

**Equations and Relations:**

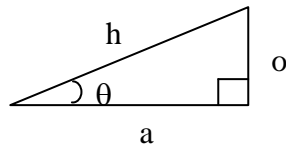
Trigonometry:

$$\sin \theta = o/h$$

$$\cos \theta = a/h$$

$$\tan \theta = o/a$$

$$a^2 + o^2 = h^2$$



Common Prefixes:

G	giga	$10^9$
M	mega	$10^6$
k	kilo	$10^3$
c	centi	$10^{-2}$
m	milli	$10^{-3}$
$\mu$	micro	$10^{-6}$
n	nano	$10^{-9}$

1. Physicists often use a dimensional analysis of an equation as a first test of its validity:

a) You remember the following equations for the circumference and area of a circle and the volume and surface area of a sphere.  $\frac{4}{3}\pi r^3$ ,  $4\pi r^2$ ,  $2\pi r$ ,  $\pi r^2$ . However, you don't remember which equation to use when. Use a dimensional analysis to determine which equation to use.

b) Can dimensional analysis determine whether the area of a circle is  $\pi r^2$  or  $2\pi r^2$ ?

Explain.

c) Which of the following equations is dimensionally consistent? (the variables represent the following parameters  $x$ = displacement in  $m$ ,  $v$ = velocity in  $m/s$ ,  $a$ = acceleration in  $m/s^2$ )

(A)  $v = at$ , (B)  $v = at + \sqrt{2ax} + v_0$ , (C)  $t = a/v$ , (D)  $t = t_0 + \frac{x}{v} + \frac{x^2}{v}$ , (E)

$$t = t_0 + \frac{x}{v} \sqrt{\frac{x^2}{a^2 t_0^4}}$$

d) Velocity is related to acceleration and distance by the following expression,  $v^2 = 2ax^p$ . Find the power  $p$  that makes this equation dimensionally consistent.

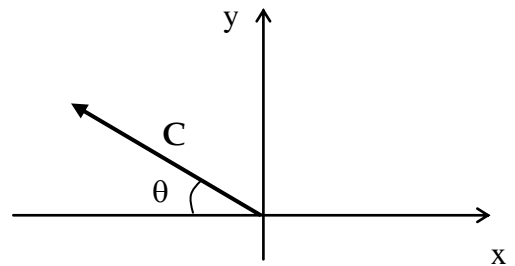
e) The time  $T$  required for one complete oscillation of a mass  $m$  on a spring of force constant  $k$  is

$$T = 2\pi \sqrt{\frac{m}{k}}$$

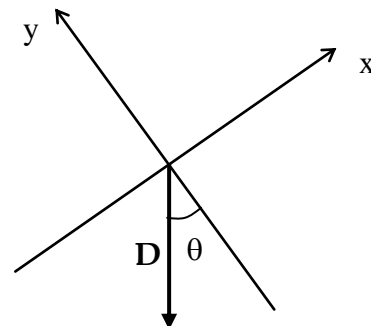
Find the dimensions  $k$  must have for this equation to be dimensionally correct.

2. The conversion of units is essential, but often a source of unnecessary mistakes. Here are some practice problems:
- A has a mass of 130.2 carats, where 1 carat = 0.20 g. What is the weight of this diamond in pounds?
  - Suppose 10.0 cubic meter of oil is spilled into the ocean. Find the area of the resulting slick, assuming that it is one molecule thick, and that each molecule occupies a cube 0.50  $\mu\text{m}$  on a side.
  - Convert the following units: (A) 123  $\text{m}^3$  in  $\text{mm}^3$ ,  $\mu\text{m}^3$ ,  $\text{km}^3$ , liters,  $\text{inch}^3$ , (B) 18  $\text{m}^2$  in  $\text{nm}^2$ ,  $\text{km}^2$ ,  $\text{cm}^2$ ,  $\text{mm}^2$ , (C) 1.13  $\text{mg}/\text{m}^3$ , 145  $\mu\text{g}/\text{cm}^3$  in  $\text{Kg}/\text{m}^3$
3. It is always good to check if the order of magnitude of your solutions is reasonable. Sometimes you will not be able to find a equation to calculate are value precisely, but you can still get a fairly good estimate of real answer.
- Milk is often sold by the gallon in plastic containers. Estimate the number of gallons of milk that are purchased in the United States each year. What approximate weight of plastic does this represent?
  - Which answers are physically reasonable:
    - A long distance runner can run from Portland to Seattle in 10 hours.
    - A hairdryer uses 100 KW
    - The tallest human is 2.3135 m
    - A hair has a diameter of 1 mm
    - 150 apples have a mass of 1000 kg
    - A race car could drive the distance from the earth to the moon in roughly two months
    - 10 Million hydrogen atoms have a mass of 1 gram.

