

Equations and Relations:

Frequency, period, angular frequency

$$f = \frac{1}{T}$$
$$\omega = 2\pi f$$

Position, velocity and acceleration of a harmonic oscillator

$$x = A \cos\left(\frac{2\pi}{T} t\right) = A \cos(\omega t)$$
$$v = -A\omega \sin(\omega t)$$
$$a = -A\omega^2 \cos(\omega t)$$
$$v_{\max} = A\omega$$
$$a_{\max} = A\omega^2$$

Spring Period

$$T = 2\pi \sqrt{\frac{m}{k}}$$

Energy

$$E = \frac{1}{2}k A^2$$
$$U = \frac{1}{2}k A^2 \cos^2(\omega t)$$
$$K = \frac{1}{2}k A^2 \sin^2(\omega t)$$

Simple pendulum period

$$T = 2\pi \sqrt{\frac{L}{g}}$$

Physical pendulum period

$$T = 2\pi \sqrt{\frac{I}{m g d}}$$

Underdamping

$$A = A_0 e^{-bt/2m}$$

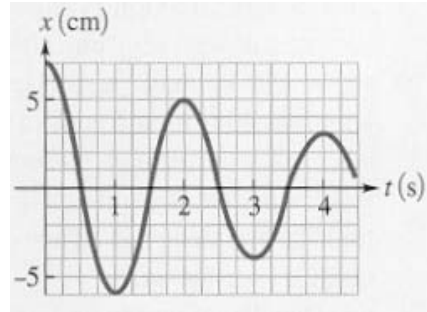
1. A mass of 1 kg is attached to a spring and undergoes simple harmonic oscillations with a period of 1 s. What is the force constant of the spring?
2.
 - a. If a simple pendulum has period $T = 1.0$ s and you double its length, what is its new period in terms of T ?
 - b. If a simple pendulum has a length $L = 1.0$ m and you want to triple its frequency, what should be its length?
 - c. Suppose a simple pendulum has a length L and period T on earth. If you take it to a planet where the acceleration of freely falling objects is ten times what it is on earth, what should you do to the length to keep the period the same as on earth?
 - d. If you do *not* change the simple pendulum's length in part (c), what is its period on that planet in terms of T ?
 - e. If a pendulum has a period T and you triple the mass of its bob, what happens to the period (in terms of T)?

3. What is the period of a pendulum formed by placing a horizontal axis (a) through the end of a meterstick (100-cm mark)? (b) through the 75-cm mark? (c) through the 60-cm mark? Assume $g = 9.80 \text{ m/s}^2$.

(Hint: $I_{cm} = \frac{1}{12}ml^2$, Parallel axis theorem: $I = I_{cm} + md^2$)

- *A pendulum clock runs too slow and loses time. What adjustment should be made?*
 - *How can the principle of the pendulum be used to compute (a) length, (b) mass, and (c) time?*
 - *A pendulum is mounted in an elevator that moves upward with constant acceleration. Is the period greater than, less than, or the same as when the elevator is at rest? Why?*
4. A 1.5-kg mass oscillates at the end of a spring in SHM. The amplitude of the vibration is 0.15 m, and the spring constant is 80 N/m. If the mass is displaced 15 cm,
- (a) what are the magnitude and direction of the acceleration and force on the mass?
- If the system is now operated on a frictionless horizontal surface,
- (b) what is total energy?
 - (c) what is the maximum velocity?
 - (d) what is the maximum acceleration?
- *What effect will doubling the amplitude A of a body moving with SHM have on (a) the period, (b) the maximum velocity, and (c) the maximum acceleration?*
 - *Explain how the period of a mass-spring system can be independent of amplitude, even though the distance traveled during each cycle is proportional to the amplitude.*
 - *Explain why the velocity in SHM is greatest when the magnitude of the acceleration is the least.*
 - *A mass hanging vertically from a spring and a simple pendulum both have a period of oscillation of 1 s on Earth. An astronaut takes the two devices to another planet where the gravitational field is stronger than that of Earth. For each of the two systems, state whether the period is now longer than 1 s, shorter than 1 s, or equal to 1 s. Explain your reasoning.*

5. A mass is vibrating at the end of a spring of force constant 225 N/m . The figure shows a graph of its position x as a function of time t .
- At what times is the mass not moving?
 - How much energy did the system originally contain?
 - How much energy did the system lose between $t = 1.0 \text{ s}$ and $t = 4.0 \text{ s}$?



Additional Questions

- A bungee jumper leaps from a bridge and comes to a stop a few centimeters above the surface of the water below. At that lowest point, is the tension in the bungee cord equal to the jumper's weight? Explain why or why not.
- A ball is dropped from a height h onto the floor and keeps bouncing. No energy is dissipated, so the ball regains the original height h after each bounce. Sketch the graph for $y(t)$ and list several features of the graph that indicate that this motion is *not* SHM.