## Rotational kinematics

## Equations and Relations:

Newton's Laws:

- 1st: An object will stay at rest or in motion with constant velocity unless acted on by a net force.
- $2 \mathrm{nd}: \quad \Sigma F_{x}=m a_{x}, \quad \Sigma F_{y}=m a_{y}$
- 3rd: Forces come in pairs. If A exerts a force on $B$, then $B$ exerts a force on $A$ with the same magnitude but in the opposite direction.


## Motion with Constant Acceleration:

$v=v_{0}+a t$
$x=x_{0}+v_{0} t+\frac{1}{2} a t^{2}$
$v^{2}=v_{0}^{2}+2 a\left(x-x_{0}\right)$
$g=9.80 \mathrm{~m} / \mathrm{s}^{2} \approx 10 \mathrm{~m} / \mathrm{s}^{2}$

$$
\begin{aligned}
& \text { Linear } \rightarrow \text { Rotation } \\
& x \rightarrow \theta \\
& \mathrm{x} \rightarrow \omega \\
& \mathrm{a} \rightarrow \alpha \\
& \text { Angular velocity: } \\
& \omega_{a v}=\frac{\Delta \theta}{\Delta t} \\
& v=\omega r \\
& s=\theta r \\
& \text { Uniform Circular Motion: } a_{c}=v^{2} / r
\end{aligned}
$$

1. Consider a clockwise-rotating object, such as the odd-shaped one shown below. The large black mark near the center of the object represents the point around which it is rotating, and the line through that point is called the axis of rotation of the object. $\mathrm{P}_{1}$ and $P_{2}$ represent arbitrary points on the object's surface.

(a) Describe the trajectories of the points $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$. What is different about these trajectories?
(b) Will the points $P_{1}$ and $P_{2}$ be moving with the same velocities? If not, what are the differences?
(c) Supposing the object is rotating at constant speed, are the points $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$ accelerating? If so, are their accelerations the same, or not?
(d) Now imagine that the object is speeding up in its rotation (rotating faster and faster with time). Carefully sketch the acceleration vectors for the points $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$ on the diagram.
(e) What are the quantities that the points $\mathrm{P}_{1}$ and $\mathrm{P}_{2}$ have in common?
2. A soccer ball of diameter 30 cm rolls without slipping at a linear speed of $2.6 \mathrm{~m} / \mathrm{s}$. Through how many revolutions has the soccer ball turned as it moves a linear distance of 28 m ?
3. An elevator cable winds on a drum of radius 90.0 cm that is connected to a motor.
(a) If the elevator is moving down at $0.50 \mathrm{~m} / \mathrm{s}$, what is the angular speed of the drum?
(b) If the elevator moves down 6.0 m , how many revolutions has the drum made?
4. Translate the following quantities into plain language that someone with very little knowledge of mathematics could easily understand. That is, describe what these quantities really mean in non-technical terms.
(a) One radian (1 rad)
(b) Twelve and a half radians per second ( $12.5 \mathrm{rad} / \mathrm{s}$ )
(c) Six radians per second squared $\left(6 \mathrm{rad} / \mathrm{s}^{2}\right)$


What are the dimensions of a radian?
5. Frankie is playing on a playground merry-go-round. He pushes on the railing at its edge and gives the merry-go-round a constant angular acceleration of $1 / 2 \mathrm{rad} / \mathrm{s}^{2}$ for 10 seconds. The diameter of the merry-go-round is 2 meters.

(a) Find the number of rotations made after 10 seconds.
(b) How far has Frankie run after 10 seconds?
(c) Find the angular speed of the rotating merry-go-round after 10 seconds.
(d) How fast is Frankie running after 10 seconds? Is this realistic?
6. A CD with a diameter of 12.0 cm starts from rest and with a constant angular acceleration of $1.0 \mathrm{rad} / \mathrm{s}^{2}$ acquires an angular velocity of $5.0 \mathrm{rad} / \mathrm{s}$. The CD continues rotating at $5.0 \mathrm{rad} / \mathrm{s}$ for 15.0 seconds and then slows to a stop in 12.0 seconds with a constant angular deceleration. Calculate for a point 4.0 cm from the center at the time 2.0 seconds, 12 seconds, and 25 seconds from the start:
a.) The tangential velocity
b.) The centripetal acceleration

c.) The angular distance traveled by the point
d.) Calculate the above parameter for a point 8 cm from the center.
$>$ Where should you go on a rotating object to have the smallest speed?
$>$ Which point on a rotating object has the smallest acceleration?

## Additional Questions

1. Can an object have zero translational acceleration, and at the same time have nonzero angular acceleration? If your answer is no, explain why not. If your answer is yes, give a specific example.
2. Two boys sit on a merry-go-around. One is 1 m from the axis of rotation and the other is 3 m from it. Which boy has a larger
a.) Linear speed
b.) Angular acceleration
c.) Angular speed
d.) Centripetal acceleration
