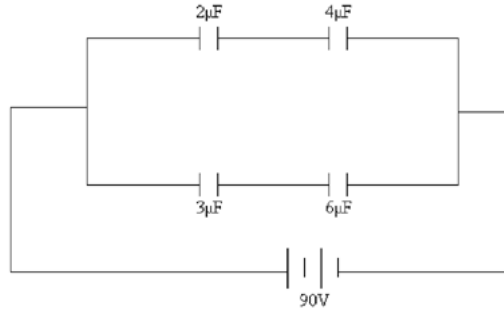
$$\tau = RC$$

1. The potential difference between the plates of a parallel plate capacitor with the plate separation of 6 cm is 60 V. What is the electric field between the plates of this capacitor?
2. A + 7.0- μC charge is moved from a negative to a positive plate of a parallel plate capacitor. In moving this charge 0.50 mJ of energy is used. What is the potential difference between the plates of this capacitor?
3. A parallel plate capacitor with plate separation of 4.0 cm has a plate area of $4.0 \times 10^{-2} \text{m}^2$. What is the capacitance of this capacitor with air between these plates?

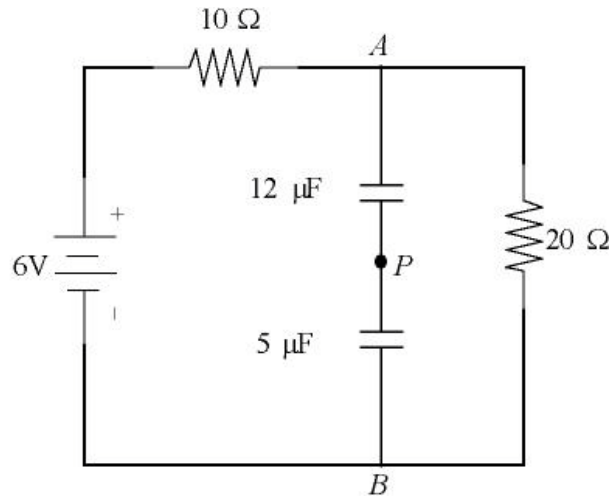
4. Two parallel conducting plates, each of area 0.30 m^2 , are separated by a distance of $2.0 \times 10^{-2} \text{ m}$ of air. One plate has charge $+Q$, while the other plate has charge $-Q$. An electric field of 5000 N/C is directed to the left in the space between the plates, as shown in the diagram above.
- Indicate on the diagram which plate is positive and which is negative.
 - Determine the potential difference between the plates.
 - Determine the capacitance of this arrangement of plates.
 - If an electron is initially located at a point midway between the plates, determine the magnitude of the electrostatic force on the electron at this location and state its direction.
 - If the electron is released from rest at this location midway between the plates, determine its speed just before striking one of the plates. Assume that gravitational effects are negligible.



5. In one kind of computer keyboard, each key is attached to one plate of a parallel plate capacitor; the other plate is fixed in position. The capacitor is maintained at a constant potential difference of 5.0 V by an external circuit. When the key is pressed down, the top plate moves closer to the bottom plate, changing the capacitance and causing charge to flow through the circuit.
- If each plate is square of side 6.0 mm and the plate separation changes from 4.0 mm to 1.2 mm when a key is pressed, how much charge flows through the circuit?
 - Does the charge on the capacitor increase or decrease? Assume that there is air between the plates instead of a flexible insulator.
- *A fully charged parallel plate capacitor is disconnected from the power supply. The plates are then pulled apart (the charge remains on the plates). What happens to:*
- The capacitance of the capacitor*
 - The electric field between the plates*
 - The potential difference*
 - The energy stored by the capacitor*
6. A $5.0 \mu\text{F}$ and a $7.0 \mu\text{F}$ capacitor are connected in series across an 8.0-V DC source. What is the charge on the $5.0 \mu\text{F}$ capacitor?
7. Four capacitors are connected across a 90-V DC source as shown in the Figure below. What is the charge on the $4.0\text{-}\mu\text{F}$ capacitor?



8. Capacitors C_1 and C_2 are connected to a battery whose voltage is V . Recall that the electrical energy stored by each capacitor is $\frac{1}{2} C_i V_i^2$, where V_i is the voltage across capacitor C_i .
- (a) If the capacitors are connected in series, is the total energy stored by them greater than, less than, or equal to the total energy stored by C_1 and C_2 connected in parallel?
- (b) The battery voltage is $V = 64.0 \text{ V}$ and the capacitances are $C_1 = 1.95 \mu\text{F}$ and $C_2 = 4.50 \mu\text{F}$. Determine the total energy stored by the two capacitors when connected in (a) series and in (b) parallel.
9. A circuit contains two resistors (10Ω and 20Ω) and two capacitors ($12 \mu\text{F}$ and $5 \mu\text{F}$) connected to a 6 V battery as shown in the diagram. The circuit has been connected for a long time.



- (a) Calculate the total capacitance of the circuit.
- (b) Calculate the current in the 10Ω resistor.
- (c) Calculate the potential difference between points A and B .
- (d) Calculate the charge stored on one plate of the $5 \mu\text{F}$ capacitor.
- If the wire is cut at point P , will the potential difference between points A and B increase, decrease, or remain the same? Explain your reasoning.

10. A capacitor $C=10\mu\text{F}$ is charged through a resistor $R=1\text{K}\Omega$.
- In terms of the time constant τ when will the charge on the capacitor be half its maximum value?
 - What is that time in seconds?
 - If the applied voltage is 10 V, what is the current and voltage at the time?
 - What is the current and voltage after the capacitor is charged for 15ms?
 - What is maximum charge on the capacitor?

Now the fully charged capacitor is discharged through the $1\text{K}\Omega$ resistors

- What is the current and voltage after the capacitor is discharged for 25ms?
 - What is the current and voltage after the capacitor is discharged for $t=\tau$.
- *Plot the voltage for charging and discharging process as a function of time. Indicate the times that you calculated in the problem above.*
- *The time constant is given by $\tau=RC$. Verify that the units of this equation are consistent.*

Additional questions

- The electric field inside a parallel plate capacitor is constant everywhere. Is this also true for the potential?
- A proton is released from the positive plate of a parallel plate capacitor. At the same time an electron is released from the negative plate. Which particle strikes the other plate of the capacitor first?