CHAPTER 2
Special Relativity

Space time diagrams are skipped

- 2.1 The Need for Ether
- 2.2 The Michelson-Morley Experiment
- 2.3 Einstein's Postulates
- 2.4 The Lorentz Transformation
- 2.5 Time Dilation and Length Contraction
- 2.6 Muon observation on earth (experimental verification special relativity)
- 2.7 Twin Paradox (NOT a problem of special relativity)
- 2.8 Addition of Velocities
- 2.9 Doppler Effect

- Summary up to that point & Conclusions Michelson-Morley experiment

- 2.10 Relativistic Momentum
- 2.11 Relativistic Energy
- Pair Production and Annihilation
- 2.13 Computations in Modern Physics
- 2.14 Electromagnetism and Relativity

- **Chapter 15, General Relativity**

It was found that there was no displacement of the interference fringes, so that the result of the experiment was negative and would, therefore, show that there is still a difficulty in the theory itself…

- Albert Michelson, 1907

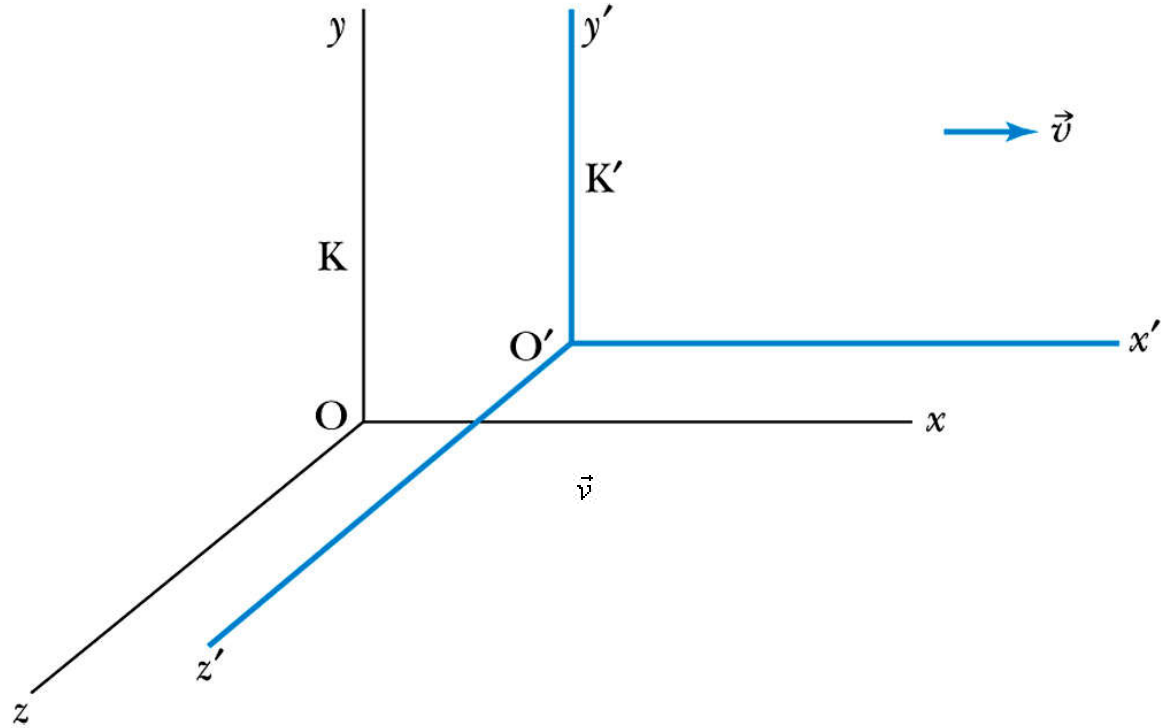
The "paradox" is only a conflict between reality and your feeling of what reality "ought to be." R. P. Feynman

Newtonian (Classical) Relativity / Invariance due to Galileo Galilei, 1564 - 1642

All of mechanics (i.e. Newton's laws) is independent on the inertial reference frame in which it is happening. Not only do we not feel that the Earth is moving (around the sun while spinning), we also cannot prove this by mechanical experiments.

So the Earth may well be moving around the sun despite the catholic church's burning of Giordano Bruno (1548-1600) on the stake for that belief.

