CHAPTER 3: **Energy in bound system is quantized, light comes in quanta of energy / particles called photons**

- 3.1 Discovery of the Electron (1897)
- 3.2 Determination of precise value of Electron Charge
- 3.3 Quantization of charge and mass
- Discovery of X Rays (1895), X-ray diffraction (1912) and X-ray crystallography (1913)
- 3.4 Line Spectra
- 3.5 Blackbody Radiation
- 3.6 Photoelectric Effect
- 3.7 X-Ray Production and diffraction on crystals
- 3.8 Compton Effect
- 4.7: Frank-Hertz experiment, chapter 4

As far as I can see, our ideas are not in contradiction to the properties of the photoelectric effect observed by Mr. Lenard.

\- Max Planck, 1905

Lenard got the Nobel prize that year for his work on cathode rays, but missed out on two of the most important discoveries in this kind of research (X-rays and the electron)

Pair production was already discussed in chapter 2 (special relativity)

other loose ends in Classical Physics

besides $c = 1/srt(\varepsilon\mu)$ and $m_{inertial} \approx m_{dynamic}$

Modern Physics

Stefan (1879)
Wien (1893)
Rayleigh (1900)
(also Jeans, 1905)

Thermal Radiation

How come we can't derive this from Maxwellian waves?

Roentgen (1895) : X-rays
Becquerel (1896) : radioactivity

waves or particles?

Balmer (1884)
Rydberg (1890)
Zeeman (1896)

Studies of spectral lines

How come they are characteristic of atoms?

J.J. Thompson (1897) electron

Atoms have parts!

Planck (1900) $E_n = nh\nu$ → Einstein (1905) "photon" →

wave particle duality f[or] massless particles

→ Rutherford (1911) "nucleus" → Bohr (1913) model of atom → de Broglie (1924) wave-particle duality → Quantum Mechanics: Heisenberg (1925) Schrödinger (1926) Dirac (1927) →

the correct theory of matter at last

then applications, PH 312

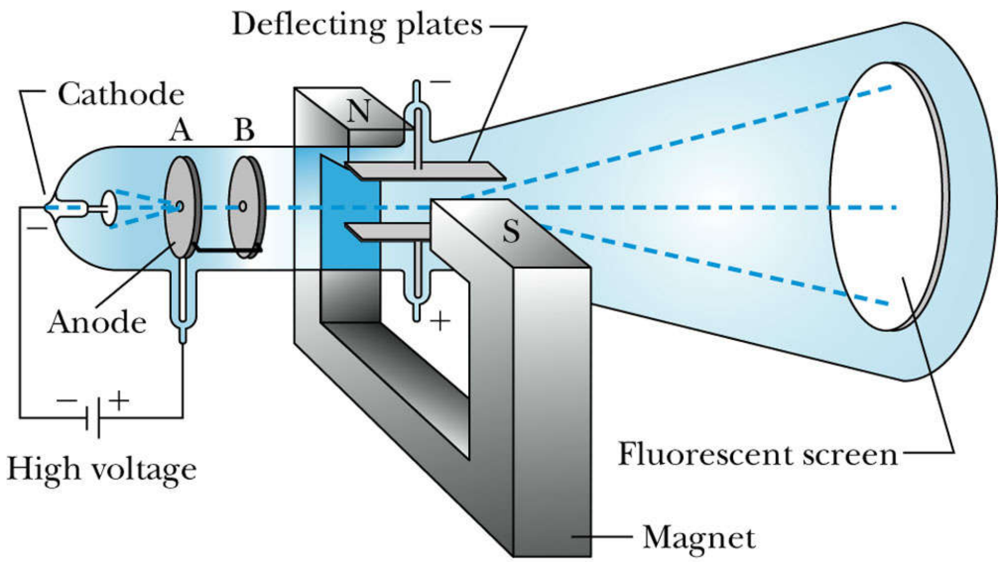
Cathode Ray Experiments

- In the 1890s scientists and engineers were familiar with "cathode rays". These rays were generated from one of the metal plates in an evacuated tube across which a large electric potential had been established.
- It was surmised that cathode rays had something to do with atoms. Different metals were used as cathode materials
- It was known that cathode rays were deflected by magnetic and electric fields, could to some extent penetrate matter (but the tube itself required high vacuum for reproducible operations

Nobel prize 1905 Philip Eduard Anton von Lenard, who later became a Nazi, and would not find academic employment in the new Germany after WWII

Apparatus of J.J. Thomson's Cathode-Ray Experiment

- Thomson used an evacuated cathode-ray tube to show that the cathode rays were negatively charged particles (electrons) by deflecting them in electric and magnetic fields.



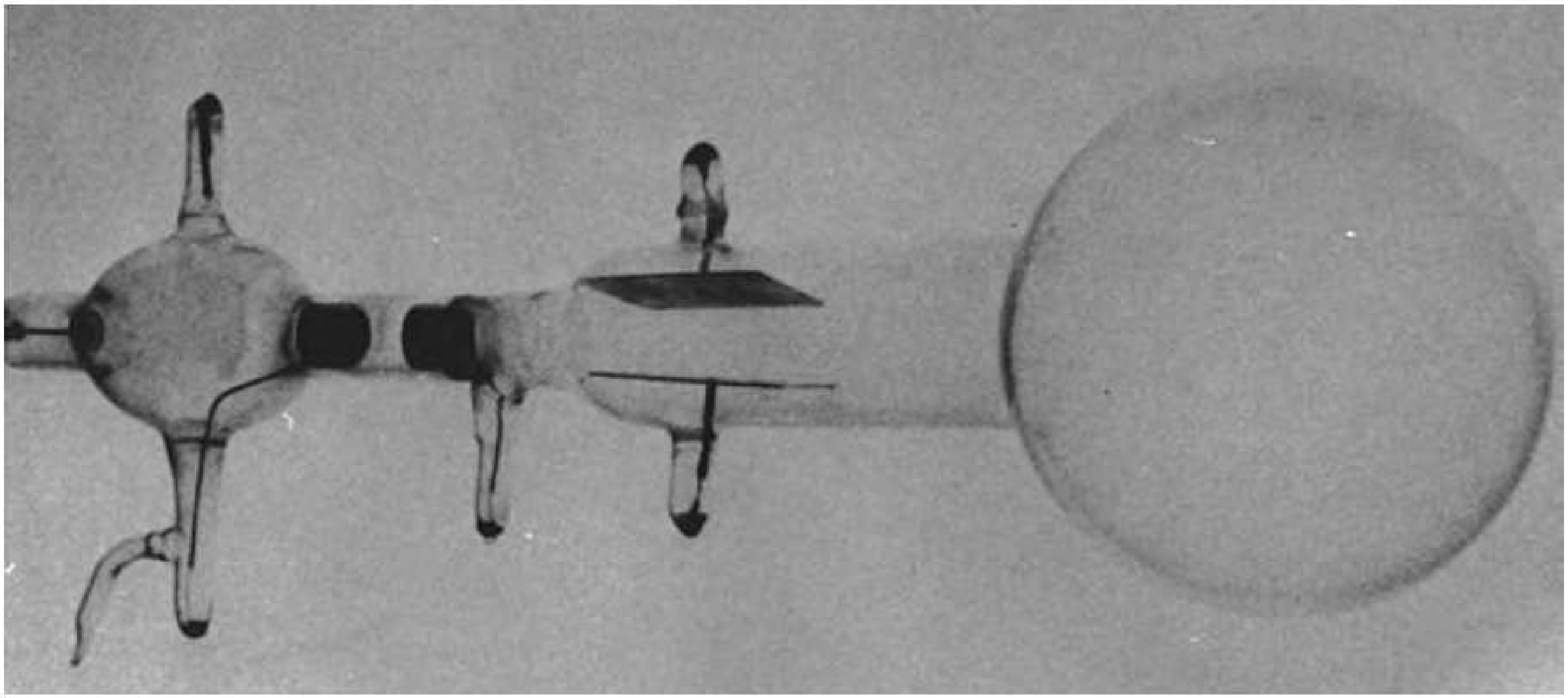
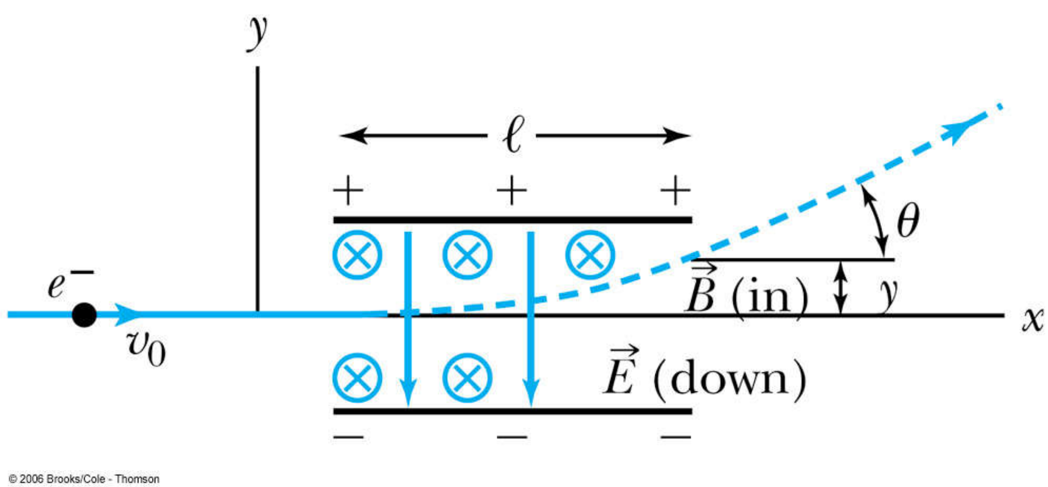


Figure 4.4 The original e/m_e tube used by J. J. Thomson. (*After Figure 1.3, p. 7, R. L. Sproull and W. A. Phillips,* Modern Physics, *3rd ed., New York, John Wiley & Sons, 1980*).

J. J. Thomson's Experiment

- Thomson's method of measuring the ratio of the electron's charge to mass was to send electrons through a region containing a magnetic field perpendicular to an electric field.



Calculation of *e*/*m*

- An electron moving through the electric field is accelerated by a force: $F_y = ma_y = qE$

- Electron angle of deflection: $$\tan\theta = \frac{v_y}{v_x} = \frac{a_y t}{v_0} = \frac{qE}{m}\frac{\ell}{v_0^2}$$

- The magnetic field deflects the electron against the electric field force. $\vec{F} = q\vec{E} + q\vec{v}\times\vec{B} = 0$

- The magnetic field is adjusted until the net force is zero.

$$\vec{E} = -\vec{v}\times\vec{B} \qquad\qquad v = \frac{\left|\vec{E}\right|}{\left|\vec{B}\right|} = v_0$$

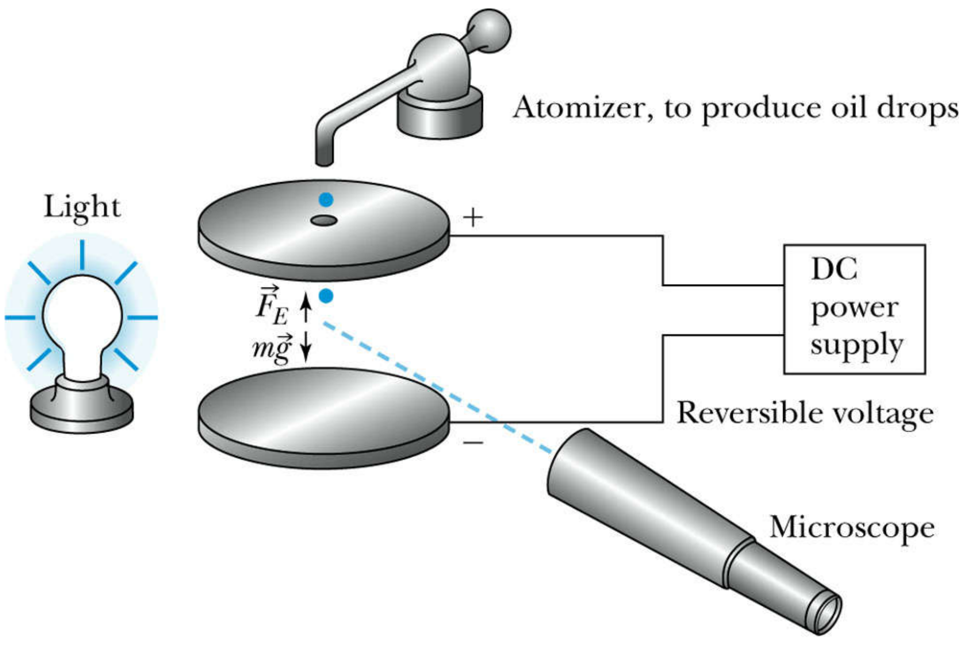
- Charge to mass ratio: $$\frac{q}{m} = \frac{v_0^2\tan\theta}{E\ell} = \frac{E\tan\theta}{B^2\ell}$$

Discovery of the Electrons

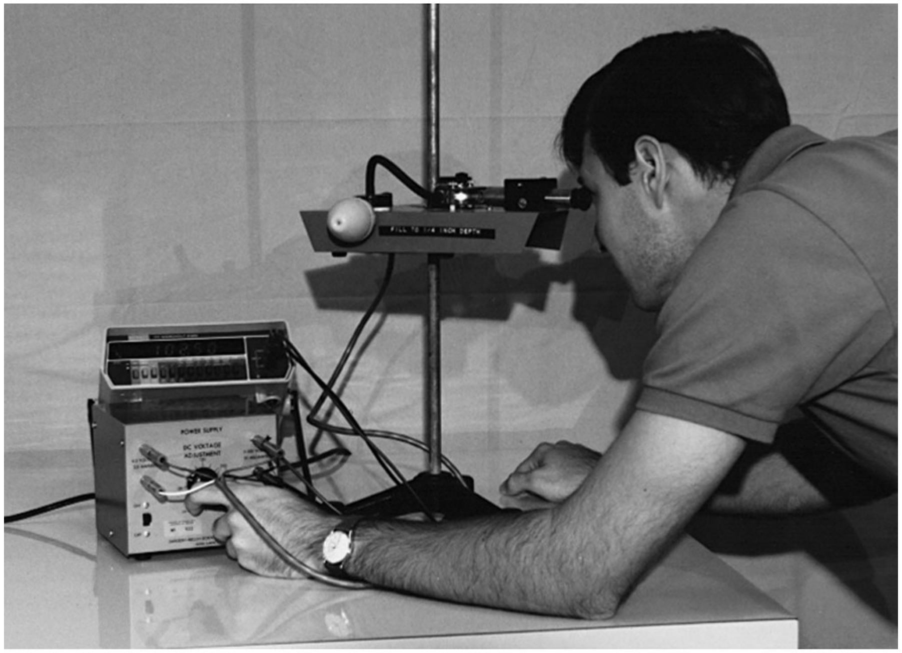
- **Electrons were discovered by J. J. Thomson 1897**
  - Observed that cathode rays were charged particles with a mass to charge ratio of about 1/2000 of the mass of an ionized hydrogen atom, i.e. a proton.
  - Same particle independent on what metal cathode it was extracted from, so atoms are not indivisible … should have an internal structure

3.2: Determination of precise value of Electron Charge

Millikan oil drop experiment



(a)



Calculation of the oil drop charge

- Used an electric field and gravity to suspend a charged oil drop.

$$\vec{F}_y = q\vec{E} = -m\vec{g}$$

- Mass is determined from Stokes's relationship of the terminal velocity to the radius and density.

$$m = \frac{4}{3}\pi r^3 \rho$$

- Magnitude of the charge on the oil drop.

$$q = \frac{mgd}{V}$$

- Thousands of experiments showed that there is a basic quantized electron charge.

$$q = 1.602 \times 10^{-19}\ \mathrm{C}$$

Quantization of charge and mass

- Current theories predict that charges are quantized in units (called **quarks**) of $e/3$, $2e/3$ and e, but quarks are not directly observed experimentally. The charges of particles that have been directly observed are quantized in units of e.
- The measured atomic weights are not continuous—they have only discrete values, which are close to integral multiples of a unit mass.

Discovery of X Rays

X rays were discovered by Wilhelm Conrad Röntgen in 1895

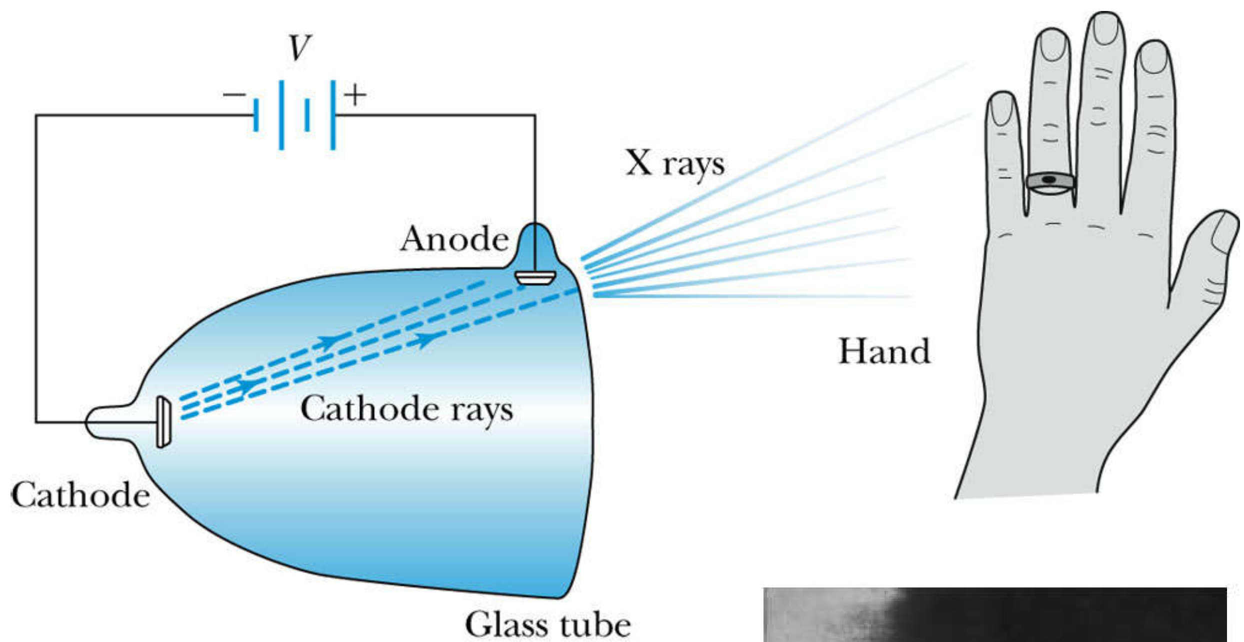
- Observed X rays emitted by cathode rays bombarding glass, found that they cannot be affected by electromagnetic fields, i.e. are not charged particles
- Could not get diffraction effects, which would prove that they are Maxwellian waves, today we know this is due to their very small wavelength, very high frequency, very high energy

Observation of X Rays

- Wilhelm Conrad Röntgen studied the effects of cathode rays passing through various materials. He noticed that a phosphorescent screen near the tube glowed during some of these experiments. These rays were unaffected by magnetic fields and penetrated materials much more than cathode rays.
- He called them **x rays** and deduced that they were produced by the cathode rays bombarding the glass walls of his vacuum tube.

Röntgen's X-Ray Tube

- Röntgen constructed an x-ray tube by allowing cathode rays to impact the glass wall of the tube and produced x rays. He used x rays to image the bones of a hand on a phosphorescent screen.



An x ray of Mrs. Roentgen's hand taken by Roentgen shortly after his discovery.

In the late afternoon of 8 November 1895, Röntgen carefully constructed a black cardboard. Before setting up the barium platinocyanide screen to test his idea, Röntgen darkened the room to test the opacity of his cardboard cover.

