## Portland State University

General Physics Workshop
Problem Set 6

## Momentum

## Equations and Relations:

Newton's Laws:

- $\quad 1$ st: An object will stay at rest or in motion with constant velocity unless acted on by a net force.
- $\quad 2 \mathrm{nd}: \Sigma F_{x}=m a_{x}, \quad \Sigma F_{y}=m a_{y}$
- $\quad 3 \mathrm{rd}$ : Forces come in pairs. If A exerts a force on B , then B exerts a force on A with the same magnitude but in the opposite direction.
Friction: $f_{s} \leq \mu_{s} N, \quad f_{k}=\mu_{k} N$
Uniform Circular Motion: $a_{c}=v^{2} / r$
Momentum: $\vec{p}=m \vec{v}, \quad \Sigma \vec{F}=\Delta \vec{p} / \Delta t$
Work \& Energy: $\quad W=\mathrm{F}_{\|} \mathrm{d}=\mathrm{Fd}_{\|}$,
$W_{\text {net }}=\Delta K E$
$W_{n c}=\Delta K E+\triangle P E$
$K E=\frac{1}{2} m v^{2}, P E_{g r a v}=m g y$
Springs: $F_{s p}=k x, \quad P E_{s p}=\frac{1}{2} k x^{2}$

Statics: $\Sigma F_{x}=0, \Sigma F_{y}=0, \Sigma \tau=0$

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\text { Inelastic collision: } v_{f}=\frac{m_{1} v_{1, i}+m_{2} v_{2, i}}{m_{1}+m_{2}}
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\text { Elastic collision: } v_{1, f}=\left(\frac{m_{1}-m_{2}}{m_{1}+m_{2}}\right) v_{0}
$$

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v_{2, f}=\left(\frac{2 m_{1}}{m_{1}+m_{2}}\right) v_{0}
$$

Center of mass: $X_{c m}=\frac{\sum m x}{\sum m}$

1. Maria and Shawn are ice skating when Maria decides to throw a snowball at Shawn. She stops near him and throws the snowball horizontally as hard as she can.
(a) Make a free-body diagram (FBD) for the snowball while Maria is throwing it (while her arm is moving forward with the snowball in her hand). Is there a net force on the snowball during this time?
(b) Make a FBD for Maria while she is throwing the snowball. Is there a net force on her during this time? You can neglect friction between her skates and the ice.

Why are conserved quantities useful for solving physics problems?
$>$ Can there be a net vertical force on the snowball if it is thrown horizontally? If not, what force is balancing its weight?
$>$ Can there be a net vertical force on Maria? What would happen if there were one?
(c) Group your two previous FBDs together by drawing a single large circle around both. Is there a net force on this entire system while Maria is throwing the snowball?
(d) Answer the following questions by referring to parts a)-c) above.
i) Is the momentum of the snowball conserved as Maria throws it?
ii) Is Maria's momentum conserved as she throws the snowball?
iii) Is the momentum of the system of Maria plus the snowball conserved?

Don't erase your diagrams, because you will continue to analyze the same situation in the next problem.


How do you decide if momentum is conserved in a particular situation?
$>$ When are two forces on different objects equal in magnitude?
$>$ Newton's $2^{\text {nd }}$ Law is $F_{\text {net }}=m a$. Can you rewrite this law in terms of momentum to give a mathematical basis to your answer to the previous question?
2. In the previous problem, you should have concluded that for the system of Maria plus the snowball, momentum is conserved as she throws the snowball. Now let's see how to use that information to determine what happens to Maria after she throws the snowball. You will need the following information: Maria's mass is 50 kg , the mass of the snowball is $1 / 2 \mathrm{~kg}$, and the snowball leaves her hand at a speed of $20 \mathrm{~m} / \mathrm{s}$.
(a) Make 2 sketches, one representing the "initial state" (before the snowball is thrown) and one representing the "final state" (just after the snowball is thrown). Label the initial and final velocities of both Maria and the snowball with appropriate symbols.
(b) Write an equation (symbols only) for the conservation of momentum for Maria and the snowball in the horizontal direction.
(c) Solve this equation algebraically (still without numbers) for the final velocity of Maria after the throw.
(d) Use the given information to determine Maria's actual velocity just after she releases the snowball.
3. The head of a golf club is in contact with a 46 gram golf ball for 0.50 milliseconds, and as a result, the ball flies off at $70 \mathrm{~m} / \mathrm{s}$. Find the average force that was acting on the ball during the impact.
4. You are relaxing after a delicious Japanese meal by watching some sumo wrestling on TV. You observe that two sumo wrestlers typically run horizontally at each other, collide and stick together, and you decide to analyze such a collision to practice your physics. The masses and initial speeds of a particular pair of wrestlers are shown on the diagram below. During their collision, the only horizontal forces acting on the two wrestlers are the forces they exert on each other.

(a) Make a free-body diagram for each wrestler during their collision:


(b) Is the combined momentum of the two wrestlers conserved during their collision? Clearly explain why or why not.
(c) Determine the speed and direction of the two wrestlers (now stuck together) immediately after their collision.
5. A $800-\mathrm{kg}$ car stopped at an intersection is rear-ended by a $2000-\mathrm{kg}$ truck moving with a speed of $30 \mathrm{~km} / \mathrm{h}$. If the car was in neutral and its brakes were off, so that the collision is approximately elastic, find the final speed of both vehicles after the collision.
6. Two shuffleboard disks of equal mass, one orange and the other yellow, are involved in a perfectly elastic glancing collision. The yellow disk is initially at rest and is struck by the orange disk moving with a speed of $5 \mathrm{~m} / \mathrm{s}$. After the collision, the orange disk moves along a direction that makes an angle of $37^{\circ}$ with its initial direction of motion and the velocity of the yellow disk is perpendicular to that of the orange disk (after the collision). Determine the final speed of each disk.
7. A 13 gram bullet traveling $230 \mathrm{~m} / \mathrm{s}$ penetrates a 2.0 kg block of wood and emerges going 170 $\mathrm{m} / \mathrm{s}$. If the block is stationary on a frictionless surface when hit, how fast does it move after the bullet emerges?

$>$ Two equal-mass bullets traveling with the same speed strike a target. One of the bullets is rubber and bounces off, the other is metal and penetrates, coming to rest in the target. Which exerts the greater impulse on the target?
8. Where is the center of mass of:
a) a uniform square
b) a uniform cube
c) a uniform equilateral triangle
d) a bowling ball
e) a uniform ring
f) a uniform boomerang
g) a tank half filled with water and half oil
h) a thermometer at very low temperature
9. The Earth has a mass of $5.98 \times 10^{24} \mathrm{~kg}$, the Moon has a mass of $7.35 \times 10^{22} \mathrm{~kg}$, and their center-to-center distance is $3.85 \times 10^{8} \mathrm{~m} . \mathrm{a}$.) How far from the center of the Earth is the Earth-Moon center of mass? b.) Is the Earth-Moon center of mass above or below the surface of the Earth? By what distance?


## Additional Questions

1. A satellite powered by a so-called "ion propulsion engine" was recently sent into space. It essentially works by spitting atoms out the back of the engine, which somehow propels the satellite forward. Can you explain how this engine works from the point of view of conservation of momentum? For which system is momentum conserved?
2. Discuss as a group what the terms "conserve", "momentum" and "conservation of momentum" mean.
3. Highway patrol officers routinely assume that momentum is conserved during the collisions of automobiles. Explain why you think this is or is not a reasonable assumption.
4. Passengers in cars are required to wear seatbelts, but children in a school bus are not. Can you explain this fact by thinking about momentum?