Inertial Frames K and K’



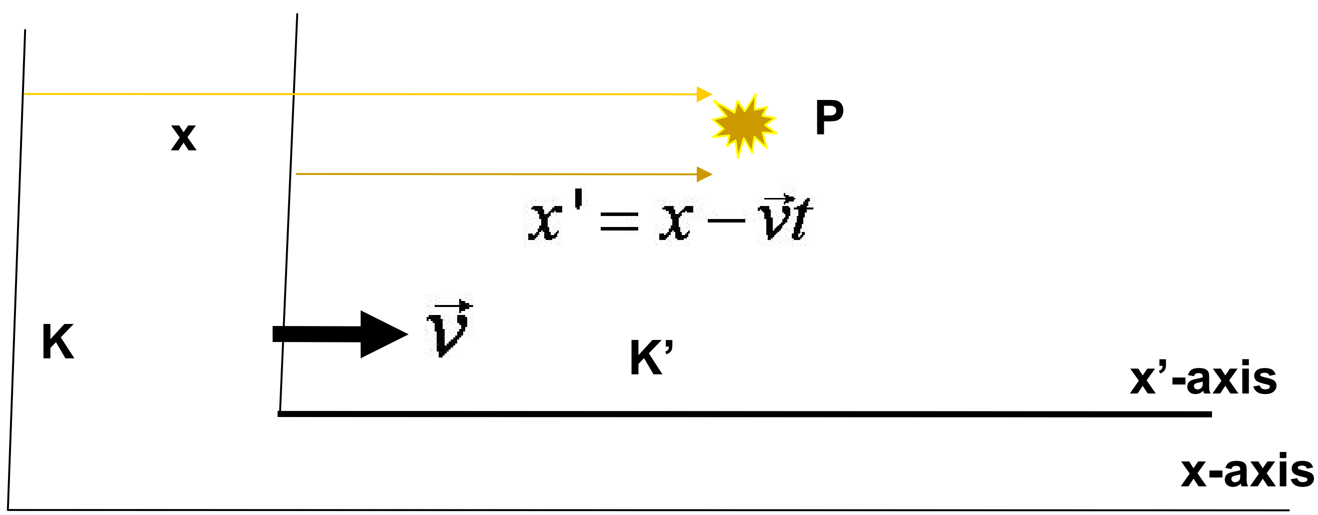
- K is at rest and K’ is moving with velocity $\vec{v}$
- Axes are parallel
- K and K’ are said to be *INERTIAL COORDINATE SYSTEMS (frames of reference)*

The Galilean Transformation

For a point P

- In system K: P = (*x*, *y*, *z*, *t*)
- In system K’: P = (*x*’, *y*’, *z*’, *t*’)

v = constant, no acceleration, Newton’s first law



Space and time are separate, both are fundamental frames in which physical processes are happening

Conditions of the Galilean Transformation

- Parallel axes
- K' has a constant relative velocity in the *x*-direction with respect to K

$$x' = x - \vec{v}t$$

$$y' = y$$

$$z' = z$$

$$t' = t$$

- **Time** (*t*) for all observers is a *Fundamental invariant*, i.e., the same for all inertial observers

The Inverse Relations

Step 1. Replace $\vec{v}$ with $-\vec{v}$.

Step 2. Replace “primed” quantities with “unprimed” and “unprimed” with “primed.”

$$x = x' + \vec{v}t$$

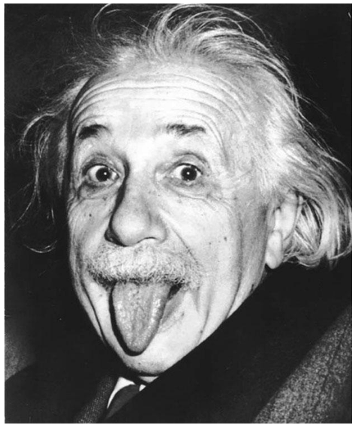
$$y = y'$$

$$z = z'$$

$$t = t'$$

The Transition to Modern Relativity

- Although Newton's laws of motion had the same form under the Galilean transformation, Maxwell's equations did not !!!
- So some corrections should be needed for their validity on Earth, some other corrections for their validity on Mars, …



$$v = c = 1/\sqrt{\mu_0 \varepsilon_0}$$

is a constant according to Maxwell, speed of light supposed to be with respect to the medium in which light is traveling in ??

- In 1905, the 26 years young Albert Einstein proposed a fundamental connection between space and time and that Newton's mechanics laws are only an approximation.

2.1: The Need for Ether

- The wave nature of light "requires" that there existed a propagation medium called the *luminiferous ether* or just **ether**. (Assuming that light is just like the other waves known to classical physics.)
  - Ether had to have such a low density that the planets could move through it without loss of energy
  - It also had to have an enormously high elasticity to support the high velocity of light waves
  - No such material was known or seemed to exist

Maxwell's Equations

- In Maxwell's theory the speed of light, in terms of the permeability and permittivity of free space (a vacuum on earth), is given by

$$v = c = 1/\sqrt{\mu_0 \varepsilon_0}$$

- So if Maxwell's equations are correct (and experiments indicate that they are on earth), and the speed of light is this constant on earth, planets, stars, … that move with respect to the earth need to have different speeds of light?

An Absolute Reference System

- Ether was proposed as an absolute reference system in which the speed of light was this constant and from which other measurements could be made.
- The Michelson-Morley experiment was an attempt to show the Earth's movement through the ether (and thereby it's existence).
- Seemed to be unreasonable that the ether would be attached to the earth, which would then be a very special place in all of the universe.