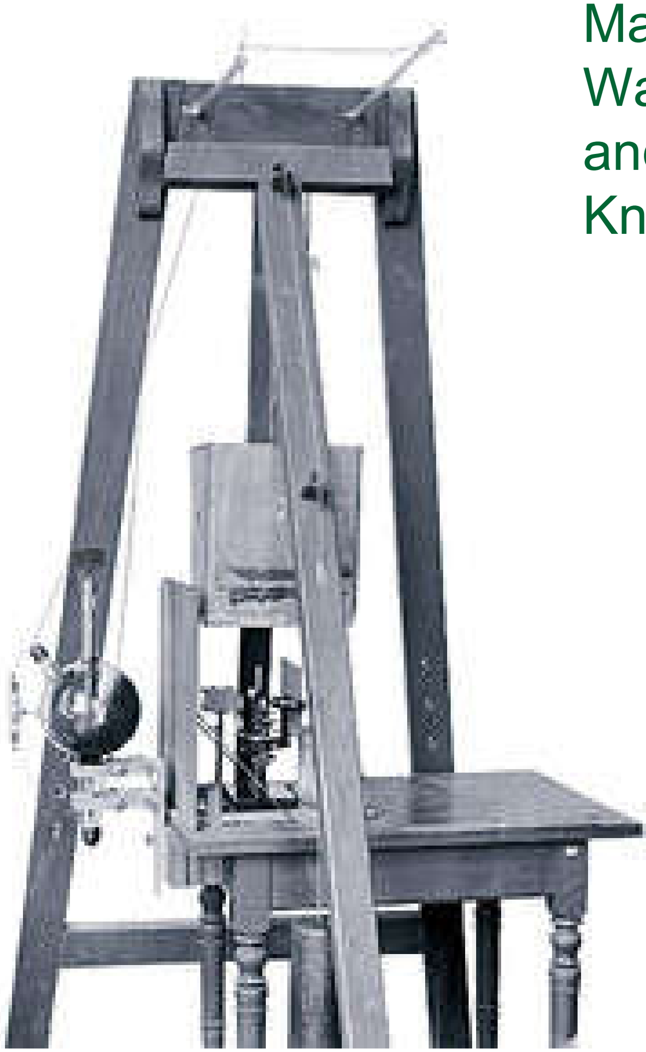
… he determined that the cover was light-tight and turned to prepare the next step of the experiment. It was at this point that Röntgen noticed a faint shimmering from a bench a few feet away from the tube. To be sure, he tried several more discharges and saw the same shimmering each time.

Striking a match, he discovered the shimmering had come from the location of the barium platinocyanide screen he had been intending to use next.

Nov./Dec 1895, he worked flat out, had a paper out by Christmas 1895,

Becquerel misunderstood the paper and discovered radioactivity in early 1896 as a result

First ever Nobel Prize in Physics, 1901



Max Laue,
Walter Friedrich,
and Paul
Knipping, 1912

With the benefit of Laue talking to Arnold Sommerfeld's PhD student Peter Paul Ewald

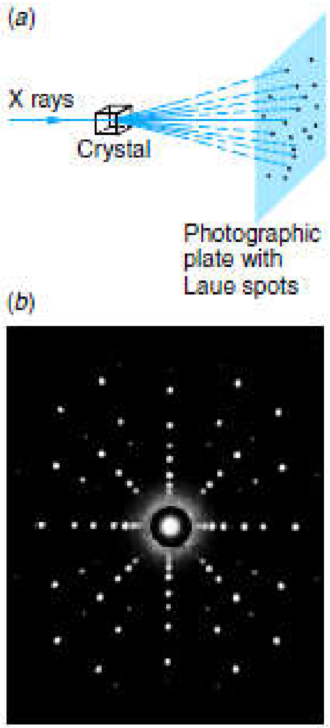


Figure 3-11 (*a*) Schematic sketch of a Laue experiment. The crystal acts as a three-dimensional grating, which diffracts the x-ray beam and produces a regular array of spots, called a *Laue pattern*, on photographic film or an x-ray-sensitive charge-coupled device (CCD) detector. (*b*) Laue x-ray diffraction pattern using a niobium boride crystal and 20-keV molybdenum x rays. [*General Electric Company.*]

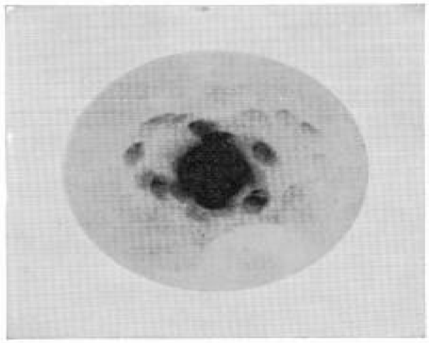


Fig. 4–4(1). Friedrich & Knipping's first successful diffraction photograph.

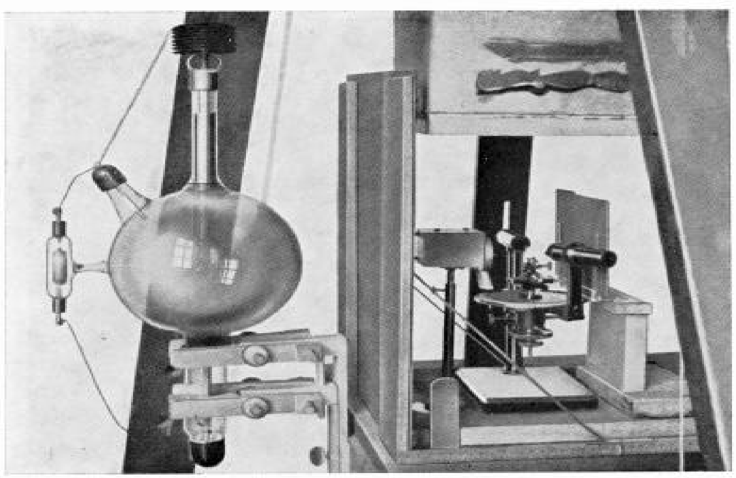
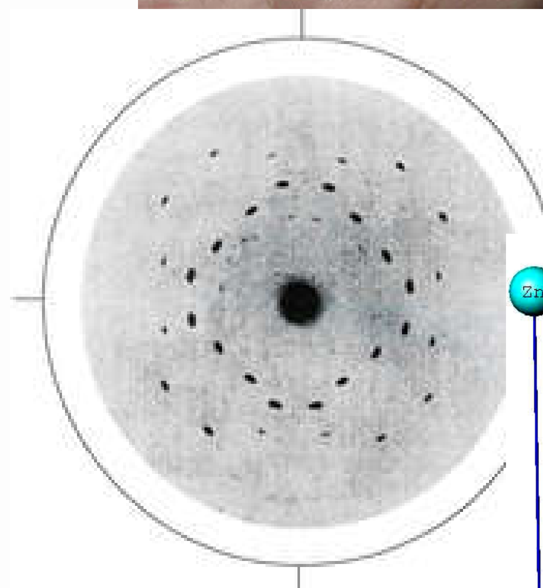
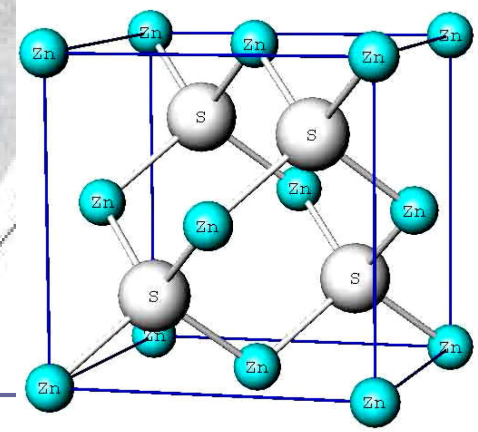
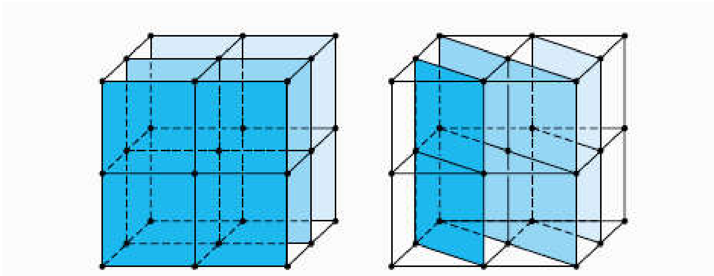


Fig. 4–4(2). Friedrich & Knipping's improved set-up.







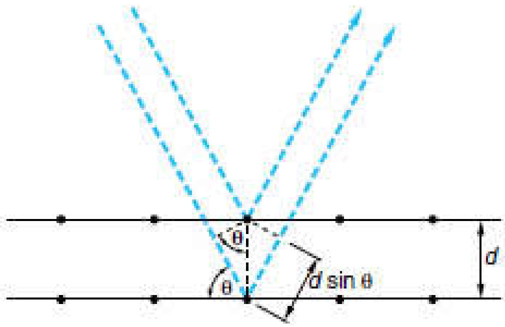


Figure 3-13 Bragg scattering from two successive planes. The waves from the two atoms shown have a path length difference of $2d \sin\theta$. They will be in phase if the Bragg condition $2d \sin\theta = m\lambda$ is met.

$$2d \sin\theta = m\lambda \quad \text{where } m = \text{an integer}$$

Nobel prizes

1914 to Max von Laue

1915 to Wilhelm Henry Bragg and Wilhelm Lawrence Bragg

http://iycr2014.org/

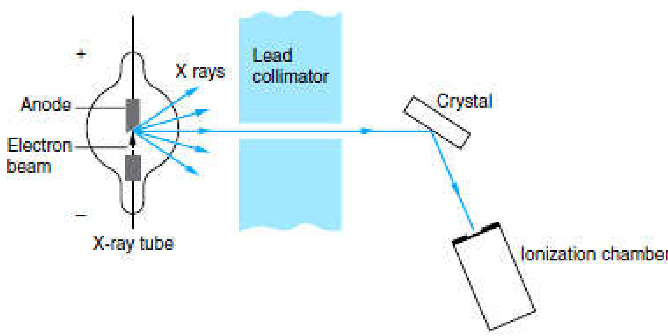
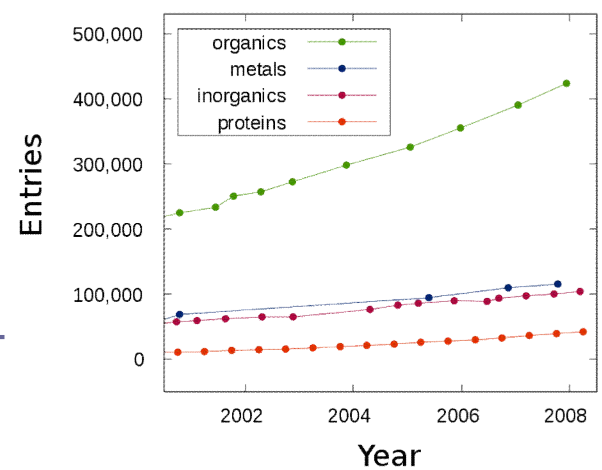


Figure 3-14 Schematic diagram of a Bragg crystal spectrometer. A collimated x-ray beam is incident on a crystal and scattered into an ionization chamber. The crystal and ionization chamber can be rotated to keep the angles of incidence and scattering equal as both are varied. By measuring the ionization in the chamber as a function of angle, the spectrum of the x rays can be determined using the Bragg condition $2d \sin\theta = m\lambda$, where d is the separation of the Bragg planes in the crystal. If the wavelength λ is known, the spacing d can be determined.



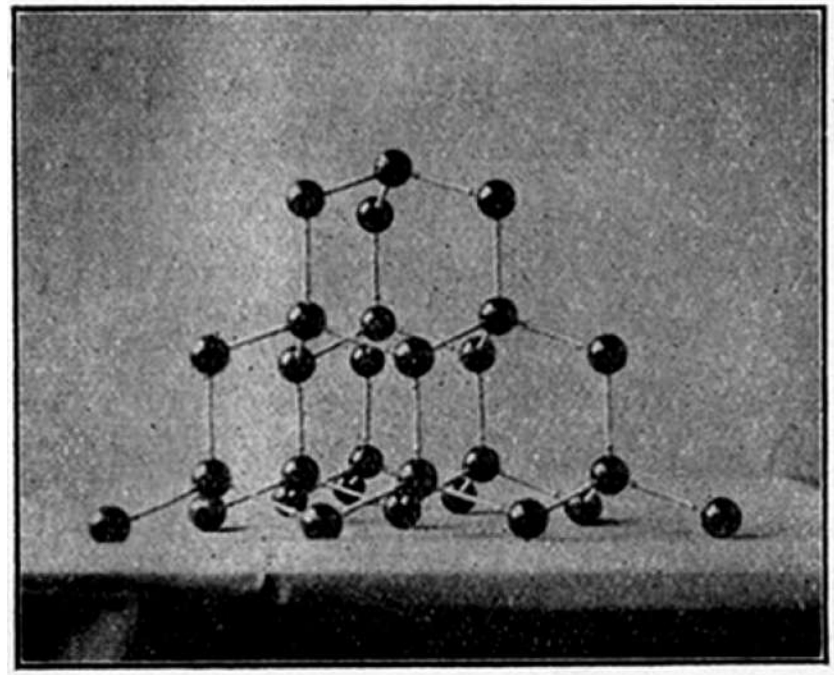
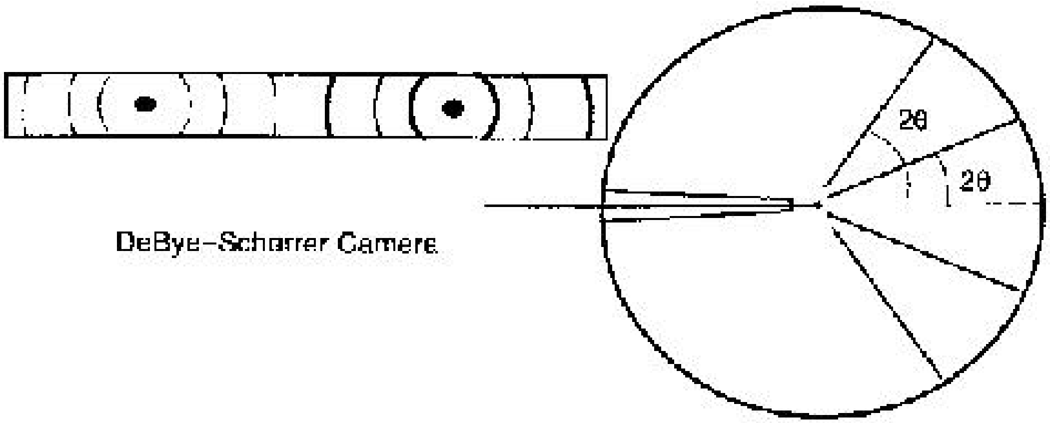


Fig. 7.—View perpendicular to a (111) axis.

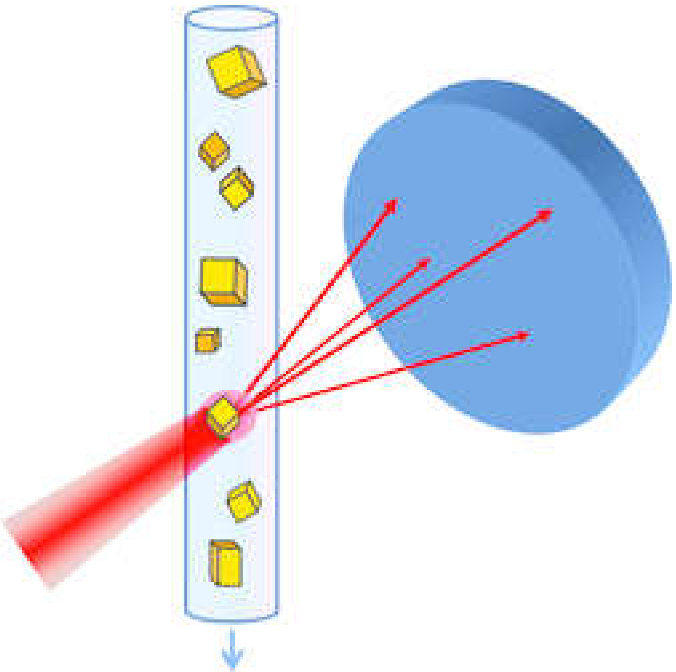
Left: Hard sphere model of the diamond structure (space filling ≈ 34 %) by W. H. Bragg, Museum of the Royal Institution, London, photo by André Authier. (Source: http://blog.oup.com/2013/08/100th-anniversary-first-crystal-structure-determinations-bragg/#sthash.50BE8wiT.6mYKaBKB.dpuf.) **Right:** Ball and stick model of the diamond structure. (Source: FIG. 7 in W. H. Bragg and W. L. Bragg, The structure of diamond, *Proc. R. Soc. Lond. A* **89**, 277; published September 22, 1913.)

Left: Reconstruction of the double helix model of deoxyribose nucleic acid containing some of the original metal plates; **Right:** used by Francis Crick and James Dewey Watson in 1953, (Source: Science Museum, http://www.sciencemuseum.org.uk/images/i045/10313925.aspx)

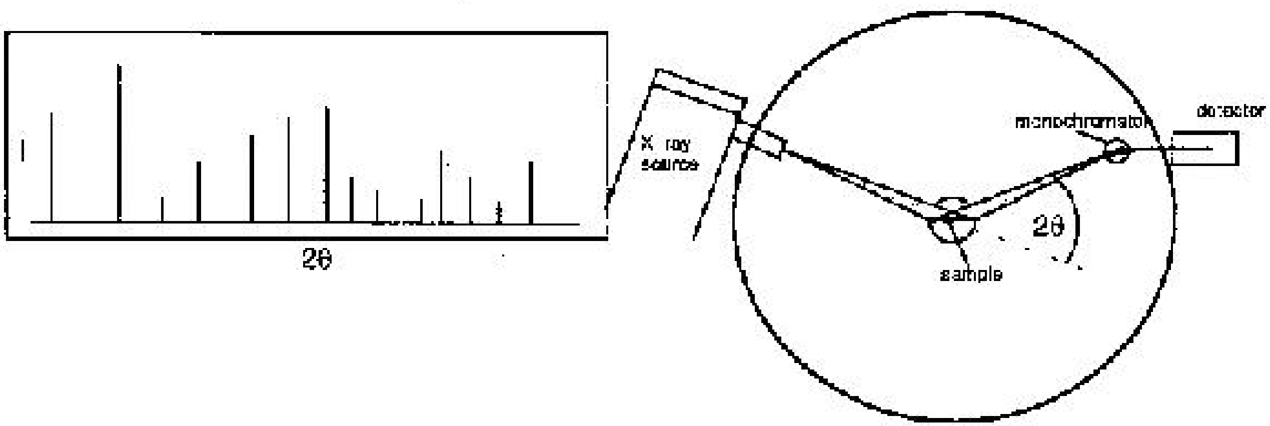
5.4. X-ray Powder Diffraction Instruments

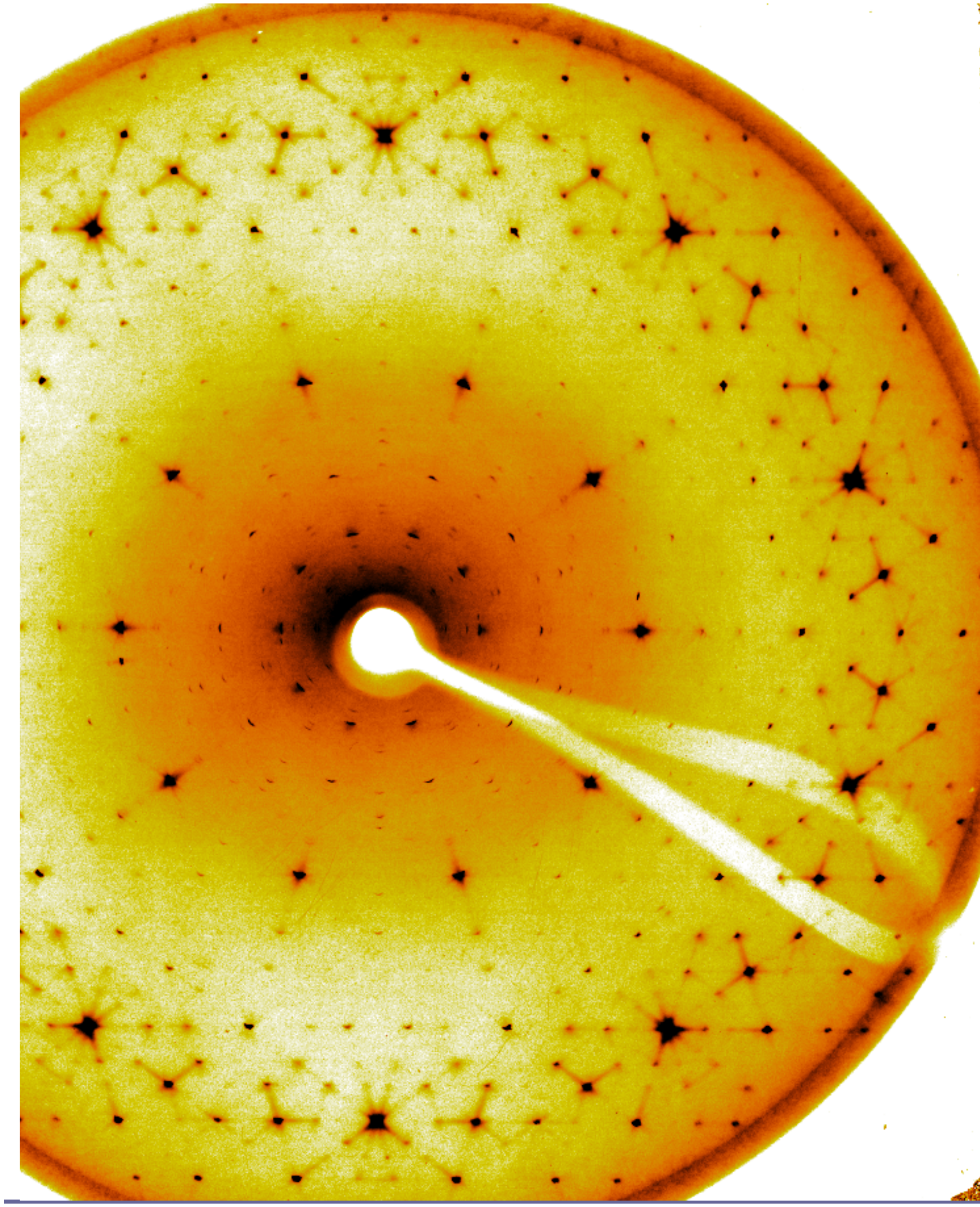


DeBye-Scherrer Camera



Diffractometer





This is actually a quasicrystal, you can tell from the 10 fold rotation symmetry

The image is a flat section through reciprocal/Fourier space

As a Fourier transform is a good mathematical model for the diffraction of X-rays by crystals (most ordinary condensed matter) and quasicrystals (some extraordinary condensed matter)

3.3: Line Spectra (characteristic of atoms)

- Chemical elements were observed to produce unique wavelengths of light when burned or excited in an electrical discharge.

