Portland State University
General Physics Workshop
Problem Set 9

Equilibrium and gravity

Equations and Relations:

Newton's Laws:
1st: An object will stay at rest or in motion with constant velocity unless acted on by a net force.
2nd: $\Sigma F_x = ma_x$, $\Sigma F_y = ma_y$
3rd: Forces come in pairs. If A exerts a force on B, then B exerts a force on A with the same magnitude but in the opposite direction.

Friction: $f_s \le \mu_s N$, $f_k = \mu_k N$

Uniform Circular Motion: $a_c = v^2 / r$

Momentum: $\vec{p} = m\vec{v}$, $\Sigma \vec{F} = \Delta\vec{p} / \Delta t$

Work & Energy: $W = F_{\parallel} d = F d_{\parallel}$

$$W_{net} = \Delta KE$$
$$W_{nc} = \Delta KE + \Delta PE$$
$$KE = \tfrac{1}{2}mv^2,\ PE_{grav} = mgy$$

Springs: $F_{sp} = kx$, $PE_{sp} = \tfrac{1}{2}kx^2$

Angular velocity:

$$\omega_{av} = \frac{\Delta\theta}{\Delta t}$$
$$v = \omega r$$
$$s = \theta r$$

Torque: $\tau = rF\sin\theta$

Rotational work: $W = \tau\theta$

Statics: $\Sigma F_x = 0,\ \Sigma F_y = 0,\ \Sigma\tau = 0$

Rotational kinetic energy: $K = \frac{1}{2}I\omega^2$

Moment of inertia: $I = \sum m_i r_i^2$

Newton's second law of rotation: $\sum\tau = I\alpha$

Angular momentum: $L = I\omega$

Center of mass: $X_{cm} = \frac{\sum mx}{\sum m}$

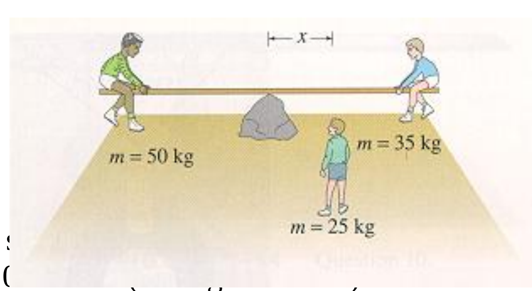
Kepler's 3rd law:

$$T = \left(\frac{2\pi}{\sqrt{GM_s}}\right) r^{3/2}$$

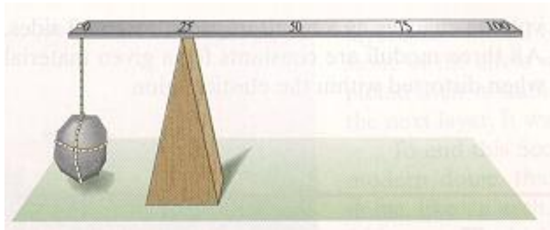
Universal law of gravity:

$$F = G\frac{m_1 m_2}{r^2}$$

1. Three boys are trying to balance on a seesaw, which consists of a fulcrum rock, acting as a pivot at the center, and a very light board 3.6 m long. Two boys are already on either end. One has a mass of 50 kg, and the other a mass of 35 kg. Where should the third boy, whose mass is 25 kg, place himself so as to balance the seesaw?

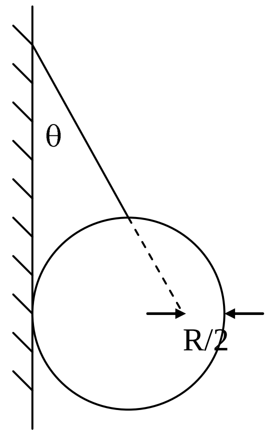


2. A uniform meter s[...] uilibrium when a 40 g rock is suspended at the ([...] f the meter stick (a) greater than, (b) equal to, or (c) less than, the mass of the rock. Explain



Calculate the mass of the meter stick

3. A rubber ball of M = 500g is attached to a wall by a light string, as shown in the diagram to the right (θ = 30º). The ball is in contact with the wall, and there is friction between the wall and the ball. Note that the string is attached in such a way that the line of force of the tension intersects the equatorial plane of the ball at a point halfway between the ball's center and its outer edge.

