Portland State University
General Physics Workshop
Problem Set 8 **Relativity**

Equations and Relations:

Time dilation

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2 / c^2}}$$

Length contraction

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

Relativistic addition of velocities

$$v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$$

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Relativistic energy

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} = m c^2$$

Rest energy

$$E_0 = m_0 c^2$$

Relativistic kinetic energy

$$K = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0 c^2$$

Schwarzschild radius

$$R = \frac{2GM}{c^2}$$

1. A spaceship moves at a constant velocity of 0.40*c* relative to an Earth observer. The pilot of the spaceship is holding a rod, which he measures to be 1.0 m long.
   (a) The rod is held perpendicular to the direction of motion of the spaceship. How long is the rod according to the Earth observer?
   (b) After the pilot rotates the rod and holds it parallel to the direction of motion of the spaceship, how long is it according to the Earth observer?

2. A rectangular plate of glass, measured at rest, has sides 30.0 cm and 60.0 cm.
   (a) As measured in a reference frame moving parallel to the 60.0-cm edge at speed 0.25*c* with respect to the glass, what are the lengths of the sides?
   (b) How fast would a reference frame have to move in the same direction that the plate of glass viewed in that frame is square?

3. A spaceship travels at constant velocity from Earth to a point 510 ly away as measured in Earth's rest frame. The ship's speed relative to Earth is 0.99*c*. A passenger is 20 yr old when departing from Earth in the year 2000.

(a) How old is the passenger when the ship reaches its destination, as measured by the ship's clock?
(b) If the spaceship sends radio signal reports back to earth every 12 hours (by their clocks), at what interval are the reports *sent* to Earth, according to Earth clock?
(c) If the ship sends a radio signal back to Earth as soon as it reaches its destination, in what year, by Earth's calendar, does the signal reach Earth?

- *While on board a starship, you intercept signals from four spaceships that are moving either directly toward or directly away from you. The signals have the same proper frequency f_0. The speed and direction (both relative to you) of the spaceships are (a) 0.3c toward, (b) 0.6c toward, (c) 0.3c away, and (d) 0.6c away. Rank the spaceships according to the frequency you receive, greatest first.*

- *A sprinter crosses the start line (event 1) and runs at constant velocity until he crosses the finish line (event 2). In what reference frame would an observer measure the proper time interval between these two events? In what reference frame would an observer measure the proper length of the track from start line to finish line?*

- *If the speed of light would be infinitely large, would you observe the effects of length contraction and time dilation?*

4. Electron A is moving west with speed 0.6*c* relative to the lab. Electron B is also moving west with speed 0.8*c* relative to the lab. What is the speed of electron B in a frame of reference which electron A is at rest?

5. An observer on earth notices two spaceships approach at speeds of 0.75c and 0.5c respectively. What is the relative speed between the spaceships as measured by a passenger of one of the spaceships.

   - *You are moving at a speed 0.1c relative to Tom who shines a light toward you. At what speed do you see the light passing you by?*

6. A constant force is applied to a particle initially at rest. Sketch qualitative graphs of the particle's speed, momentum, and acceleration as functions of time. Assume that the force acts long enough so the particle achieves relativistic speeds.

7. An electron is accelerated from rest through a potential difference of 1×10^8V.

(a) What is the relativistic kinetic energy of the electron?
(b) What is its speed?
(c) What relativistic kinetic energy and speed would a proton have after being accelerated by the same potential difference?
(d) What is the relativistic momentum of the proton and the electron?

(e) What potential difference do you need to accelerate an electron to 0.85c?

- *Why is it harder to accelerate a proton to a speed close to the speed of light, than is to accelerate an electron to the same speed?*

8. A nuclear power plant generates 10×10^9 W of power. Assuming 100% efficiency, by how much does the mass of the fuel change in one day to produce this much energy?

9. Find the radii to which the sun and the earth must be compressed for them to become black holes

Additional Questions

1. You are in a special compartment on a train that admits no light, sound, or vibration. Is there any way you can tell whether the train is at rest or moving at constant nonzero velocity? Explain.

2. You are enclosed in a box with six opaque walls. Are there any experiments you can perform inside the box to prove that you are
   (a) moving with constant linear velocity
   (b) accelerating
   (c) rotating with constant angular velocity?