- Emitted light is passed through a diffraction grating with thousands of lines per ruling and diffracted according to its wavelength λ by the equation:

$$d \sin\theta = n\lambda$$

where *d* is the distance of line separation and *n* is an integer called the order number.

Note the significant difference of this formula to the Bragg equation

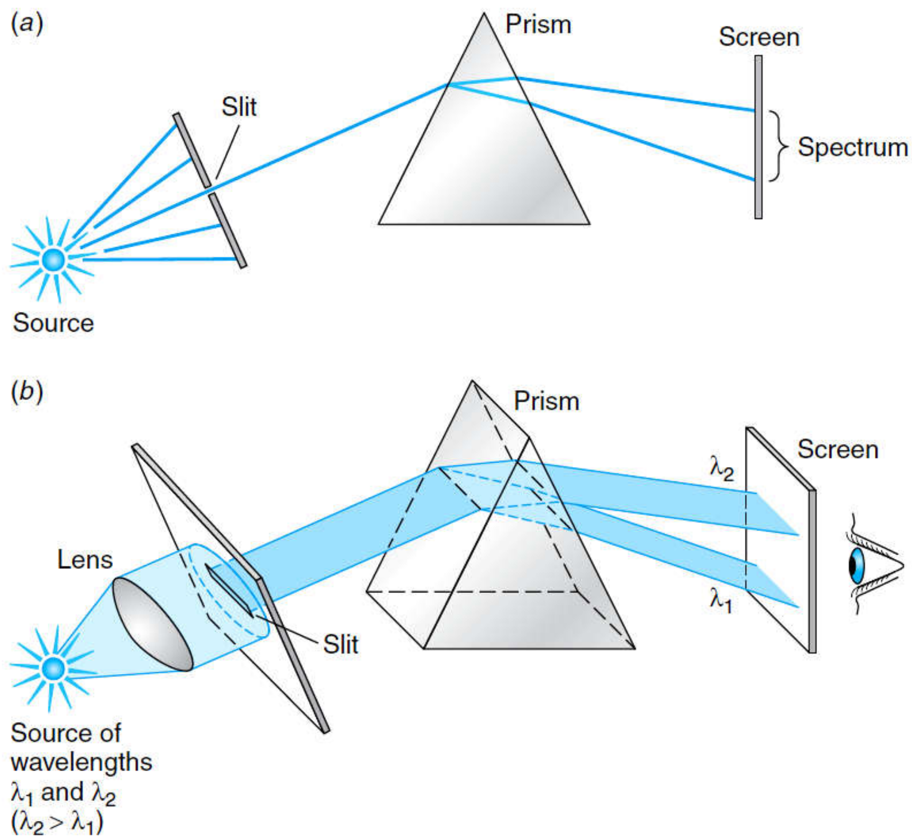


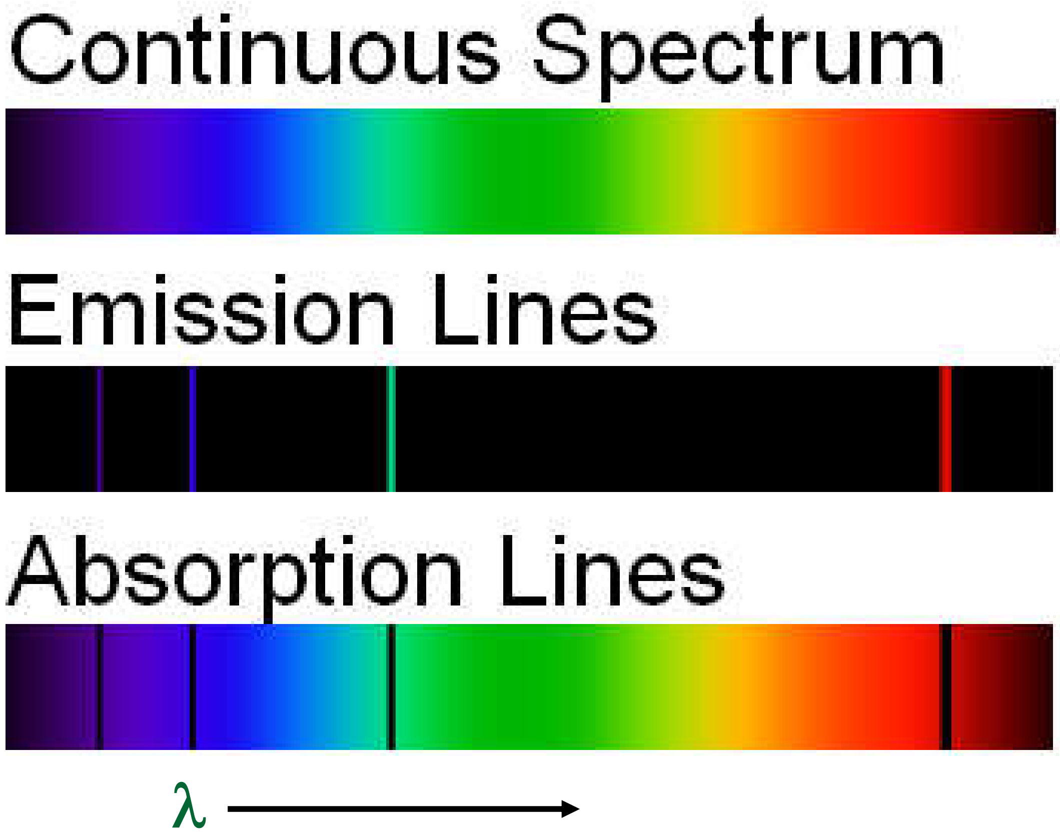
Figure 4-1 (*a*) Light from the source passes through a small hole or a narrow slit before falling on the prism. The purpose of the slit is to ensure that all the incident light strikes the prism face at the same angle so that the dispersion by the prism causes the various frequencies that may be present to strike the screen at different places with minimum overlap. (*b*) The source emits only two wavelengths, $\lambda_2 > \lambda_1$. The source is located at the focal point of the lens so that parallel light passes through the narrow slit, projecting a narrow line onto the face of the prism. Ordinary dispersion in the prism bends the shorter wavelength through the larger total angle, separating the two wavelengths at the screen. In this arrangement each wavelength appears on the screen (or on film replacing the screen) as a narrow line, which is an image of the slit. Such a spectrum was dubbed a "line spectrum" for that reason. Prisms have been almost entirely replaced in modern spectroscopes by diffraction gratings, which have much higher resolving power.

If the material is very dilute there are discrete spectra: Ångström (~ 1860)

1: this chapter, we need to learn more about the nature of light

1

2



dense material, i.e. hot “black body”, big- bang background radiation, sun if one does not look too carefully

There is ***typically*** more emission lines than absorption lines, explanation later on when we understand atoms

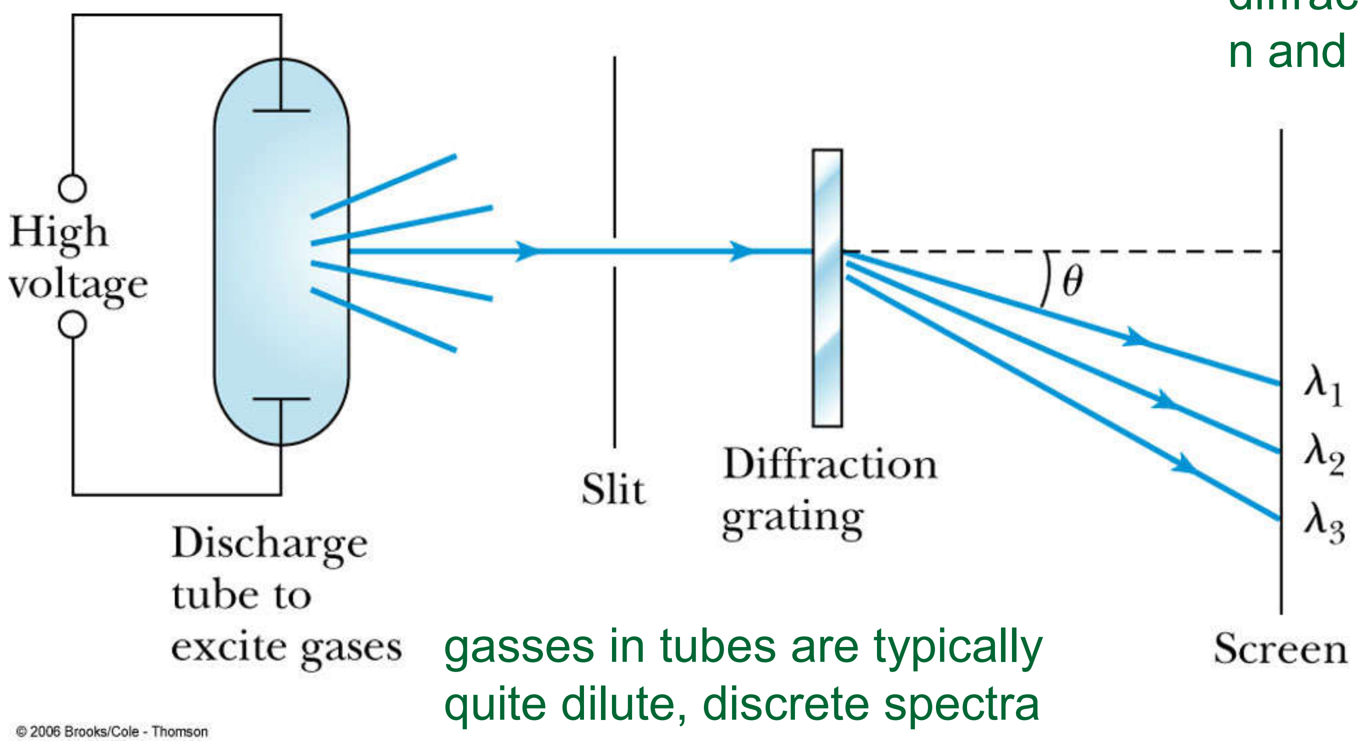
For every absorption line, there is an emission line, but **typically** not the other way around

2: next chapter where we will use new insights on the nature of light

Optical Spectrometer

$$d \sin\theta = n\lambda$$

where d is “line spacing on the diffraction grating”, n and integer



gasses in tubes are typically quite dilute, discrete spectra

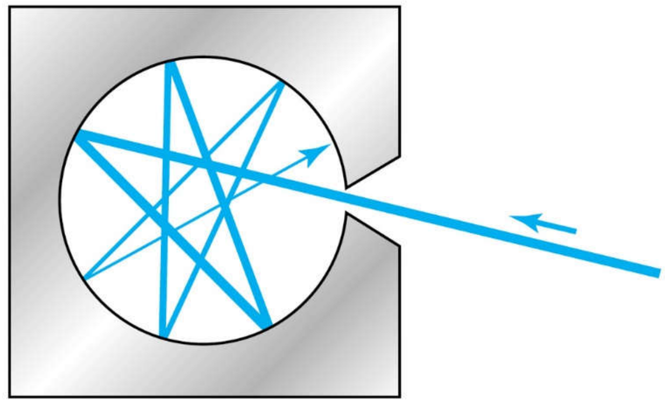
- Diffraction creates a *line spectrum* pattern of light bands and dark areas on the screen.
- The line spectrum serves as a fingerprint of the gas that allows for unique identification of chemical elements and material composition.

crystals are a different case, d_{hkl} is spacing between atomic planes, easiest to understand in 3D reciprocal space

$$2d \sin\theta = m\lambda \quad \text{where } m = \text{an integer}$$

3.5: Blackbody Radiation

- When matter is heated, it emits radiation.
- A blackbody is a cavity in a material that only emits thermal radiation. Incoming radiation is absorbed in the cavity. Material is dense, so we expect a continuous spectrum



emissivity ϵ (ϵ = 1 for idealized black body)

- Blackbody radiation is interesting because the radiation properties of the blackbody are independent of the particular material! Physicists can study the distribution of intensity versus wavelength (or frequency) at fixed temperatures. Principle of a pyrometer to measure temperatures remotely.

Stefan-Boltzmann Law

- Empirically, total power radiated (per unit area and unit wavelength = m^{-3}) increases with temperature to power of 4: also intensity (funny symbol in Thornton-Rex)

would be very interesting to know this function (intensity as a function of λ and T)

$$R(T) = \int_0^\infty \mathcal{I}(\lambda, T)\, d\lambda = \epsilon\, \sigma T^4$$

Watt per m^2

- This is known as the **Stefan-Boltzmann law**, with the constant *σ* experimentally measured to be 5.6705×10^{-8} W / ($m^2 \cdot K^4$), Boltzmann derived the "form" of the empirical formula from classical physics statistics, it was not understood what the constant of proportionality actually meant

- The **emissivity** ϵ (ϵ = 1 for an idealized black body) is simply the ratio of the emissive power of an object to that of an ideal blackbody and is always less than 1.

power = current times voltage, time derivative of mechanical work, unit of watt

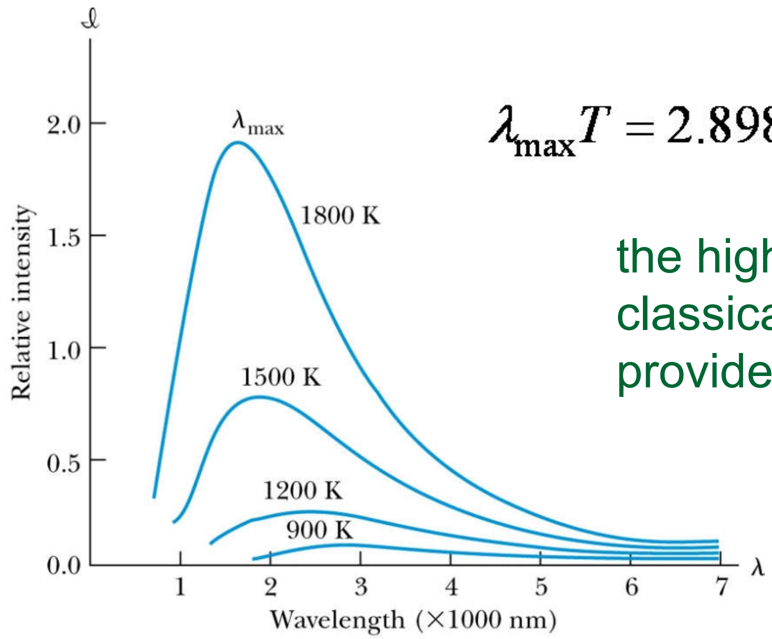
Energy density $u(T) = {}^{4}/_{c}\, \varepsilon\, \sigma\, T^4$ in Ws m^{-3}

Wien's Displacement Law

- The intensity $\mathcal{I}(\lambda, T)$ is the total power radiated per unit area per unit wavelength at a given temperature.
- **Wien's displacement law**: The maximum of the distribution shifts to smaller wavelengths as the temperature is increased.

$\mathcal{I}$

Wm^{-2}



$$\lambda_{max}T = 2.898 \times 10^{-3}\ \mathrm{m \cdot K}$$

the higher T, the shorter λ, no classical physics justification provided

Most interesting, what is the mathematical function that describes all of these curves??

Unit of intensity Watt per area

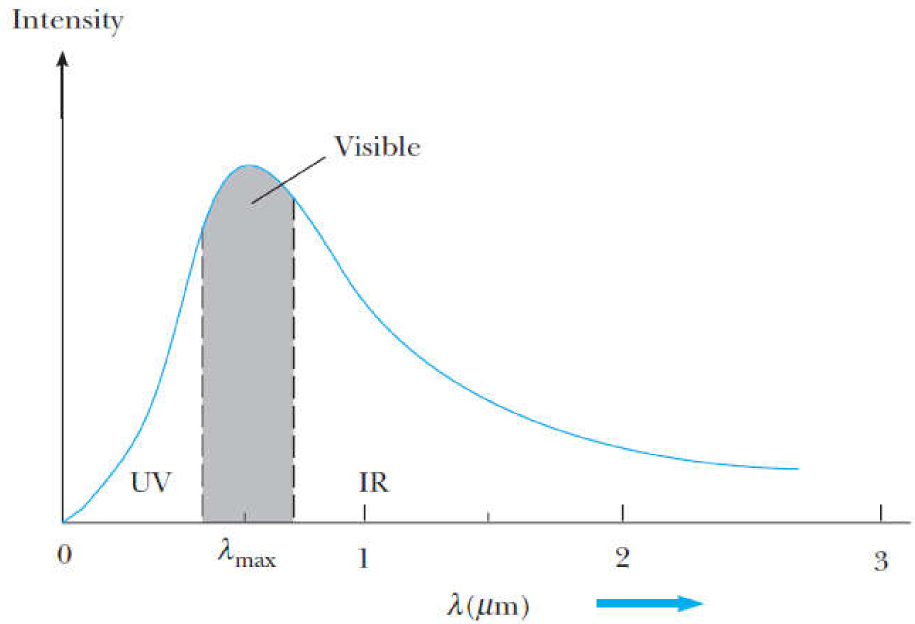


Figure 4.14 Intensity versus wavelength for a body heated to 6000 K.

So that must be approximately the black body radiation from the sun.

Why are our eyes particularly sensitive to green light of 550 nm? Because life evolved on earth receiving radiation from the sun that peaks at this particular wavelength.

Wien’s black body radiation formula with two free fitting parameters, to be determined by experiment

Watt-second (energy) Per m^3 after multiplication of intensity (Wm^{-2}) with 4/c, (factor 4 from steradiant = 4π sr for surface of a sphere)

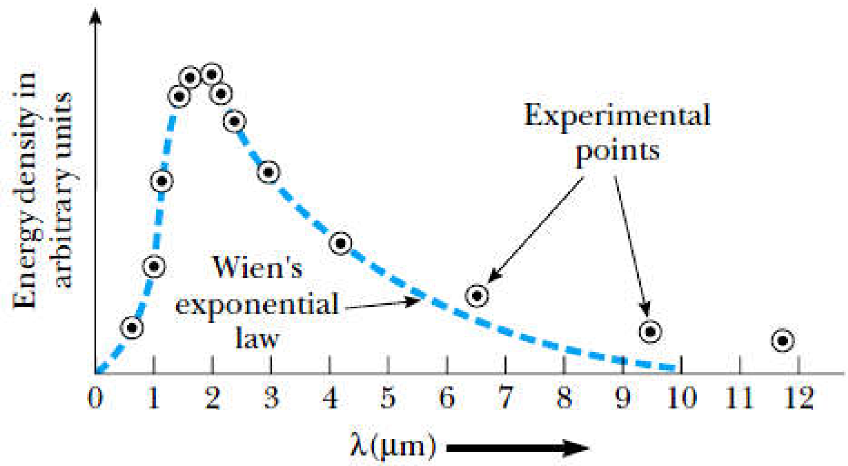


Figure 3.5 Discrepancy between Wien’s law and experimental data for a blackbody at 1500 K.

