Portland State University
General Physics Workshop
Problem Set 8 **Rotational acceleration and torque**

Equations and Relations:

Newton's Laws:
1st: An object will stay at rest or in motion with constant velocity unless acted on by a net force.
2nd: $\Sigma F_x = ma_x$, $\Sigma F_y = ma_y$
3rd: Forces come in pairs. If A exerts a force on B, then B exerts a force on A with the same magnitude but in the opposite direction.

Friction: $f_s \le \mu_s N$, $f_k = \mu_k N$

Uniform Circular Motion: $a_c = v^2 / r$

Momentum: $\vec{p} = m\vec{v}$, $\Sigma\vec{F} = \Delta\vec{p} / \Delta t$

Work & Energy: $W = F_{\parallel}d = Fd_{\parallel}$

$W_{net} = \Delta KE$

$W_{nc} = \Delta KE + \Delta PE$

$KE = \frac{1}{2}mv^2$, $PE_{grav} = mgy$

Springs: $F_{sp} = kx$, $PE_{sp} = \frac{1}{2}kx^2$

Angular velocity:

$$\omega_{av} = \frac{\Delta\theta}{\Delta t}$$

$v = \omega r$

$s = \theta r$

Torque: $\tau = rF\sin\theta$

Rotational work: $W = \tau\theta$

Statics: $\Sigma F_x = 0, \Sigma F_y = 0, \Sigma\tau = 0$

Rotational kinetic energy: $K = \frac{1}{2}I\omega^2$

Moment of inertia: $I = \sum m_i r_i^2$

Newton's second law of rotation: $\sum \tau = I\alpha$

Angular momentum: $L = I\omega$

Center of mass: $X_{cm} = \frac{\sum mx}{\sum m}$

1. Discuss the following terms, and then write a clear definition of each *in words*:
   (a) Angular acceleration
   (b) Torque
   (c) Moment of inertia

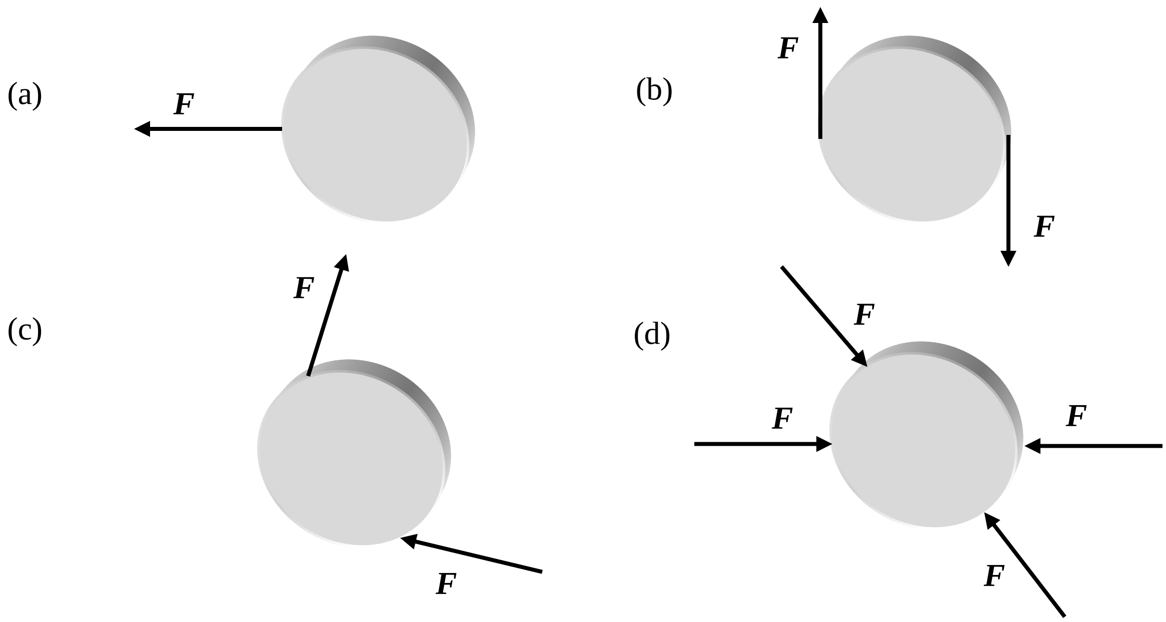
Check Point

- *Can you describe the analogy between Newton's 2nd Law (ΣF = ma) and the torque equation (Στ = Iα)?*
- *Why is it harder to do sit-ups with your hands behind your head than with them crossed over your chest?*

2. A person exerts a force of 45 N on the end of a door 84 cm wide. What is the magnitude of the torque if the force is exerted (a) perpendicular to the door, and (b) at a 60° angle to the face of the door?

3. Consider a circular object that is free to rotate about its central axis, like a car's steering wheel. An important question is, what exactly causes such an object to start rotating, or more precisely to change its rate of rotation? Is it a net force, which is the cause of changes in linear motion, or is it something else?

The diagrams below show such an object with various forces applied to it. In each case, will the object rotate? Will it accelerate linearly? Or perhaps it will do both, or neither. In each case, carefully describe the resulting motion of the object, assuming that the forces shown are the *only* forces acting upon it.