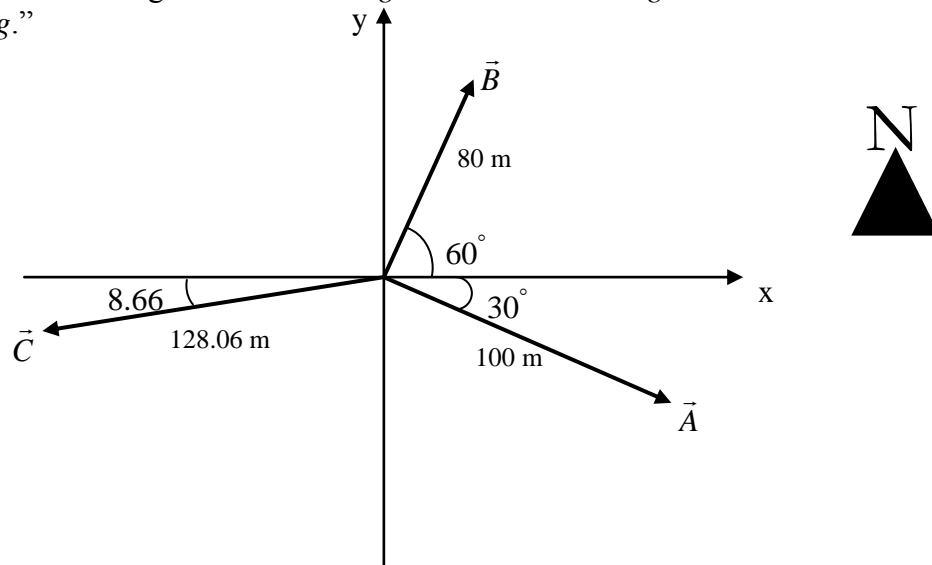
4. The vector **C** is shown on the right.
- Show the x-component  $C_x$  on the diagram. Is the x-component positive or negative?
  - Show the y-component  $C_y$  on the diagram. Is the y-component positive or negative?
  - In terms of  $C$  and  $\theta$ , write expressions for  $C_x$  and  $C_y$ .
  - Find a coordinate system in which the y-component of **C** is zero.



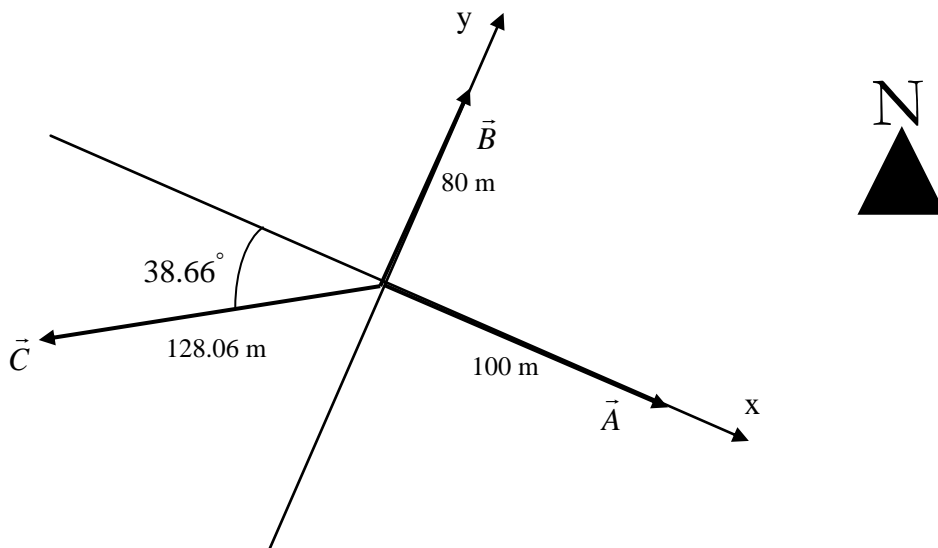
5. The vector **D** is shown on the right.
- Show the x-component  $D_x$  on the diagram. Is the x-component positive or negative?
  - Show the y-component  $D_y$  on the diagram. Is the y-component positive or negative?
  - In terms of  $D$  and  $\theta$ , write expressions for  $D_x$  and  $D_y$ .



6. A treasure map drawn by a physicist is shown below. With the map is a note that states the following: “Start at the big oak tree, walk along the vector sum  $\vec{A} + \vec{B} + \vec{C}$  and dig.”



- (a) Determine where to dig, using both graphical and algebraic methods.
- (b) Imagine that the author of the map had used a different coordinate system, rotated clockwise by  $30^\circ$  relative to the first one, as shown below. Add the vectors algebraically using this new map. Are your results any different? Was one map easier to use than the other?



**Check  
Point**

- Does changing coordinate systems change the length of a vector?
- What principles can be used to determine whether a coordinate system will be convenient for a particular problem or not?
- Can a coordinate system change the physical outcome of a situation?

7. A treasure map directs you to start at a palm tree and walk due north for 10.0 m. You are then to turn  $90^\circ$  and walk 15.0 m; then turn  $90^\circ$  again and walk 5.00 m. Give the distance from the palm tree, and the direction relative to north, for each of the four possible locations of the treasure.

**Check  
Point**

- If each component of a vector is doubled, **(a)** how does the magnitude of the vector change? **(b)** What happens to the direction of the vector?
- Given that  $\vec{A} + \vec{B} = \vec{C}$ , and that  $A^2 + B^2 = C^2$ , how are  $\vec{A}$  and  $\vec{B}$  oriented relative to one another?
- Given that  $\vec{A} + \vec{B} = \vec{C}$ , and that  $A + B = C$ , how are  $\vec{A}$  and  $\vec{B}$  oriented relative to one another?
- Given that  $\vec{A} - \vec{B} = \vec{C}$ , and that  $A - B = C$ , how are  $\vec{A}$  and  $\vec{B}$  oriented relative to one another?
- Given that  $\vec{A} + \vec{B} = 0$ , **(a)** how does the magnitude of  $\vec{B}$  compare with the magnitude of  $\vec{A}$ ? **(b)** How does the direction of  $\vec{B}$  compare with the direction of  $\vec{A}$ ?

Additional Questions

- 1) How do you determine whether one particular component of a vector is positive or negative in a given coordinate system? Describe a method in words that someone with no mathematical background could successfully use.
- 2) Rain is falling vertically downward and you are running for shelter. To keep driest, should you hold your umbrella vertically, tilted forward, or tilted backward? Explain.
- 3) The corners of a square with sides 2.5 m long lie on a circle. **(a)** Is the radius of the circle greater than, less than, or equal to the length of a side of the square? Explain. **(b)** Find the radius of the circle.