Wilhelm Wien, Physics Nobel prize 1911

$$u(f, T) = Af^3 e^{-\beta f/T}$$

1893

Problem: two fitting constants for which people struggled to find physical meaning, only Max Planck in 1900 provided some kind of an explanation of ß in the limit of very short wavelengths, to be revisited later

A continuous spectrum of electromagnetic radiation, for the very low temperature of the universe (approx. 3 K) far away from stars

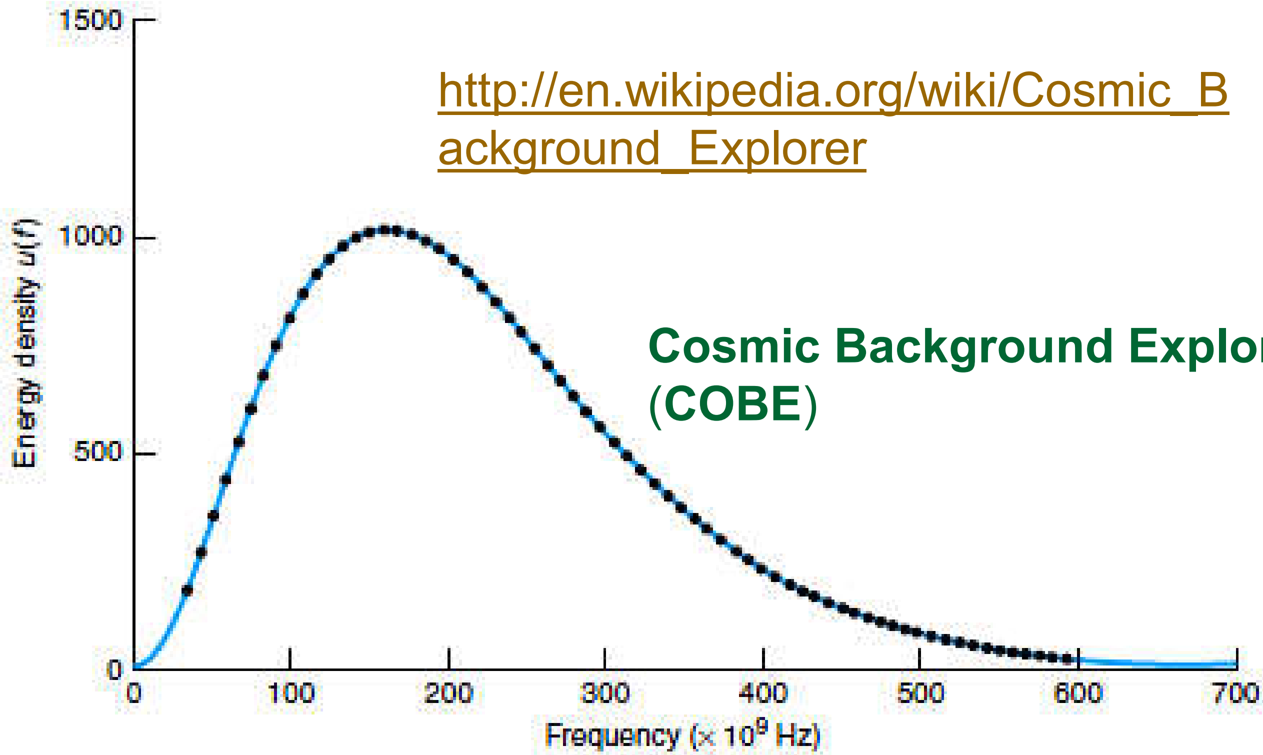
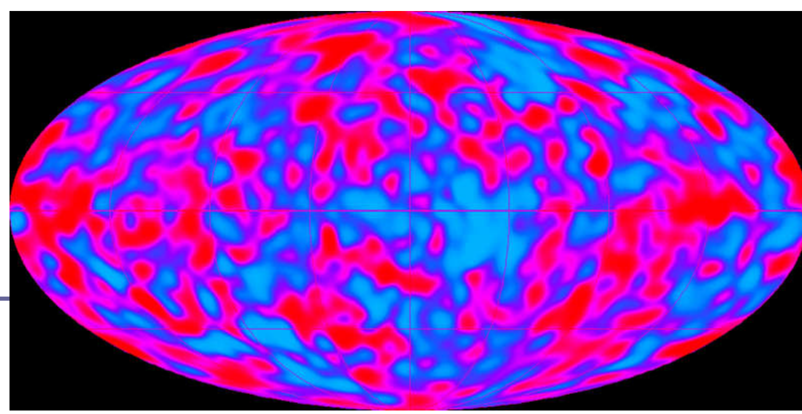


Figure 3-7 The energy density spectral distribution of the cosmic microwave background radiation. The solid line is Planck's law with $T = 2.725$ K. These measurements (the black dots) were made by the COBE satellite.

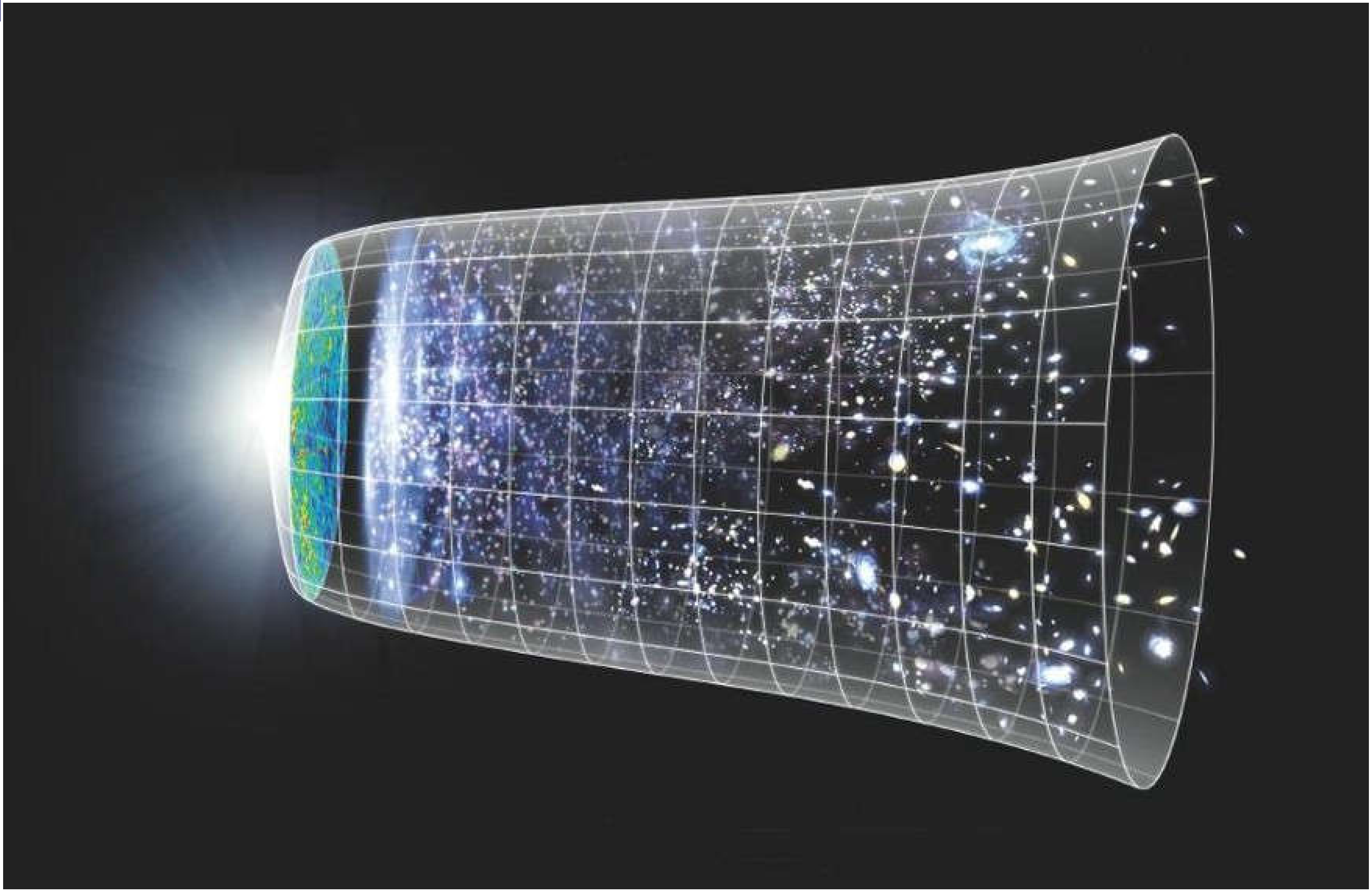
http://en.wikipedia.org/wiki/Cosmic_Background_Explorer

Cosmic Background Explorer (**COBE**)

Leaders of COBE project, George Smoot and John Mather, received 2006 Physics Nobel prize, now even better measurements



Very slight anisotropy, i.e. slightly different blackbody radiation temperature



Rayleigh-Jeans Formula

- Lord Rayleigh (John Strutt) and James Jeans used the classical theories of electromagnetism and thermodynamics to show that the blackbody spectral distribution should be, 1905

$$\mathcal{I}(\lambda, T) = \frac{2\pi c k T}{\lambda^4}$$

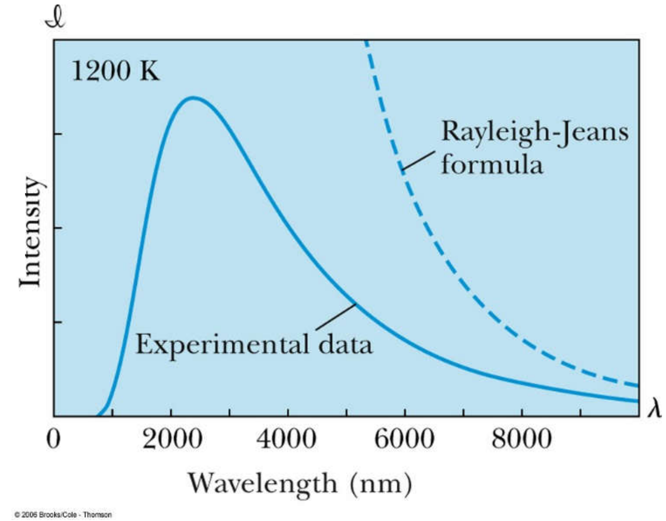
$$u(\lambda, T)\, d\lambda = \frac{8\pi}{\lambda^4} k_B T \, d\lambda$$

$$u(f, T)\, df = \frac{8\pi f^2}{c^3} k_B T \, df$$

With k as Boltzmann constant:

1.38065 10^{-23} J/K

kT at room temperature (293 K) ≈ 25 meV



- It approaches the data at longer wavelengths, but it deviates very badly at short wavelengths. This problem for small wavelengths became known as “the ultraviolet catastrophe” and was one of the outstanding problems that classical physics could not explain around 1900s, **implication: light may not be a Maxwellian wave**

http://hyperphysics.phy-astr.gsu.edu/hbase/mod6.html

Max Planck's Radiation Law

- Planck had the very best experimental data (over the widest available wavelength range available, he started out with Boltzmann's statistical methods, but that didn't work. By a curve fit over the best available data, he obtained the following formula that describes the totality of the blackbody radiation data very well, to provide a theoretical explanation, he assumed that the radiation in the cavity was emitted (and absorbed) by some sort of "oscillators" that were contained in the walls.

$$\mathcal{I}(\lambda, T) = \frac{2\pi c^2 h}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1}$$ Planck's radiation law

- **This formula/result flies in the face of classical physics because**:
  1) The oscillators (of electromagnetic origin) can only have certain discrete energies determined by $E_n = nhf$, where n is an integer, f is the frequency, and h is now called Planck's constant.
  $h = 6.6261 \times 10^{-34}$ J·s (Ws^2)
  2) The oscillators can absorb or emit energy only in discrete multiples of the quanta of energy given by $$\Delta E = hf$$

and a Nobel Prize for M. Planck, 1918

a hint on quantized energy in atoms

How does this sound to a classical physicist?

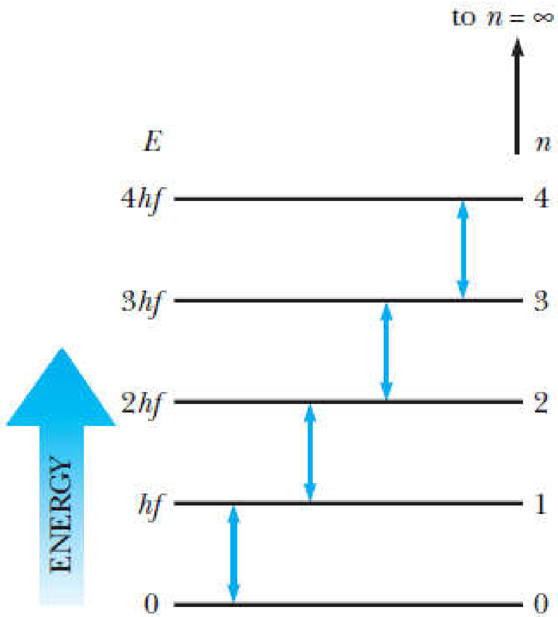
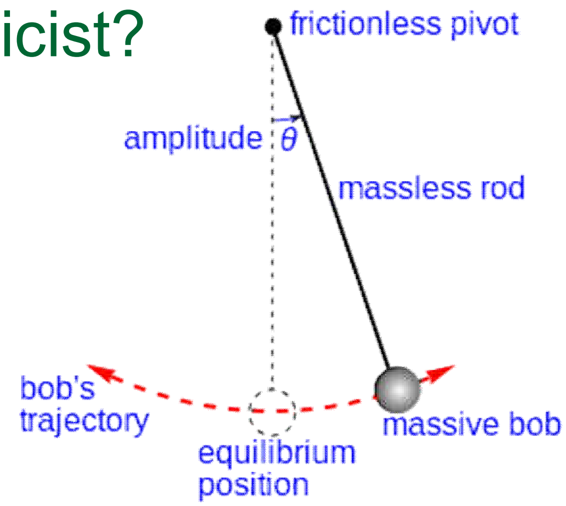


Figure 3.9 Allowed energy levels according to Planck's original hypothesis for an oscillator with frequency *f*. Allowed transitions are indicated by the double-headed arrows.



Approx. simple harmonic motion for small swings, g is a "constant" of the bob Earth (bound) system, mechanical energy can have any value

Transitions in energy of a harmonic quantum oscillator are just E = hf, this assumption will be confirmed by standard 3D Quantum Mechanics at the end of this course. Planck didn't know about zero point energy and Heisenberg's uncertainty principle, so his zero level is off.

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4 + \cdots\right)$$

$$T \approx 2\pi\sqrt{\frac{L}{g}} \qquad \theta_0 \ll 1 \quad \text{rad}$$

Versions of Planck's black body radiation law

$$u(\lambda,T)=\frac{8\pi hc}{\lambda^5\exp\left(\frac{hc}{\lambda k_B T}\right)-1}$$

$$u(\lambda,\, T)\, d\lambda = \frac{8\pi hc\; d\lambda}{\lambda^5\left(e^{hc/\lambda k_B T}-1\right)}$$

unit of energy/volume

Conversion factor = $^{c}/_{4}$

$$u(f,T)=\frac{8\pi hf}{\lambda^4\exp\left(\frac{hf}{k_B T}\right)-1}$$

$$\mathcal{I}(\lambda,T)=\frac{2\pi c^2 h}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1}$$

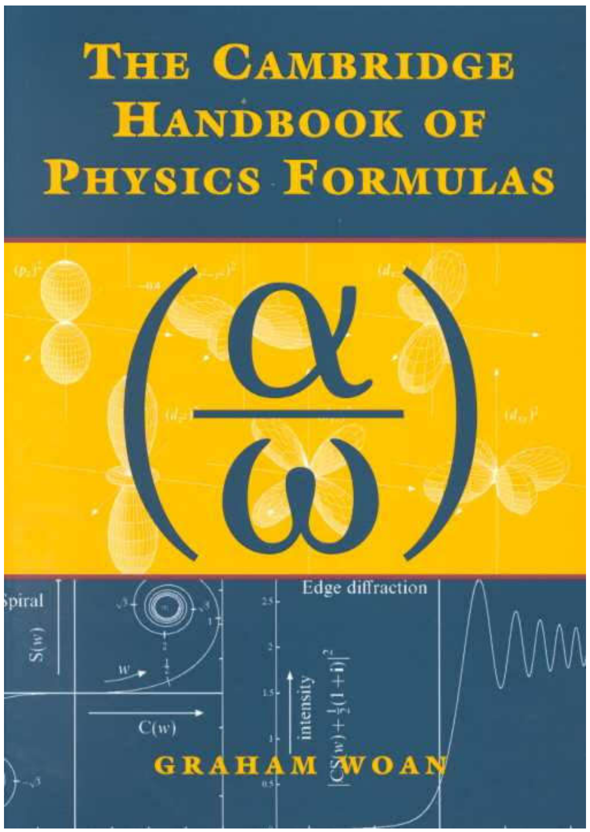
Watt per area

$$u(f,T)=\frac{8\pi\cdot E_{photons}}{\lambda^4\exp\left(\frac{E_{photons}}{E_{thermal}}\right)-1}$$

$$\mathcal{I}(\lambda,T)= {}^{4}/_{c}\; u(\lambda,T)$$

In Serway et al. and other books u(f,T) is defined differently, but also correctly

$$u(f,T)\,df = \frac{8\pi f^2}{c^3}\left(\frac{hf}{e^{hf/k_B T}-1}\right)df$$



$$B_\nu(T) = \frac{2h\nu^3}{c^2}\{\exp(\frac{h\nu}{k_B}) - 1\}^{-1}$$

$$B_\lambda(T) = B_\nu(T)\frac{d\nu}{d\lambda}$$

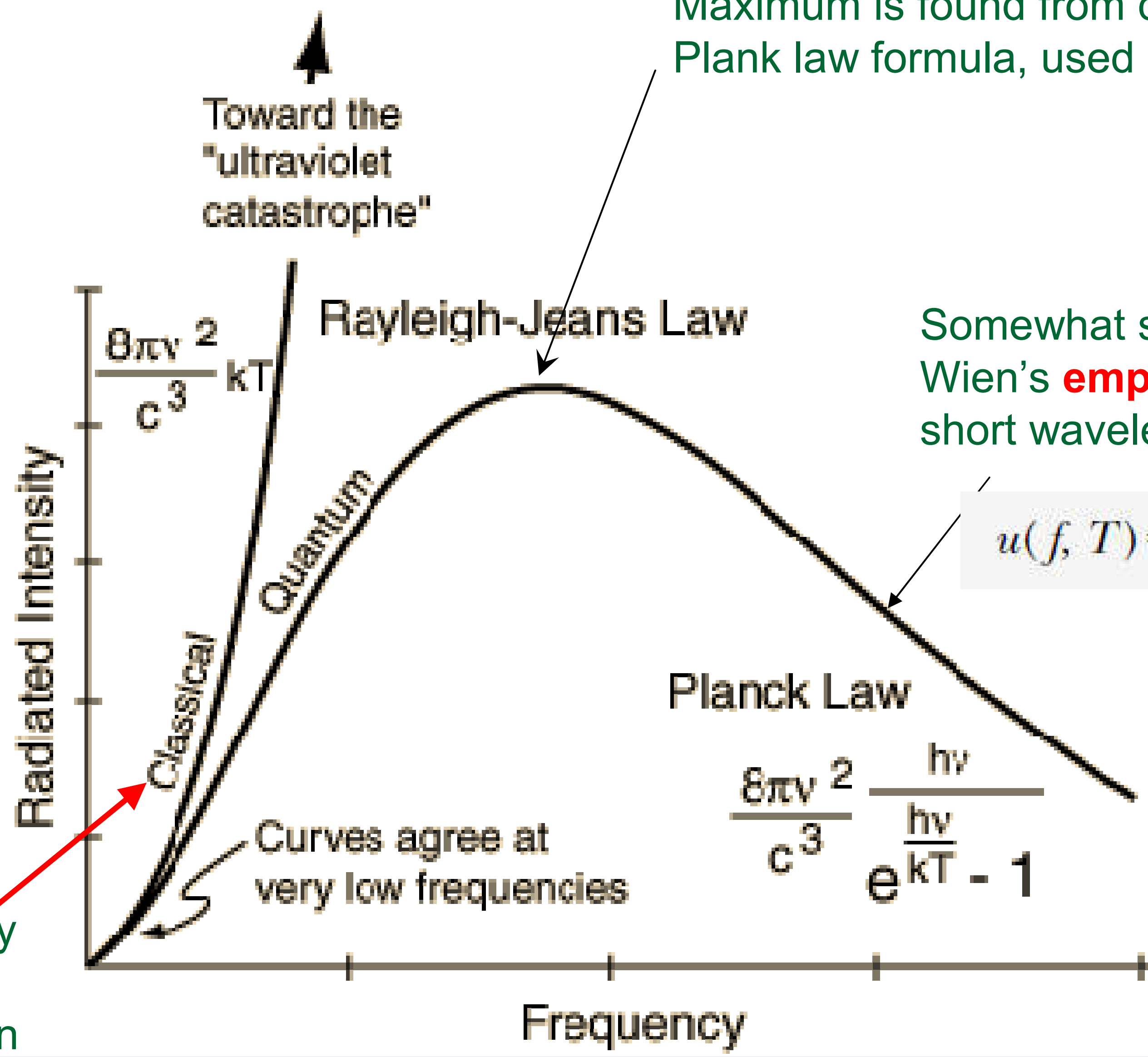
$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\{\exp(\frac{hc}{\lambda k_B T}) - 1\}^{-1}$$