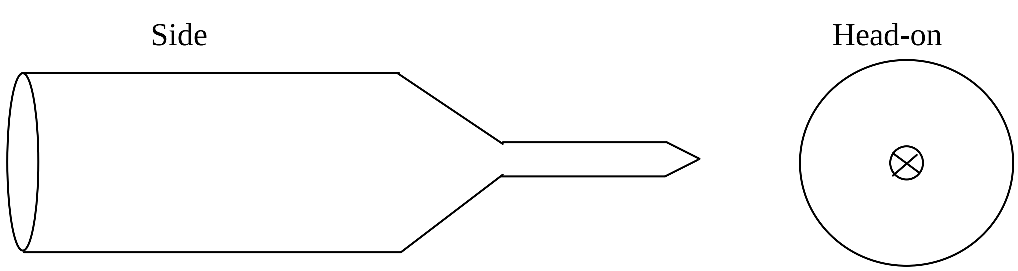


Referring to your analysis of the above diagrams, what can you conclude about the cause of rotational (or angular) acceleration? [In physics we refer to this cause as a net torque.]

Check Point

➢ *What do you suppose resists angular acceleration?*

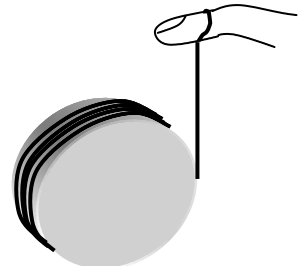
4. A Phillips' head screwdriver looks like this:



Explain how the design of the screwdriver allows a relatively small force exerted by your hand to result in a much larger force exerted on a screw.

Check Point

- *Can you think of other examples where a mechanical advantage is gained using the same principle (a varying lever arm)?*
- *Does the length of a screwdriver affect the force it exerts on a screw?*



5. Consider a yo-yo with its string wound up, ready for action. Suppose that we know nothing about this yo-yo except that it has the shape of a thin cylinder, with the string wrapped around the outside of the cylinder. (So this is even simpler than a regular yo-yo.) If you release the yo-yo from rest, the string will unwind and it will accelerate downward, but at what rate?

    (a) Make a free-body diagram and choose a convenient coordinate system for the yo-yo as it falls.
    (b) Apply Newton's 2nd Law ($\Sigma F = ma$) to your FBD. Assuming that the yo-yo's mass and radius are known quantities, can you determine the acceleration of the yo-yo in terms of known quantities from this equation? Why or why not?
    (c) Now choose a rotational coordinate system and apply the torque equation $\Sigma\tau = I\alpha$ to the yo-yo (the moment of inertia of a disk is $\frac{1}{2}MR^2$).

Check Point

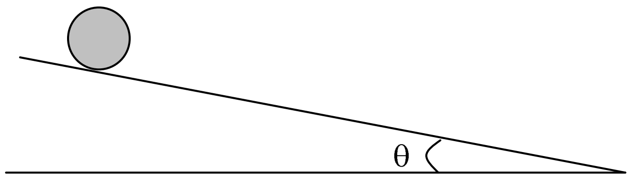
- *What would be different about your work so far if this were a real yo-yo, where the string was wrapped around an inner radius that differs from the outer radius?*
- *How do you determine the moment of inertia of an odd-shaped object like a real yo-yo?*

(d) What is the relationship between α (the angular acceleration of the yo-yo) and a (the linear acceleration of the yo-yo)?
(e) Using the results of (b), (c) and (d), determine the acceleration of the yo-yo. Does the answer depend on the mass or radius of the yo-yo? What *does* it depend on?
(f) What is the tension in the string as the yo-yo falls? Give your answer as some fraction or multiple of the weight of the yo-yo.

Check Point

- *Again, how would the problem differ for a real yo-yo?*
- *Without repeating all of your work, can you predict the downward acceleration of a ball of yarn if it is held at one end and allowed to unravel?*

6. A round ball (moment of inertia, $I=2/3\ MR^2$) is released from rest and rolls without slipping down a hill of steepness θ.



(a) Make a FBD for the ball, and choose both ordinary and rotational coordinate systems. (Note: friction can **not** be neglected in this problem.)
(b) Write out Newton's 2nd Law and the torque equation for the ball.
(c) Find an equation to calculate the acceleration of the ball down the hill.
(d) Calculate the velocity of the ball at the bottom of the ramp. The ramp is 50cm high, $\theta=30°$, M=500g, R=10cm.
(e) Use conservation of energy to calculate the same velocity

Check Point

- *Was it easier to use Newton's 2^{nd} law or conservation of energy to calculate the velocity?*
- *Use the results of part (c) to find an equation for the velocity of the following objects down a hill:*

  - *A cylinder ($I = \frac{1}{2} MR^2$)*
  - *A sphere ($I = \frac{2}{5} MR^2$)*
  - *A mass that slides without friction (what's the "effective" moment of inertia in this case?)*

<u>Additional Questions</u>

1. Consider a ball rolling down a hill. Make a free-body diagram for the ball. Which force causes the ball to rotate? Which force causes it to accelerate down the hill?

2. Imagine that you are trying to turn the wheel that opens the escape hatch on your sinking submarine. It is a matter of life and death, and the wheel seems to be stuck. You are able to exert a certain amount of force with each hand, but no more. Describe how you would use these forces to your best advantage in order to turn the wheel. Be creative; your life depends on it!

3. Is it possible to exert a net torque on an object without exerting a net force? Think of an example, or explain why it's not possible.

4. Is it possible to exert a net force on an object without exerting a net torque? Again, explain or provide an example.

5. Can a single object have more than one moment of inertia? Explain.