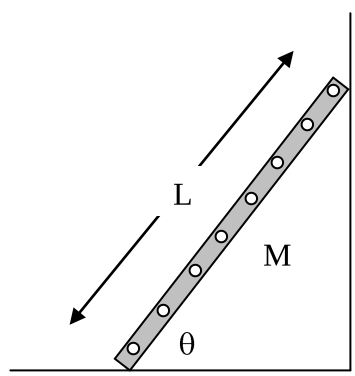


   a) Make a free-body diagram for the ball and discuss the following questions: Does friction act on the ball, and if so in which direction? How do you include a force on a FBD if the direction in which the force acts is not obvious?
   b) Determine a convenient axis about which to sum torques. What makes one axis better than another?
   c) Determine the magnitude and direction of the force of friction acting on the ball.
   d) Find the force of tension in the string

Check Point

- *Choose various axes of rotation and discuss the pros and cons of each.*
- *Carefully explain how, for a given force, you determine the direction of the torque it exerts around a particular axis.*

4. Consider a ladder of mass M and length L leaning against a wall at some angle θ, as shown in the sketch to the right. The bottom of the ladder is rubberized (so there's a lot of friction), but the top is bare aluminum and can be considered frictionless.



a) Make a free-body diagram for this ladder, including a coordinate system.
b) Write out the equations for static equilibrium of the ladder. (Indicate with a "P" on your FBD the point you are choosing to sum torques around, and show with an arrow your choice for the direction of a positive torque.)

c) In terms of the known quantities (M, L and θ), determine all of the forces exerted on the ladder by the wall and by the floor.

Check Point

- *Suppose someone is standing on the ladder. Which forces on the ladder do you expect to be different, and will they be bigger or smaller? Show explicitly how your equations will be modified.*

5. For the same ladder as in the previous problem, suppose that the coefficient of static friction between the floor and the ladder is known to be $\mu = 1/2$.

   a) What is the maximum amount of friction that the floor can exert on the ladder? Express your result as a multiple of the weight of the ladder (examples: f = 2mg or f = 0.25mg).
   b) At what angle θ is the frictional force at its maximum?
   c) What would happen to the ladder if θ were made smaller?

Check Point

- *The coefficient of friction between the ladder and the floor is a constant number. How then do you explain (in words) the fact that the ladder becomes unstable as the angle θ is decreased?*
- *Which is more likely to slip when leaned against the wall as in this problem, a tall ladder or a short ladder? Assume that the ladders have the same rubber on the bottom.*

6. A student on a piano stool rotates freely with an angular speed of 2.85 rev/s. The student holds a 1.25-kg mass in each outstretched arm, 0.760 m from the axis of rotation. The combined moment of inertia of the student and the stool, ignoring the two masses, is 6.00 kg m^2 a value that remains constant.

   (a) As the student pulls his arms inward, his angular speed increases to 3.14 rev/s. How far are the masses from the axis of rotation at this time, considering the masses to be points?
   (b) Calculate the initial and final kinetic energy of the system.

   - *Stars form when a large, rotating cloud of gas collapses. What happens to the angular speed of the gas cloud as it collapses?*

7. a) Calculate the gravitational acceleration on the surface of the moon. The moon's radius is about 1.74 x 106 m and its mass is 7.35 x 1022 kg.

   b) At what altitude above the Earth's surface is the acceleration of gravity equal to $g/2$?

   c) In each hand you hold a 0.10-kg orange. What is the gravitational force exerted by each orange on the other when their separation is 0.50 m?

   d) A geo-synchronous satellite is one that stays above the same point on the equator of the earth. Such satellites are used for purposes as cable TV transmission, for weather forecasting, and as communication relays. What is the height above the earth's surface such a satellite must orbit?

Check Point

- D*o lower orbit satellites move faster or slower?*

Additional Questions

1. What purpose does the tail rotor on a helicopter serve?

2. If the Earth were to magically expand, doubling its radius while keeping its mass the same, would the length of the day increase, decrease, or stay the same?

3. Which of the following statements is correct:

- According to Kepler's third law, the time needed for a planet to go around the sun

____ a) depends on its mass
____ b) depends on the average radius of orbit
____ c) depends on its speed of rotation
____ d) is the same for all the planets

- The speed of a planet in its elliptical orbit around the sun

____ a) is constant
____ b) is highest when it is closest to the sun
____ c) is lowest when it is closest to the sun
____ d) varies, but not with respect to its distance from the sun

- Satellite 1 is in a certain circular orbit about a planet, while satellite 2 is in a larger circular orbit. Which satellite has the longer period and greater speed?

____ a) satellite 1 and satellite 1
____ b) satellite 1 and satellite 2
____ c) satellite 2 and satellite 1
____ d) satellite 2 and satellite 2
____ e) they have the same period and speed