Surface brightness per unit frequency (Greek symbol Ny) or wavelength (λ) with respect to the projected area of the surface, again a conversion factor $^4/_c$ can be applied – as in next slide

Maximum is found from derivative of the Plank law formula, used in pyrometers

Somewhat similar to Wien's **empirical** formula, short wavelengths

$$u(f, T) = Af^3 e^{-\beta f/T}$$



Assumes light is a only a wave phenomenon

ν is frequency, also f in many books

Energy density as function of frequency and temperature

$$u(f,T)=\frac{8\pi\cdot hf}{\lambda^4}\cdot\frac{1}{\exp{}^{hf}\!/\!_{k_BT}-1}\approx\frac{8\pi\cdot hf}{\lambda^4}\cdot\frac{1}{1+({}^{hf}\!/\!_{k_BT})-1}\approx\frac{8\pi\cdot k_BT}{\lambda^4}\cdot\frac{hf}{hf}$$

$$u(f,T)_{R-J}=\frac{8\pi kT}{\lambda^4}$$

Rayleigh-Jeans formula from classical physics, for hf << kT

exp(x) ≈ 1 + x for very small x, i.e. when h → 0, d. h. classical physics (also ***for f very small*** for any T)

Small frequency means large wavelength, behave more like a wave, high frequency means it does **not** behave like a wave (it needed an Einstein to figure out what is going on)

h = 4.135667 10^{-15} eV so rather small but not zero, we do not live in a classic physics world although that is all we can comprehend directly

Total energy for a photon: E = h f, but it takes a Genius (A. Einstein) to realize that. Everybody else cannot make sense of Planck’s assumption of quantization of the energy levels of his “resonators” at that time. Note that Rayleigh-Jeans formula is given correctly only in 1905

$$u(f, T) = \frac{8\pi h f^3}{c^3}\left(\frac{1}{e^{hf/k_B T} - 1}\right)$$

if ***f*** is very large, wavelength very short (for any kind of T), the exponential is really large, so that the -1 term might be neglected to a good approximation

$$u(f, T) = A f^3 e^{-\beta f/T}$$

Wien's empirical law with two fitting parameters somewhat similar results, for very energetic photons, which behave much like classical particles, classical physics, for hf >> kT

So the Planck formula get both regions, short wavelengths (high frequencies) and large wavelengths (small frequencies) right !!

Einstein saw the deeper implications of the peculiar form of the Energy density (or Intensity per area) versus wavelength curve first in 1905

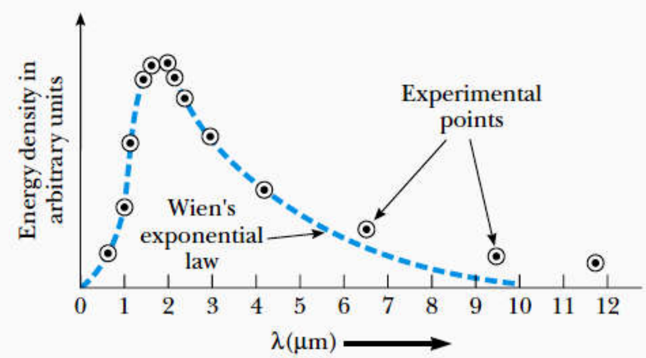


Figure 3.5 Discrepancy between Wien's law and experimental data for a blackbody at 1500 K.

Removing the -1 changes the distribution function for bosons to classical particles

EXAMPLE 3.3 Derivation of Stefan's Law from the Planck Distribution

In this example, we show that the Planck spectral distribution formula leads to the experimentally observed Stefan law for the total radiation emitted by a blackbody at all wavelengths,

$$e_{\text{total}} = 5.67 \times 10^{-8}\, T^4\ \text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-4}$$

Solution Since Stefan's law is an expression for the total power per unit area radiated at all wavelengths, we must integrate the expression for $u(\lambda, T)\,d\lambda$ given by Equation 3.20 over λ and use Equation 3.7 for the connection between the energy density inside the blackbody cavity and the power emitted per unit area of blackbody surface. We find

$$e_{\text{total}} = \frac{c}{4}\int_0^\infty u(\lambda, T)\,d\lambda = \int_0^\infty \frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda k_B T} - 1)}\,d\lambda$$

If we make the change of variable $x = hc/\lambda k_B T$, the integral assumes a form commonly found in tables:

$$e_{\text{total}} = \frac{2\pi k_B^4 T^4}{c^2h^3}\int_0^\infty \frac{x^3}{(e^x - 1)}\,dx$$

Using

$$\int_0^\infty \frac{x^3}{(e^x - 1)}\,dx = \frac{\pi^4}{15}$$

we find

$$e_{\text{total}} = \frac{2\pi^5 k_B^4}{15c^2h^3}\,T^4 = \sigma T^4$$

Finally, substituting for k_B, c, and h, we have

$$\sigma = \frac{(2)(3.141)^5(1.381 \times 10^{-23}\ \text{J/K})^4}{(15)\,(2.998 \times 10^8\ \text{m/s})^2\,(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})^3}$$
$$= 5.67 \times 10^{-8}\ \text{W}\cdot\text{m}^{-2}\cdot\text{K}^{-4}$$

N.B. there are only a very few fundamental constants, sure Stefan's constant was not one of them, k_B , c, and h are, loosely speaking these three constants are about the thermal behavior of particles, travel of light waves, and "strength" of the wave – particle coupling / duality

$$u(\lambda,T)=\frac{8\pi hc}{\lambda^5\exp\left(\frac{hc}{\lambda k_B T}\right)-1}$$

Einstein realized that Planck's formula describes something never before "seen" in physics, at very small wavelengths, we have "particle like" behavior, at very large wavelengths, we have "wave like" behavior

So light shows wave – particle duality properties, but is neither a classical wave nor a classical particle, it's a quantum mechanical entity,

Light is a stream of "mass less" particles, which are allowed by special relativity, that each individually possess total energy E = h f, which is all kinetic, rushing around with the speed of light for any observer (in an inertial frame of reference, with general relativity in any frame of reference)! Isn't that exiting?? Accelerator equation for mass less particles E = pc, with

c = λ f, we get p = h/λ as "hallmark" of wave-particle duality

$$E^2=(pc)^2+(m_0c^2)^2$$

$$E^2 = (pc)^2 + (m_0 c^2)^2$$

How come that photons do not have rest-mass? Their total energy is E = hf, with c = λf

For particles with rest mass

$$m_0 = \frac{1}{c^2}\sqrt{(\gamma m_0 c)^2 - (\gamma m_0 v)^2 c^2}$$

Because v < c, there is always a positive value, for rest-mass for relativistic particles other than light photons

For mass-less particles

$$m_0^{light?} = \frac{1}{c^2}\sqrt{(hf)^2 - (\frac{h}{\lambda})^2 c^2} = \frac{1}{c^2}\sqrt{(hf)^2 - (\frac{hf}{c})^2 c^2} = 0$$

Because $\lambda = {}^{c}/_{f}$

We have **two new equations for the total energy (which is all kinetic) and the momentum of a photon** from Einstein's realization that light particles jump around at the speed of light in discrete bundles with well defined energy and momentum, but no mass

Mass is not a continuum either, it's a bit like coins, 1¢, 2¢, 5¢, 10¢, 25¢ coins, but not 31¢ coins (but there is also binding energy when elemental particles stick together)

Charge of the (free) elemental particles, electrons, protons, positrons, … come in units of the elemental charge. So they are quantized in integers (oil drop experiment).

http://usatoday30.usatoday.com/tech/science/genetics/2008-05-08-platypus-genetic-map_N.htm

“Australia's unique duck-billed platypus is part bird, part reptile and part mammal according to its gene map.

The platypus is classed as a mammal because it has fur and feeds its young with milk, ***but they hatch from eggs***. It flaps a beaver-like tail. But it also has bird and reptile features — a duck-like bill and webbed feet, and lives mostly underwater. Males have venom-filled spurs on their heels.”

The physicist Max Born, an important contributor to the foundations of quantum theory, had this to say about the particle–wave dilemma:

> The ultimate origin of the difficulty lies in the fact (or philosophical principle) that we are compelled to use the words of common language when we wish to describe a phenomenon, not by logical or mathematical analysis, but by a picture appealing to the imagination. Common language has grown by everyday experience and can never surpass these limits. Classical physics has restricted itself to the use of concepts of this kind; by analyzing visible motions it has developed two ways of representing them by elementary processes: moving particles and waves. There is no other way of giving a pictorial description of motions—we have to apply it even in the region of atomic processes, where classical physics breaks down.
>
> Every process can be interpreted either in terms of corpuscles or in terms of waves, but on the other hand it is beyond our power to produce proof that it is actually corpuscles or waves with which we are dealing, for we cannot simultaneously determine all the other properties which are distinctive of a corpuscle or of a wave, as the case may be. We can therefore say that the wave and corpuscular descriptions are only to be regarded as complementary ways of viewing one and the same objective process, a process which only in definite limiting cases admits of complete pictorial interpretation.[16]

3.6: Photoelectric Effect

Methods of electron emission:

- Thermionic emission: Application of heat allows electrons to gain enough energy to escape.
- Secondary emission: The electron gains enough energy by transfer from another high-speed particle that strikes the material from outside.
- Field emission: A strong external electric field pulls the electron out of the material.
- **Photoelectric effect:** Incident light (electromagnetic radiation) shining on the material transfers energy to the electrons, allowing them to escape.

Electromagnetic radiation interacts with electrons within metals and gives the electrons increased kinetic energy. Light can give electrons enough extra kinetic energy to allow them to escape. We call the ejected electrons **photoelectrons**.

Experimental Setup

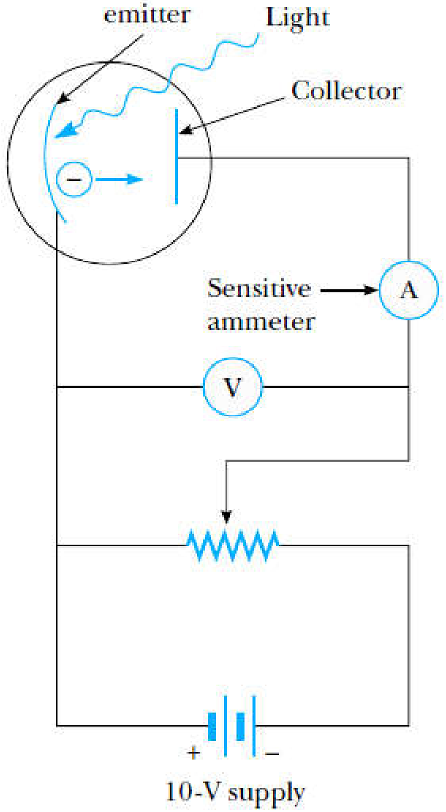


Figure 3.14 Photoelectric effect apparatus.

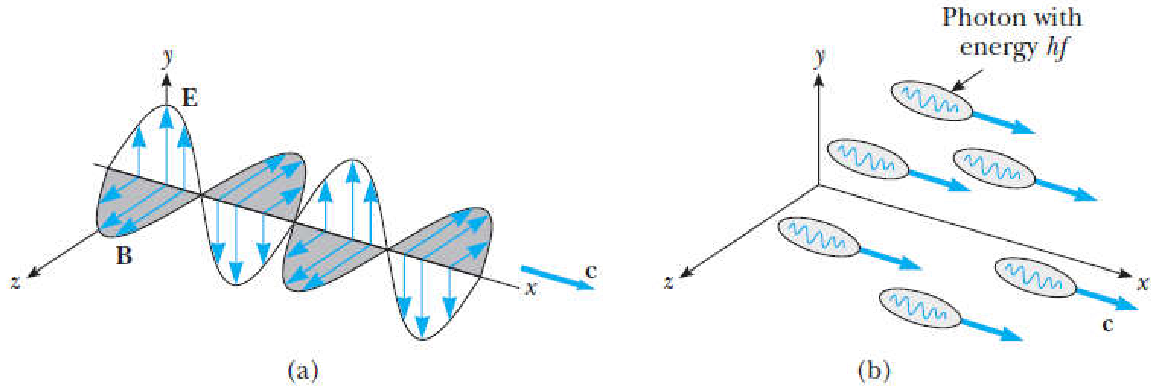


Figure 3.16 (a) A classical view of a traveling light wave. (b) Einstein's photon picture of "a traveling light wave."

variable voltage, polarity such that negative charge is applied to the collector, stopping the electrons = stopping (retarding) potential

Table 3.1 Work Functions of Selected Metals

Metal	Work Function, ϕ, (in eV)
Na	2.28
Al	4.08
Cu	4.70
Zn	4.31
Ag	4.73
Pt	6.35
Pb	4.14
Fe	4.50

How much energy is needed to get electrons out of a metal

http://www.walter-fendt.de/ph14e/photoeffect.htm

Experimental Results

1) **Only sufficiently energetic light makes an effect, there can be an enormous intensity of light that is not sufficiently energetic and no effect is observed**
2) The kinetic energies of the photoelectrons are independent of the light intensity.
3) The maximum kinetic energy of the photoelectrons, for a given emitting material, depends only on the frequency of the light.
4) The smaller the work function φ of the emitter material, the smaller is the threshold frequency of the light that can eject photoelectrons.
5) When the photoelectrons are produced, however, their number is proportional to the intensity of light (electronic light intensity measurement are based on that).
6) The photoelectrons are emitted almost instantly following illumination of the photocathode, independent of the intensity of the light.

http://lectureonline.cl.msu.edu/~mmp/kap28/PhotoEffect/photo.htm

Experimental Results

Same emitter material (V_0), same kind of light (f or ν) with 1, 2, and 3 fold intensity, **photocurrent intensity depends on incoming light intensity if sufficiently energetic light *hf* has been used**

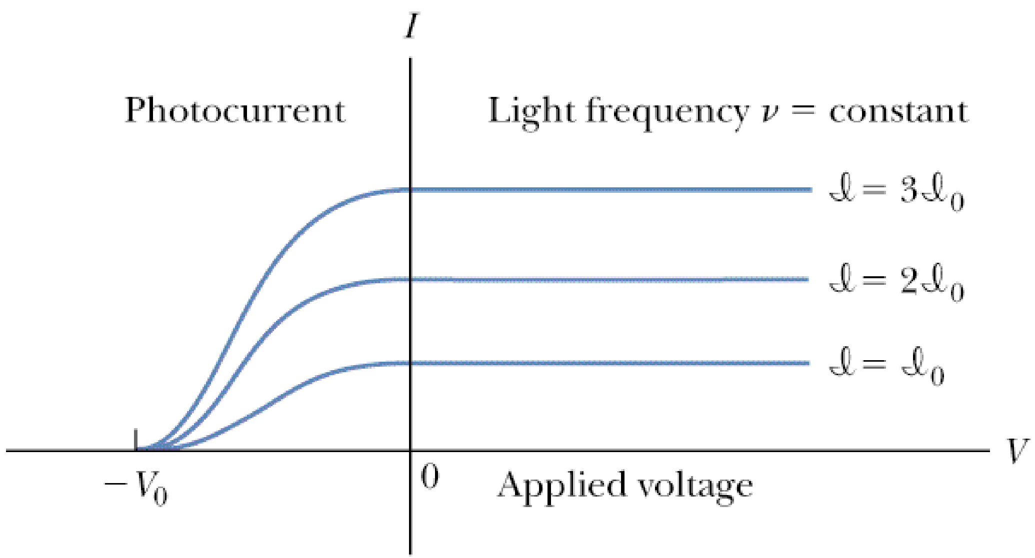
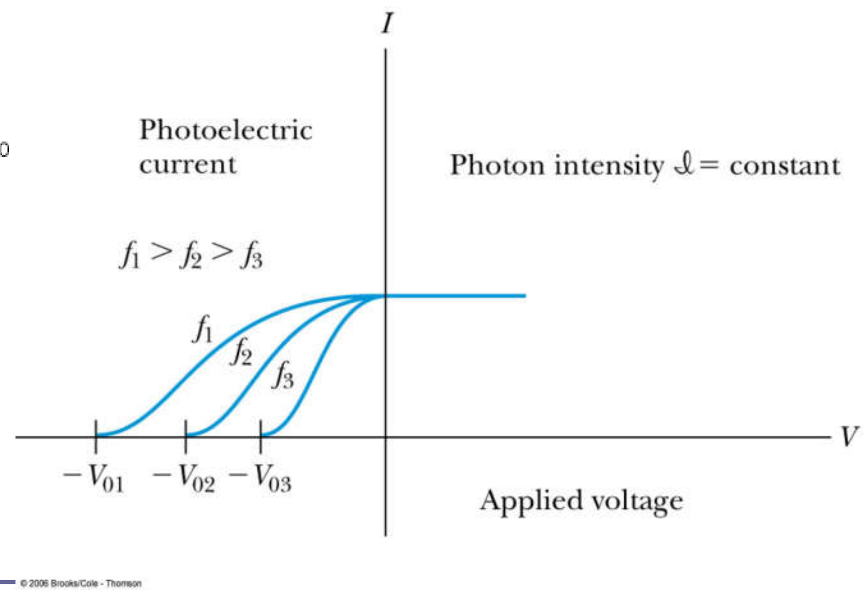


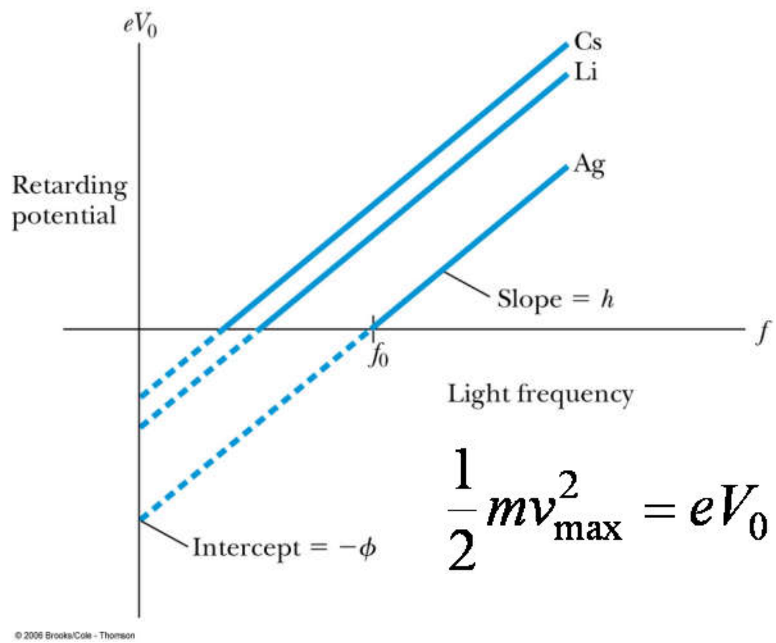
Table 3-1 Photoelectric work functions

Element	Work function (eV)
Na	2.28
C	4.81
Cd	4.07
Al	4.08
Ag	4.73
Pt	6.35
Mg	3.68
Ni	5.01
Se	5.11
Pb	4.14

Three different emitter materials, three different kinds of light (f) are required to get a photocurrent for the same intensity of the incoming light



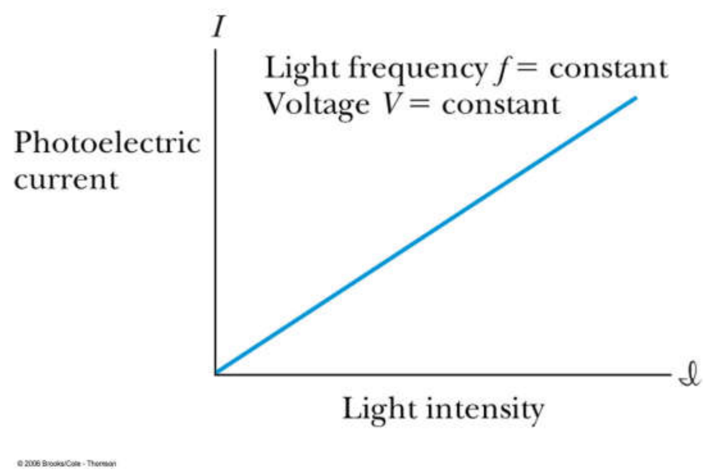
Experimental Results



$$\frac{1}{2}mv_{\max}^2 = eV_0 = hf - \phi$$

Different materials have different work functions, i.e. amounts of energy required to allow an electron to escape a metal block, note that the slope of all of these curves is just ***h***

Analogous to first graph from pervious slide (which showed just three different values of intensities of the incoming light, Photocurrent is linear function of light intensity if it is sufficiently energetic to overcome the work function (stopping potential)



https://arxiv.org/abs/1909.06286

Submitted on 13 Sep 2019

High Precision Determination of the Planck Constant by Modern Photoemission Spectroscopy

Jianwei Huang, Dingsong Wu, Yongqing Cai, Yu Xu, Cong Li, Qiang Gao, Lin Zhao, Guodong Liu, Zuyan Xu, X. J. Zhou

The Planck constant, with its mathematical symbol *h*, is a fundamental constant in quantum mechanics that is associated with the quantization of light and matter. It is also of fundamental importance to metrology, such as the definition of ohm and volt, and the latest definition of kilogram. … Through the direct use of the Einstein's photoelectric equation, the Planck constant is determined by measuring accurately … using light sources with various photon wavelengths. The precision of the measured Planck constant, $6.62610(13)\times10^{-34}$ J·s, is four to five orders of magnitude improved from the previous photoelectric effect measurements…

In 2020, the numerical value of the Planck constant has been *fixed, with finite significant figures*, just like the speed of light and the second before.

The kilogram has also been redefined. Under the present technical definition of the kilogram: one kilogram is now defined by taking the fixed numerical value of *h* to be $6.62607015 \times 10^{-34}$ when expressed in the unit J·s, and the equivalency of $E = mc^2$ and $E = hf$ for photons into account. A certain amount of photons are equivalent to rest mass of 1 kg. Future refinements will be in the number of photons and Avogadro's number (and the chemical mole) as *h* and *c* are now fixed.

Attempt of a classical interpretation

- The existence of a threshold frequency is completely inexplicable in classical theory.
- Classical theory predicts that the total amount of energy in a light wave increases as the light intensity increases.
- Classical theory would predict that for extremely low light intensities, a long time would elapse before any one electron could obtain sufficient energy to escape. We observe, however, that the photoelectrons are ejected almost immediately.
- Conclusion, light can not be a Maxwellian wave, must be something else, a stream of particles (as though through by Newton in deriving geometric (ray-) optic
- On the other hand, light shows typical wave properties, i.e. interference, so what light seems to be depends on the experiment?

Einstein’s Interpretation

- Einstein suggested that the electromagnetic radiation field is quantized into particles called **photons**. Each photon possesses the quantum of total energy (which is all kinetic as they do not have mass):

$$E = hf$$

  where *f* is the frequency of the light and *h* is Planck’s constant.

- The photon travels at the speed of light in a vacuum, defining its wavelength and frequency $\lambda f = c$

Einstein's analysis

- Conservation of total energy yields:

$$\text{Energy before (photon)} = \text{energy after (electron)}$$
$$hf = \phi + \text{K.E. (electron)}$$

where ϕ is the work function of the metal.

Explicitly the energy is (as v is small, no special relativity treatment)

$$hf = \phi + \frac{1}{2}mv_{\max}^2$$

- The retarding (stopping) potentials measured in the photoelectric effect are the opposing potentials needed to stop the most energetic electrons. $eV_0 = \frac{1}{2}mv_{\max}^2$

Light Quantum Theory Interpretation

- The kinetic energy of the electron does not depend on the light intensity at all, but only on the light frequency and the work function of the material.

$$\frac{1}{2}mv_{\max}^2 = eV_0 = hf - \phi$$

- Einstein in 1905 predicted that the stopping potential was linearly proportional to the light frequency, with a slope *h*, the same constant found by Planck.

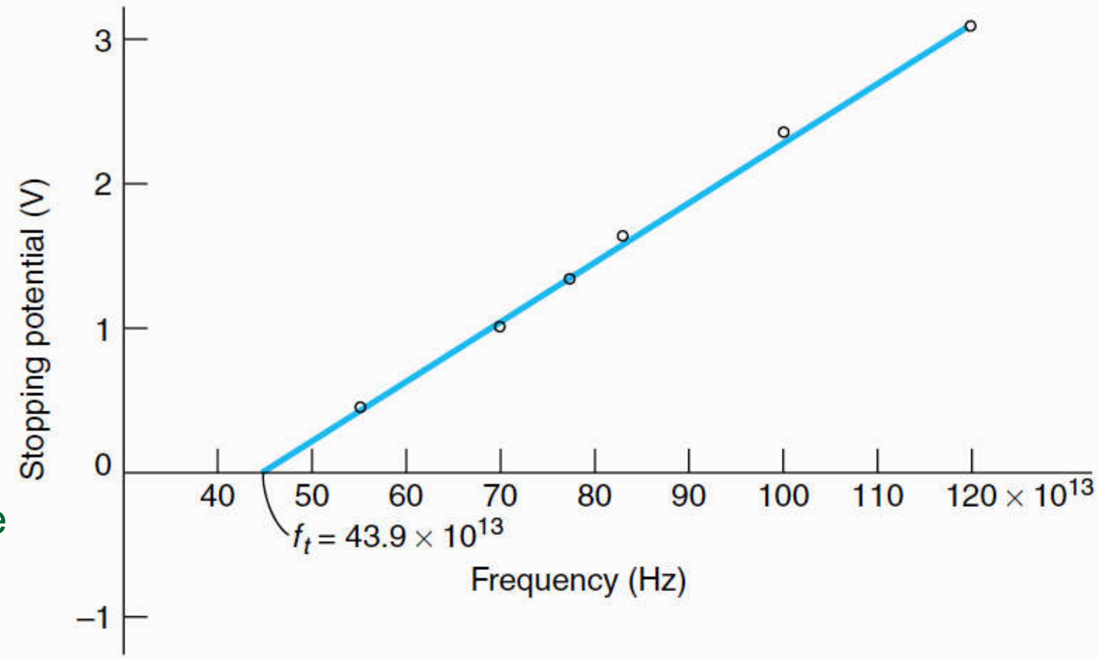
$$eV_0 = \frac{1}{2}mv_{\max}^2 = hf - hf_0 = h(f - f_0)$$

- From this, Einstein concluded that light consists of particles with energy:

$$E = hf = \frac{hc}{\lambda}$$

a Nobel Prize for A. Einstein, 1921 (none for his other great achievements as people had difficulty with special/general relativity and its experimental verification for a long time)

Figure 3-10 Millikan's data for stopping potential versus frequency for the photoelectric effect. The data fall on a straight line with slope h/e, as predicted by Einstein a decade before the experiment. The intercept on the stopping potential axis is $-\phi/e$. [*R. A. Millikan, Physical Review, 7, 362 (1915).*]



When Millikan's Nobel Prize came to pass (1923), his Nobel address contained passages that showed his continuing struggle with the meaning of his own achievement: ***"This work resulted, contrary to my own expectation, in the first direct experimental proof…of the Einstein equation and the first direct photo-electric determination of Planck's h."*** (Nobel prize for precision measurements of both elemental charge and ***h***)

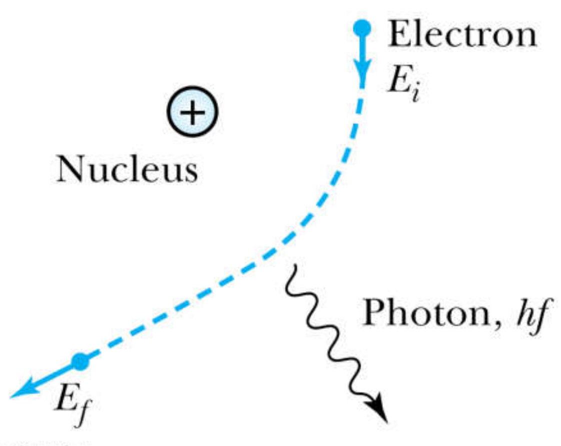
Millikan himself on a more informal occasion: "I spent ten years of my life testing that 1905 equation of Einstein's and contrary to all my expectations, I was compelled in 1915 to assert its unambiguous verification in spite of its unreasonableness since it seemed to violate everything we knew about the interference of light.

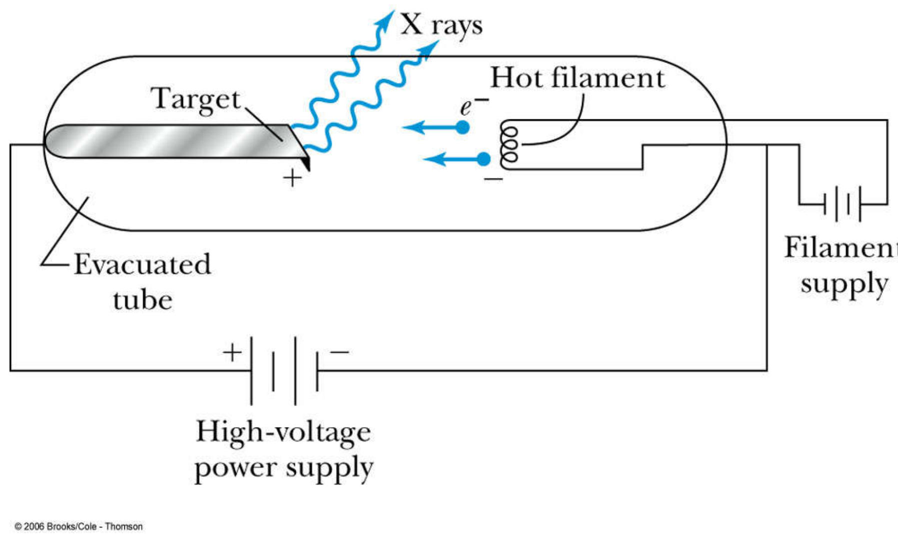
"All this nonsense makes no sense!" Millikan_Ariel in The Blegdamsvej Faust

3.7: X-Ray Production

- An energetic electron passing through matter will radiate photons and lose kinetic energy which is called **bremsstrahlung**, from the German word for “braking radiation.” Linear momentum must be conserved, but a nucleus of an atom absorbs very little energy, and it is ignored. The final energy of the electron is determined from the conservation of energy to be $E_f = E_i - hf$

- An electron that loses a large amount of energy will produce an X-ray photon of very short wavelength. Current passing through a filament produces copious numbers of electrons by thermionic emission. These electrons are focused by the cathode structure into a beam and are accelerated by potential differences of tens of thousands of volts until they impinge on a metal anode surface, producing x-rays by bremsstrahlung as they are deflected in the anode material.





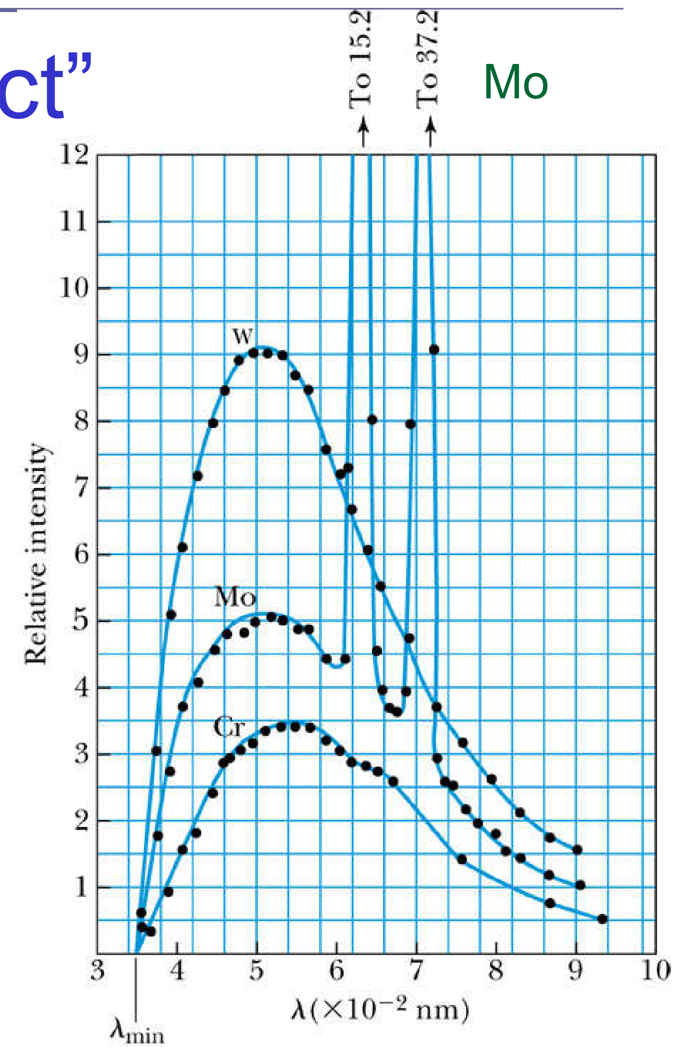
"Inverse Photoelectric Effect"

- Conservation of energy requires that the electron kinetic energy equal the maximum photon energy where we neglect the work function because it is very small. This yields the **Duane-Hunt limit** which was first found experimentally. The photon wavelength depends only on the accelerating voltage V_0 and is the same for all targets.

$$eV_0 = hf_{\max} = \frac{hc}{\lambda_{\min}}$$

$$\lambda_{\min} = \frac{hc}{e}\frac{1}{V_0} = \frac{1.240\times10^{-6}\ \mathrm{V\cdot m}}{V_0}$$

Mo



The really interesting bits are the intense and narrow "X-ray lines", which are characteristic of the anode material and are called **characteristic X-rays, one can use them either to identify the material in an electron microscope or use them for quasi-monochromatic X-ray diffraction experiments**

Duane-Hunt rule,

$$\lambda_m = \frac{1.24 \times 10^3}{V} \text{ nm}$$

the value of λ_m can be used to determine h/e

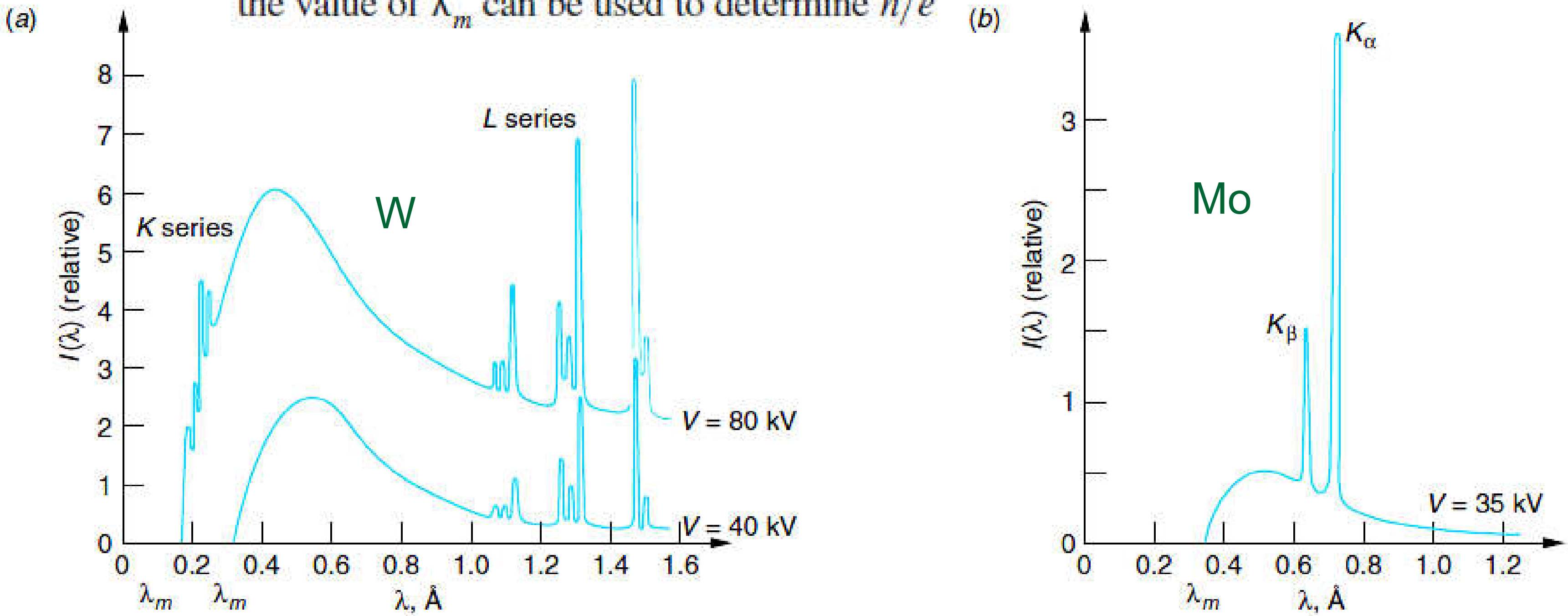
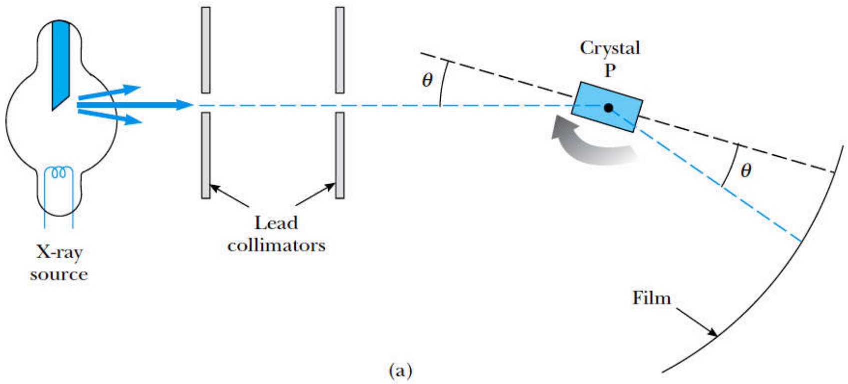


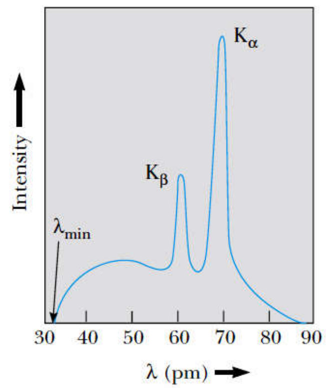
Figure 3-15 (*a*) X-ray spectra from tungsten at two accelerating voltages and (*b*) from molybdenum at one. The names of the line series (*K* and *L*) are historical and explained in Chapter 4. The *L*-series lines for molybdenum (not shown) are at about 0.5 nm. The cutoff wavelength λ_m is independent of the target element and is related to the voltage on the x-ray tube V by $\lambda_m = hc/eV$. The wavelengths of the lines are characteristic of the element.



(a)

$$2d \sin\theta = m\lambda \quad \text{where } m = \text{an integer}$$

Because of its high intensity, it makes sense to use only K_α X-rays (nearly monochromatic) for X-ray crystallography



(b)

Figure 3.21 (a) A Bragg crystal x-ray spectrometer. The crystal is rotated about an axis through *P*. (b) The x-ray spectrum of a metal target consists of a broad, continuous spectrum plus a number of sharp lines, which are due to the characteristic x-rays. Those shown were obtained when 35-keV electrons bombarded a molybdenum target. Note that 1 pm = 10^{-12} m = 10^{-3} nm.

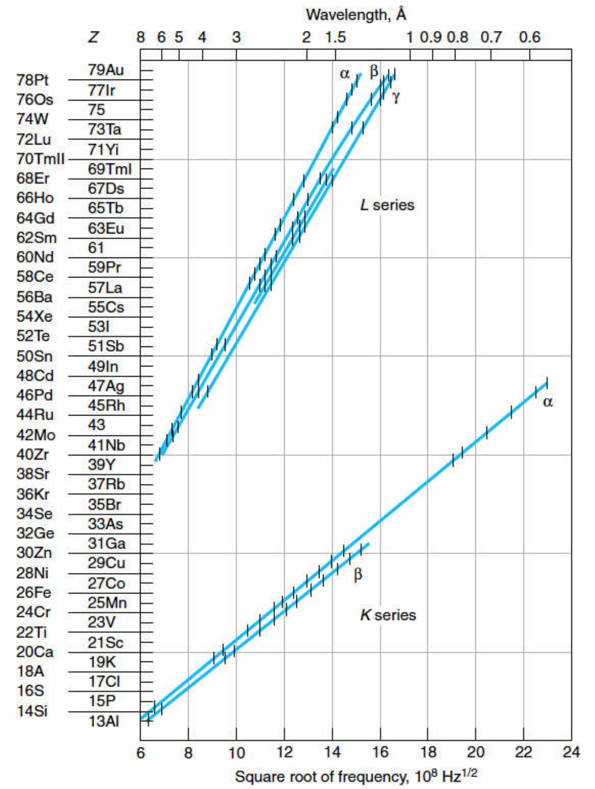


Figure 4-19 Moseley's plots of the square root of frequency versus Z for characteristic x rays. When an atom is bombarded by high-energy electrons, an inner atomic electron is sometimes knocked out, leaving a vacancy in the inner shell. The K-series x rays are produced by atomic transitions to vacancies in the $n = 1$ (K) shell, whereas the L series is produced by transitions to the vacancies in the $n = 2$ (L) shell. [*From H. Moseley, Philosophical Magazine (6), 27, 713 (1914)*]

These curves can be fitted by the empirical equation

$$f^{1/2} = A_n(Z - b) \qquad \textbf{4-34}$$

where A_n and b are constants for each characteristic x-ray line. One family of lines, called the *K series*, has $b = 1$ and slightly different values of A_n for each line in the graph. The other family shown in Figure 4-19, called the *L series*,[17] could be fitted by Equation 4-34 with $b = 7.4$.

Any good theory of matter (atoms) needs to explain this peculiar behavior !! next chapter

Apparent gaps in the periodic table of elements

(a)

(b)

(c)

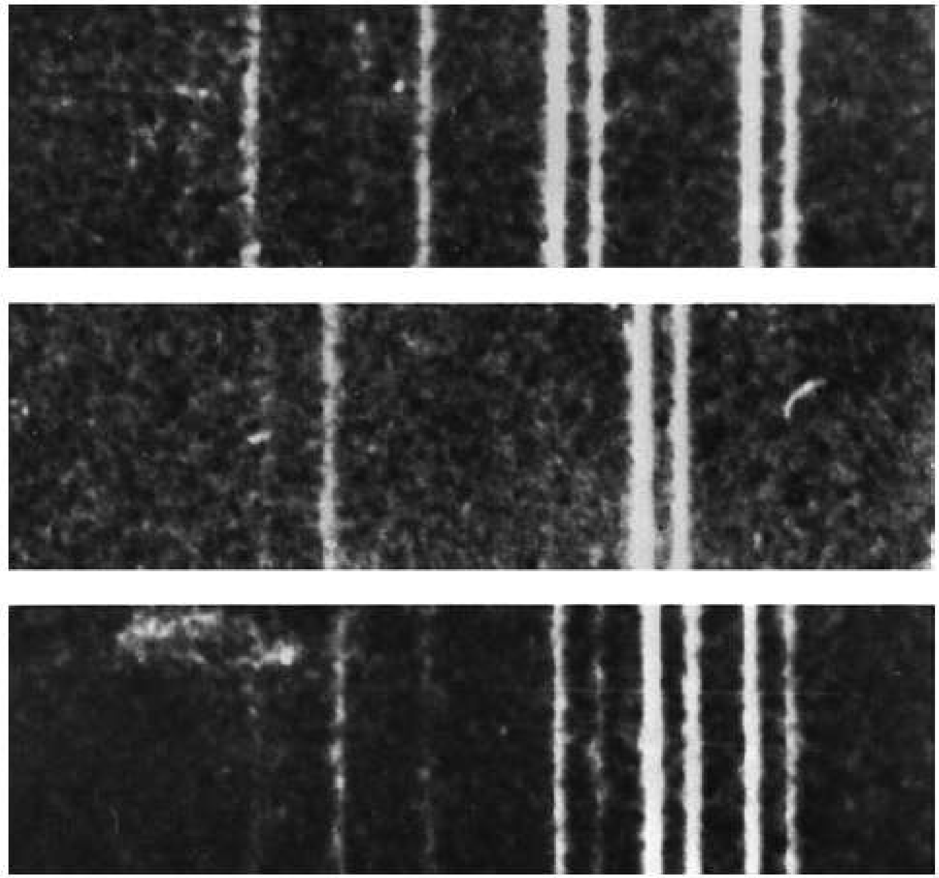


Figure 4-20 Characteristic x-ray spectra. (*a*) Part of the spectra of neodymium ($Z = 60$) and samarium ($Z = 62$). The two pairs of bright lines are the K_α and K_β lines. (*b*) Part of the spectrum of the artificially element promethium ($Z = 61$). This element was first positively identified in 1945 at the Clinton Laboratory (now Oak Ridge). Its K_α and K_β lines fall between those of neodymium and samarium, just as Moseley predicted. (*c*) Part of the spectra of all three of the elements neodymium, promethium, and samarium. [*Courtesy of J. A. Swartout, Oak Ridge National Laboratory.*]

3.8: Compton Effect

as we have seen from Millikan’s statements, the physics establishment was baffled by Einstein’s wave-particle duality ideas for light, i.e. light itself is quantized, not only the energy in a harmonic oscillator as needed by Planck in order to derive the right kind of formula for black-body radiation (where he obtained a first value of h from a simple curve fit)

The photoelectric effect would only give us the maximal kinetic energy of the photoelectrons, a more direct proof that light is pure kinetic energy of certain sizes was needed !!!

i.e. a direct collision experiment of sufficiently energetic light particles (best with very high intensity to simplify the detection) with some small particle that possesses mass, so let’s take characteristic X-rays and electrons

Classical physics prediction: loosely speaking: the E vector would pick the electron up, shake it a bit and a new wave of longer wavelengths would be created that radiates outwards in a spherical manner, i.e. no directional difference whatsoever

We cannot keep track of the electron, so no way to measure Φ, but we can do spectroscopy on the inelastically scattered X-rays as a function of angle θ

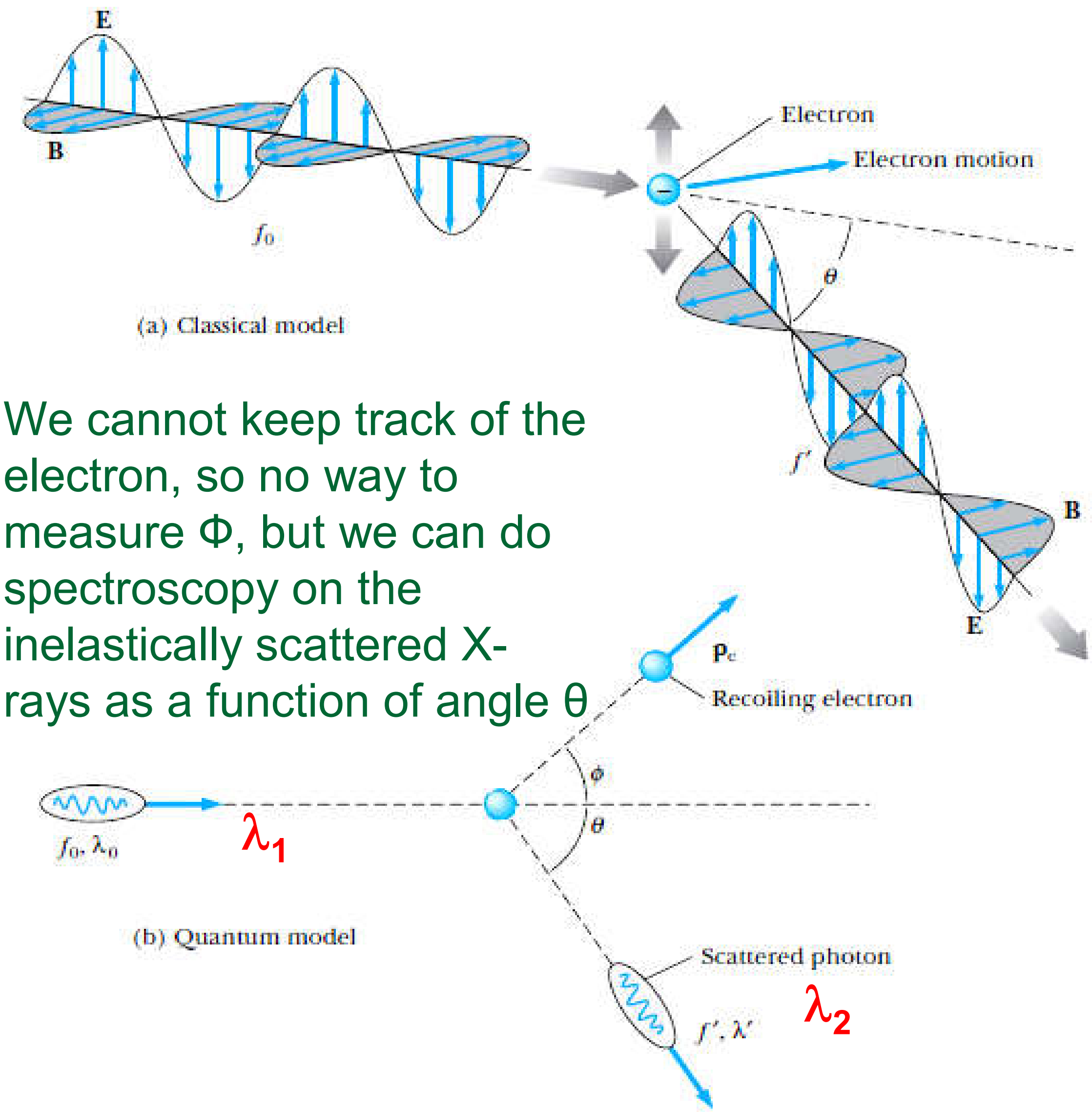


Figure 3.22 X-ray scattering from an electron: (a) the classical model, (b) the quantum model.

If light has also particle properties (in addition to being a Maxwellian wave), a photon should collide with the electron, total energy and momentum needs to be conserved, special relativity needs to be involved as the "light particle" moves with the speed of light

Derivation of Compton's Equation, applying conservation of energy and momentum to the relativistic collision of a photon and an electron, is included on the home page: www.whfreeman.com/tiplermodern physics5e. See also Equations 3-26 and 3-27 and Figure 3-18 here.

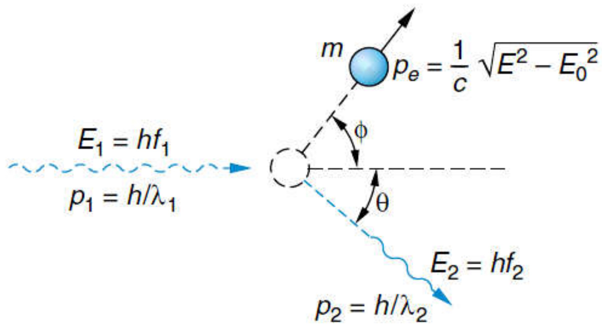


Fig. 3-21 The scattering of x rays can be treated as a collision of a photon of initial momentum h/λ_1 and a free electron. Using conservation of momentum and energy, the momentum of the scattered photon h/λ_2 can be related to the initial momentum, the electron mass, and the scattering angle. The resulting Compton equation for the change in the wavelength of the x ray is Equation 3-40.

Let λ_1 and λ_2 be the wavelengths of the incident and scattered x rays, respectively, as shown in Figure 3-21. The corresponding momenta are

according to special relativity

$$p_1 = \frac{E_1}{c} = \frac{hf_1}{c} = \frac{h}{\lambda_1}$$

and **3-41**

$$p_2 = \frac{E_2}{c} = \frac{h}{\lambda_2}$$

carbon electron can be considered to be free.

as their binding energy, a few eV

is much less than the energy of the incoming photon

17.4 keV

Conservation of momentum gives

3-41 $$\mathbf{p}_1 = \mathbf{p}_2 + \mathbf{p}_e$$

$$p_e^2 = p_1^2 + p_2^2 - 2\mathbf{p}_1 \cdot \mathbf{p}_2$$
$$= p_1^2 + p_2^2 - 2p_1 p_2 \cos\theta$$

where p_e is the momentum of the electron after the collision and θ is the scattering angle for the photon, measured as shown in Figure 3-21. The energy of the electron before the collision is simply its rest energy $E_0 = mc^2$ (see Chapter 2). After the collision, the energy of the electron is $(E_0^2 + p_e^2c^2)^{1/2}$. accelerator equation of special relativity

Conservation of energy gives

$$p_1c + E_0 = p_2c + (E_0^2 + p_e^2c^2)^{1/2}$$

New left side only as right side is simple, then cancelling E_0 dividing by c^2 and resolving for p_e^2, whereby

Transposing the term p_2c and squaring, we obtain

$$E_0^2 + c^2(p_1 - p_2)^2 + 2cE_0(p_1 - p_2) = E_0^2 + p_e^2c^2$$

or

3-42 $$p_e^2 = p_1^2 + p_2^2 - 2p_1p_2 + \frac{2E_0(p_1 - p_2)}{c}$$

$$(p_1c - p_2c + E_0)^2 = \{c(p_1 - p_2) + E_0\}^2 =$$
$$c^2(p_1 - p_2)^2 + 2c(p_1 - p_2)E_0 + E_0{}^2$$

and

$$(p_1 - p_2)^2 = p_1{}^2 - 2p_1p_2 + p_2{}^2$$

Note that we have two independent equations for p_e^2, by using them both, we get rid of p_1^2 and p_2^2 and gain an interesting cos θ term, this also clarifies that the result will be independent on the initial wavelength of the light, we will just derive a change in wavelengths that is "hiding" in the products of p_1p_2

If we eliminate p_e^2 from Equations 3-41 and 3-42, we obtain

$$\frac{E_0(p_1 - p_2)}{c} = p_1 p_2 (1 - \cos\theta)$$

Multiplying each term by $hc/p_1p_2E_0$ and using $\lambda = h/p$, we obtain *Compton's equation*:

$$E_0 = mc^2 \qquad \frac{h(p_1 - p_2)}{p_1 p_2} = (1 - \cos\Theta)\frac{hc}{mc^2} \qquad h^2/\lambda_1 - h^2/\lambda_2 = h_1^2/\lambda_1 \cdot \lambda_2 (1 - \cos\Theta)\frac{h}{mc}$$

$$\{1/\lambda_1 - 1/\lambda_2\} \cdot \lambda_1 \lambda_2 = \lambda_2 - \lambda_1$$

Just what we need for the left side (the right side is already OK), to finally obtain

$$\lambda_2 - \lambda_1 = \frac{h}{mc}(1 - \cos\theta)$$

$\lambda_2 = \lambda' > \lambda_1$ so $\Delta\lambda$ positive as it must

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta)$$

$$p = h/\lambda = \hbar k$$

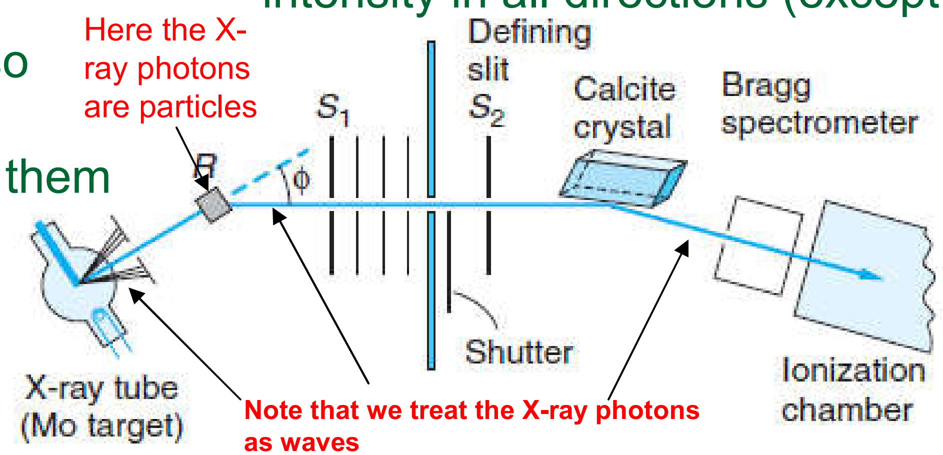
That is now just for photons, later on we will see that it is also valid for particles, it's then the De Broglie equation, next but one chapter

$h/_{mc}$ is a derived constant, Compton wavelength for a certain type of particle with (rest) mass *m*, what's interesting is the shift, the larger the mass, the smaller the shift, and 800 pound gorilla is not knocked over by a single photon

Compton verified his result experimentally using the characteristic x-ray line of wavelength 0.0711 nm from molybdenum for the incident monochromatic photons and scattering these photons from electrons in graphite. The wavelength of the scattered photons was measured using a Bragg crystal spectrometer. His experimental arrangement is shown in Figure 3-16; Figure 3-17 shows his results. The first peak at

Diffraction spots from R are undesirable, so detector gets shielded from them

Scattering is strongly forward, but with lower and lower intensity in all directions (except for the Bragg peaks)



The inelastic scattering has always two components, from the tightly bound atoms and the lightly bound electrons, the higher Φ, the easier these components are taken apart

Figure 3-16 Schematic sketch of Compton's apparatus. X rays from the tube strike the carbon block R and are scattered into a Bragg-type crystal spectrometer. In this diagram, the scattering angle is 30°. The beam was defined by slits S_1 and S_2. Although the entire spectrum is being scattered by R, the spectrometer scanned the region around the K_α line of molybdenum.

The calcite crystal picks out a certain narrow wavelength range only, the relative shift of the inelastically scattered waves for a variation of Φ is what the experiments wants to demonstrate, (note as a aside that the same book called this angle earlier θ)

Not on the same abso-lute scale

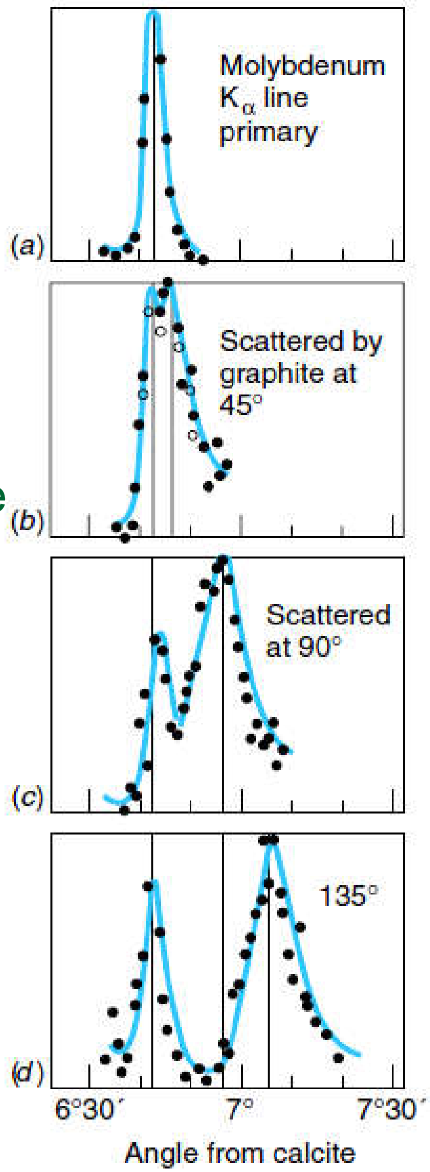


Figure 3-17 Intensity versus wavelength for Compton scattering at several angles. The left peak in each case results from photons of the original wavelength that are scattered by tightly bound electrons, which have an effective mass equal to that of the atom. The separation in wavelength of the peaks is given by Equation 3-25. The horizontal scale used by the Compton "angle from calcite" refers to the calcite analyzing crystal in Figure 3-16.

intensities drop significantly as variable angle Φ increases, but this is just an experimental difficulty

In (a) only one peak, transmitted X-rays plus inelastically scattered X-rays with have a slightly longer wavelength from both atoms and electrons overlap

In (d) finally, the inelastically scattered X-rays from the atoms and electrons clearly separate the spacing's between these two peaks is in ***good*** agreement with

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)$$

whereby θ is most definitely not the Bragg angle as the spectroscopy principle of the analysis has nothing to do with the diffraction effect within the experiment, was earlier called Φ in the derivation

$2d \sin\theta = m\lambda$ where m = an integer

and λ is very close to that of the characteristic X-ray source

Why were X-rays required for Compton's scattering experiments?

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)$$

EXAMPLE 3.8 X-ray Photons versus Visible Photons

(a) Why are x-ray photons used in the Compton experiment, rather than visible-light photons? To answer this question, we shall first calculate the Compton shift for scattering at 90° from graphite for the following cases: (1) very high energy γ-rays from cobalt, $\lambda = 0.0106$ Å; (2) x-rays from molybdenum, $\lambda = 0.712$ Å; and (3) green light from a mercury lamp, $\lambda = 5461$ Å.

Solution In all cases, the Compton shift formula gives $\Delta\lambda = \lambda' - \lambda_0 = (0.0243 \text{ Å})(1 - \cos 90°) = 0.0243 \text{ Å} = 0.00243$ nm. That is, regardless of the incident wavelength, the same small shift is observed. However, the fractional change in wavelength, $\Delta\lambda/\lambda_0$, is quite different in each case:

γ-rays from cobalt:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{0.0243 \text{ Å}}{0.0106 \text{ Å}} = 2.29$$

X-rays from molybdenum:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{0.0243 \text{ Å}}{0.712 \text{ Å}} = 0.0341$$

Visible light from mercury:

$$\frac{\Delta\lambda}{\lambda_0} = \frac{0.0243 \text{ Å}}{5461 \text{ Å}} = 4.45 \times 10^{-6}$$

Because both incident and scattered wavelengths are simultaneously present in the beam, they can be easily resolved only if $\Delta\lambda/\lambda_0$ is a few percent or if $\lambda_0 \leq 1$ Å.

(b) The so-called free electrons in carbon are actually electrons with a binding energy of about 4 eV. Why may this binding energy be ignored for x-rays with $\lambda_0 = 0.712$ Å?

Solution The energy of a photon with this wavelength is

$$E = hf = \frac{hc}{\lambda} = \frac{12\,400 \text{ eV}\cdot\text{Å}}{0.712 \text{ Å}} = 17\,400 \text{ eV}$$

Therefore, the electron binding energy of 4 eV is negligible in comparison with the incident x-ray energy.

Actually also for λ ≈ 0.0712 nm for Mo Kα used in Compton experiments about 100 years ago, highest respect to overcoming the experimental difficulties !

Nobel prize 1927 for Arthur Holly Compton, so there is no doubt that the photons (mass less particles compliant with special relativity) are indeed behaving like particles in collisions

as Einstein had predicted already in 1906: *"If a bundle of radiation causes a molecule to emit or absorb an energy packet hf, then momentum of quantity* $^{hf}/_{c}$ *is transferred to the molecule, directed along the line of motion of the bundle for absorption and opposite to the bundle for emission"*

(for photons, $p = {}^{E}/_{c}$ from special relativity 1905)

As people had enormous difficulties with this, the great master (A. E.) wrote in 1911: *"I insist on the provisional nature of this concept which does not seem reconcilable with the experimentally verified consequences of the wave theory."*

1. J. J. Thomson's experiment	Thomson's measurements with cathode rays showed that the same particle (the electron), with e/m about 2000 times that of ionized hydrogen, exists in all elements.	
2. Quantization of electric charge	$e = 1.60217653 \times 10^{-19}$ C	
3. Blackbody radiation		
Stefan-Boltzmann law	$R = \sigma T^4$	**3-4**
Wein's displacement law	$\lambda_m T = 2.898 \times 10^{-3}$ m·K	**3-5**
Planck's radiation law	$u(\lambda) = \dfrac{8\pi hc\lambda^{-5}}{e^{hc/\lambda kT} - 1}$	**3-18**
Planck's constant	$h = 6.626 \times 10^{-34}$ J·s	**3-19**
4. Photoelectric effect	$eV_0 = hf - \phi$	**3-21**
5. Compton effect	$\lambda_2 - \lambda_1 = \dfrac{h}{mc}(1 - \cos\theta)$	**3-25**
6. Photon-matter interaction	The (1) photoelectric effect, (2) Compton effect, and (3) pair production are the three ways of interaction. we dealt with the latter in the previous chapter	

We have seen in this chapter irrefutable evidence for Einstein's wave-particle duality nature of light and its consistency with special relativity

4.7: Frank-Hertz experiment, 1914

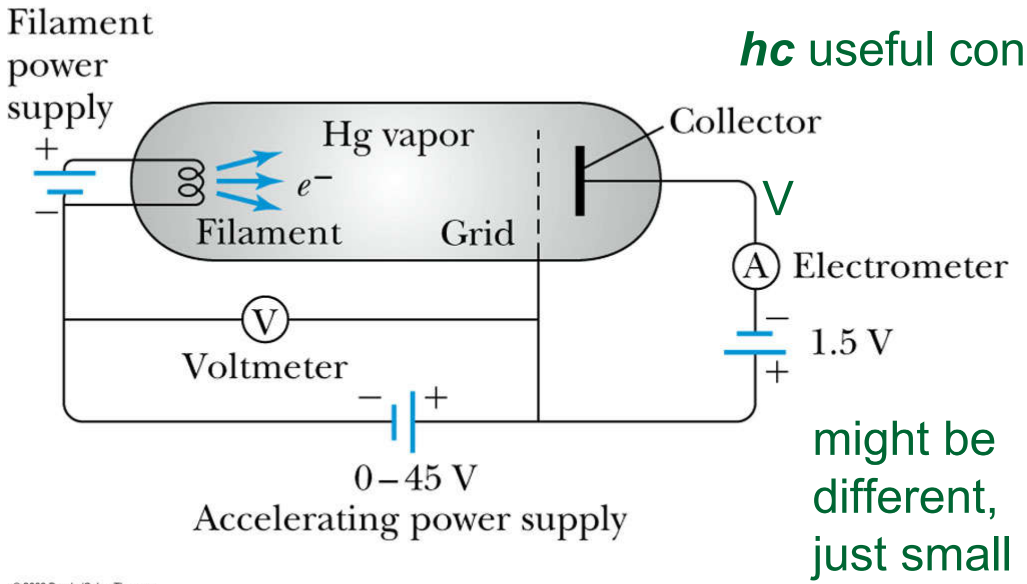
Again something peculiar hinting about quantization of energy in bound systems, (i.e. inside Hg atoms), again probed by collisions between particles, this time cathode rays (i.e. accelerated electrons) and a dilute gas of Hg, also confirming Einstein's E = hf equation and with it the existence of photons, (i.e. wave particle duality), just one year after Bohr's three paper sequence on the structure of the hydrogen atom

TOPIC	RELEVANT EQUATIONS AND REMARKS	
4. X-ray spectra		
Moseley equation	$f^{1/2} = A_n(Z - b)$	4-34
5. Franck-Hertz experiment	Supported Bohr's theory by verifying the quantization of atomic energies in absorption.	

Nobel prize in 1925

4.7: Atomic Excitation by Electrons

_hc_ useful constant combination: 1239.8 eV



Some electrons pass through the grid and reach the collector, the purpose of the voltage between the grid and collector is similar to the stopping potential in the photoelectric effect: to control kinetic energy of the electrons (but now some electrons get through as any grid has holes.

Measurement signal is a current in an electrometer in dependence of a variable acceleration voltage

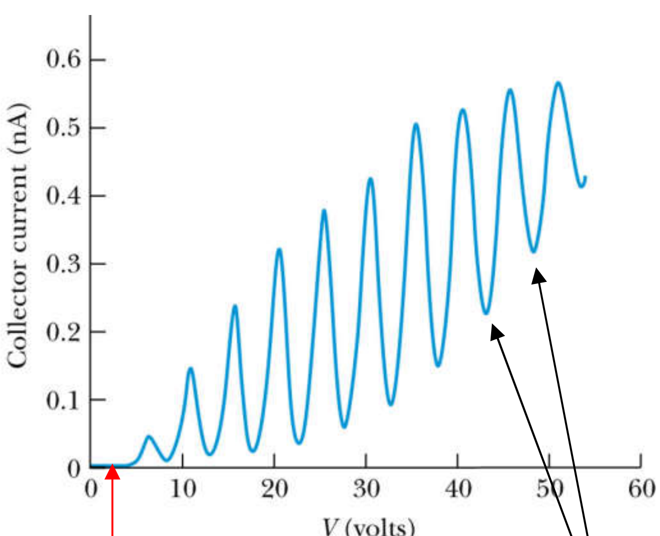
First with glass tube, was replaced by quartz glass tubing, which allows ultraviolet light to be measured outside the tube, sure ultraviolet light of $\lambda = \frac{c}{f} = \frac{hc}{hf} = \frac{hc}{eV_0} = 253\ \text{nm}$

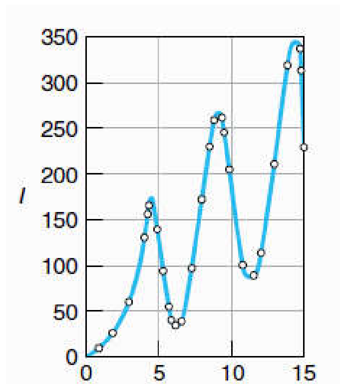
was detected above 4.9 eV (V_0) and higher. So that light's energy must correspond to an internal transition in the Hg, so atoms should have quantized energy levels !!

As acceleration voltage increases from 0 to just below approx. 4.9 eV (+1.5 eV from the collector/grid, also modifications due to work function of filament and collector), the registered current increases, more electrons get through

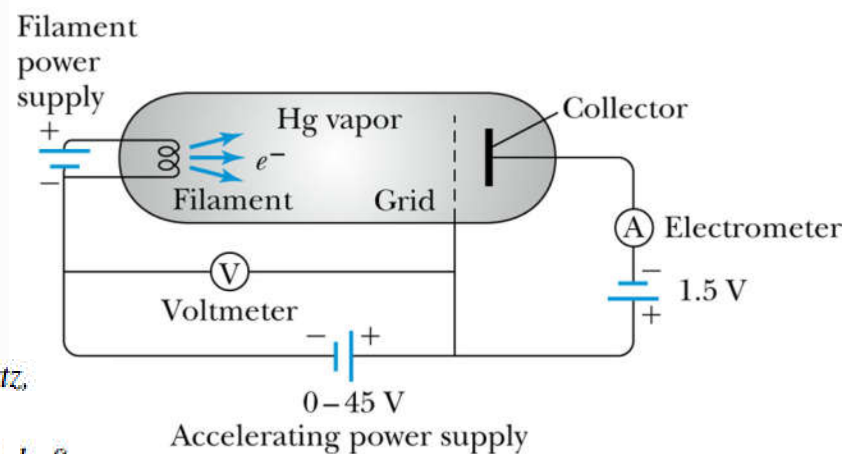
at (and above) 4.9 eV (+1.5), electrons loose essential all of their kinetic energy to the much heavier Hg atoms, the registered current drops to nearly zero, some electrons pass by Hg atoms without collision

when acceleration voltage is increased beyond 4.9 eV (+1.5 eV), the registered current increases again until it drops at two times 4.9 eV (+1.5 eV), the spacing between subsequent peaks is approx. 4.9 eV (regardless of the small work function of the collector and other corrections),





J. Franck and G. Hertz, Verband Deutscher Physiklischer Gesellschaften, 16, 457 (1914).]

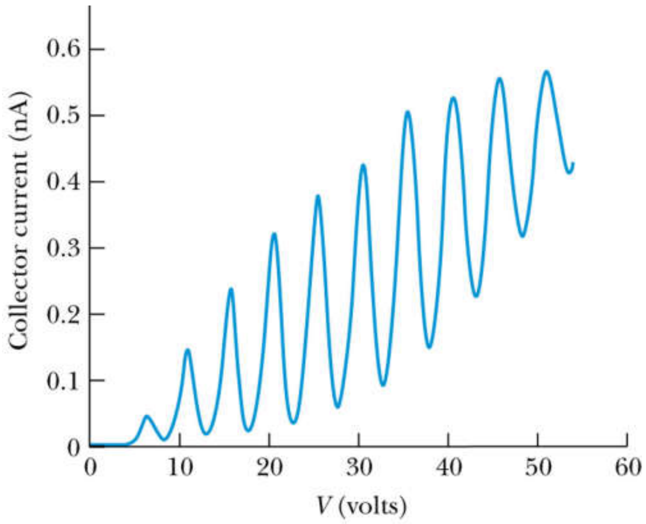


compare with sketch of experimental data

One could estimate the density of Hg atoms in the dilute gas from this

Excitation of Hg atoms by Electrons

- Explanation, only electrons with sufficient kinetic energy 4.9 eV loose it to Hg atoms, exited Hg atoms just take in that quantum of energy (in the low voltage range), after a short time they radiate it off again, the radiation intensity is zero before kinetic energy 4.9 eV is reached and exceeded.



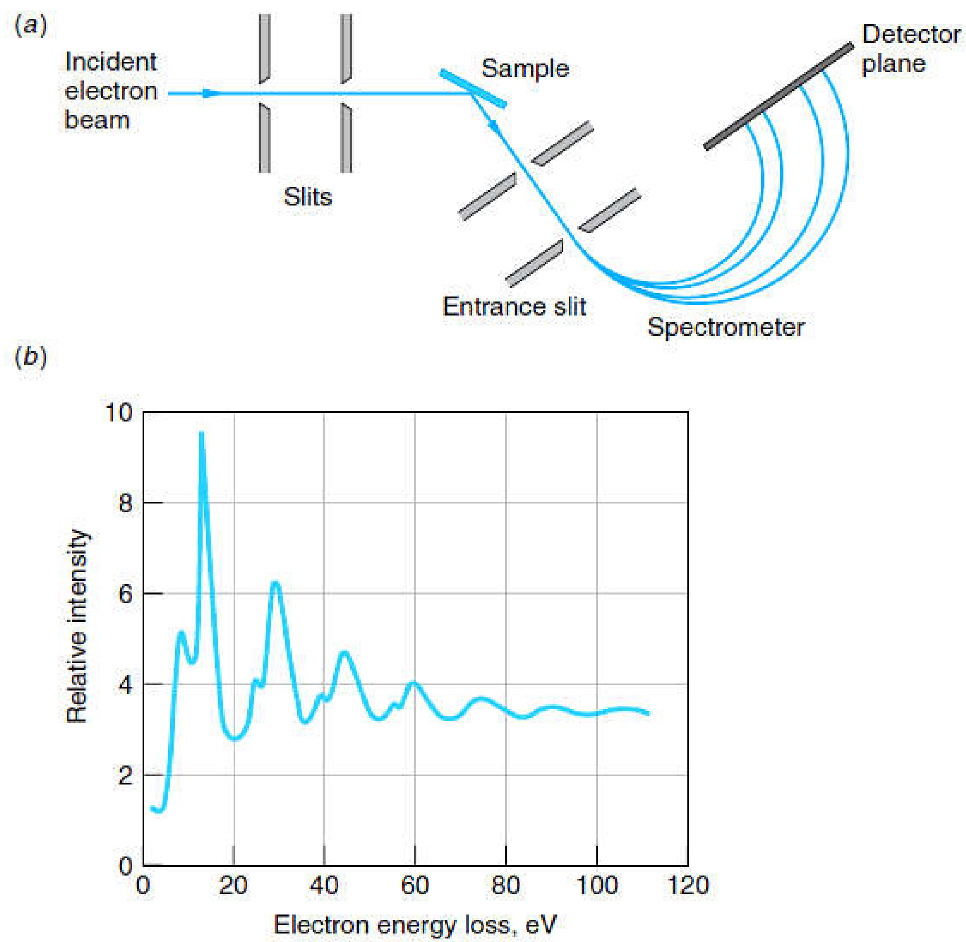
- What's important about this kind of plot is that the spacing between the maxima is just 4.9 eV (not were they actually occur, because the latter is a function of the slightly negative potential between collector and grid)

- There are multiple peaks in such a plot because a particular electron might loose 4.9 eV to a Hg atom, be accelerated again, loose again those 4.9 eV to another Hg atom, …. Do realize that Hg "takes in only 4.9 eV, no less no more" and looses 4.9 eV no less no more in radiation – how come: that must correspond to some internal transition on the basis of some internal structure which any good theory of an atom must be able to explain, next chapter

Now this “kind of effect” is used to characterize elements and compounds, e.g. in an analytical transmission electron microscope

http://en.wikipedia.org/wiki/Electron_energy_loss_spectroscopy

Figure 4-24 Energy-loss spectrum measurement. (*a*) A well-defined electron beam impinges upon the sample. Electrons inelastically scattered at a convenient angle enter the slit of the magnetic spectrometer, whose B field is directed out of the paper, and turn through radii R determined by their energy $E_{inc} - E_1$ via Equation 3-2 written in the form $R = [2m(E_{inc} - E_1)]^{1/2}/eB$. (*b*) An energy-loss spectrum for a thin Al film. [*From C. J. Powell and J. B. Swan, Physical Review, 115, 869 (1954).*]



Doing spectroscopy on the emitted light is called cathodoluminescents, i.e. another modern characterization technique in SEM where you do not have many transmitted electrons

Anybody in this class who has not heard about the Bohr model of the hydrogen atom before?????

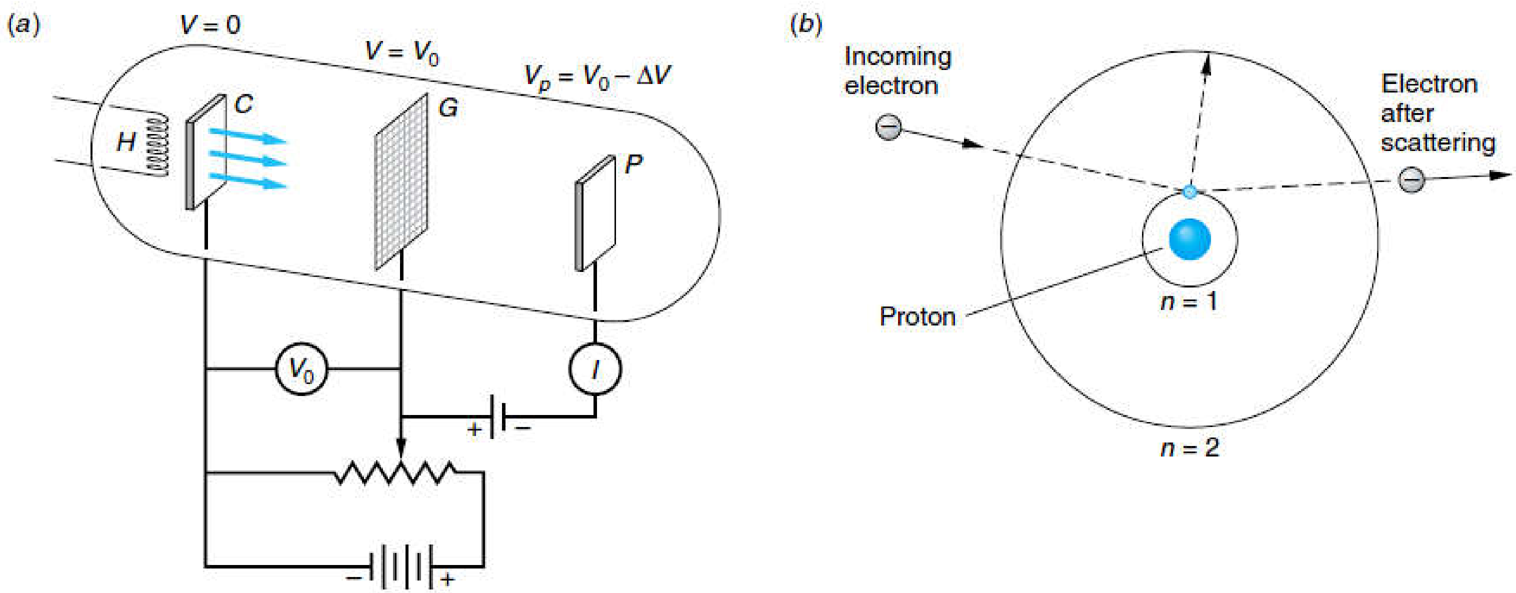


Figure 4-22 (*a*) Schematic diagram of the Franck-Hertz experiment. Electrons ejected from the heated cathode C at zero potential are drawn to the positive grid G. Those passing through the holes in the grid can reach the plate P and thereby contribute to the current I if they have sufficient kinetic energy to overcome the small back potential ΔV. The tube contains a low-pressure gas of the element being studied. (*b*) Results for hydrogen. If the incoming electron does not have sufficient energy to transfer $\Delta E = E_2 - E_1$ to the hydrogen electron in the $n = 1$ orbit (ground state), then the scattering will be elastic. If the incoming electron does have at least ΔE kinetic energy, then an inelastic collision can occur in which ΔE is transferred to the $n = 1$ electron, moving it to the $n = 2$ orbit. The excited electron will typically return to the ground state very quickly, emitting a photon of energy ΔE.

My class is about how radical that model was for its time and where its limitations are, modern physics is not separated in chapters, it all must fit together, that's how these people have progressed in the past and that's the only way to make more progress in the future, ***also about essence of models***