2.2: The Michelson-Morley Experiment

- Albert Michelson (1852–1931) was the first U.S. citizen to receive the Nobel Prize for Physics (1907),
- “***for his optical precision instruments and the spectroscopic and metrological investigations carried out with their aid"*.**

- *(interferometer* to measure the minute phase difference between two light waves traveling in mutually orthogonal directions that classical mechanics predicted.)

- **With which he didn’t get the anticipated result !!!**

Typical interferometer fringe pattern expected when the system is rotated by 90°

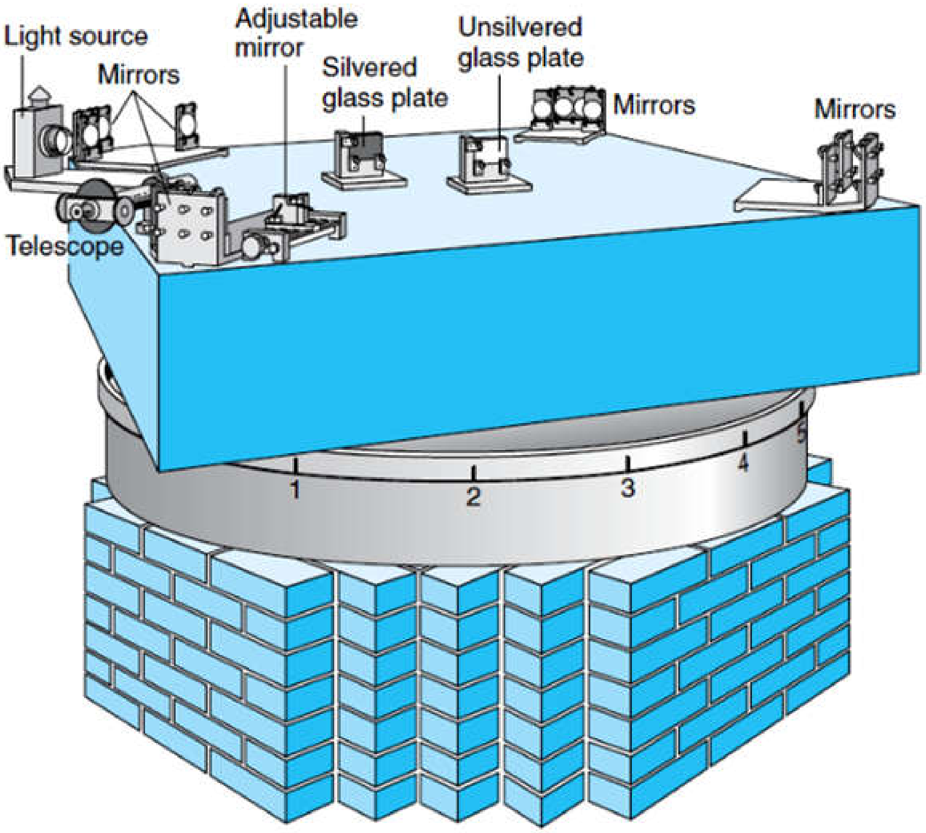
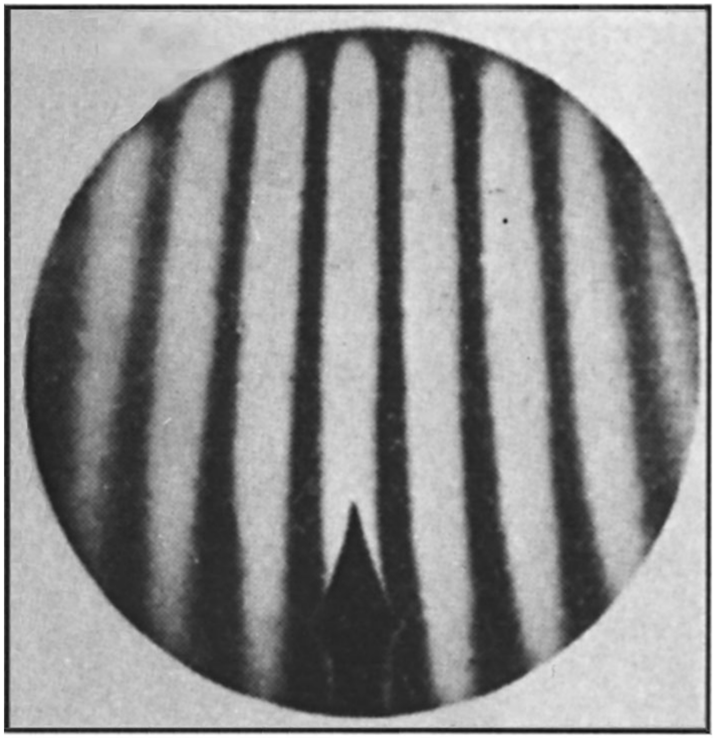
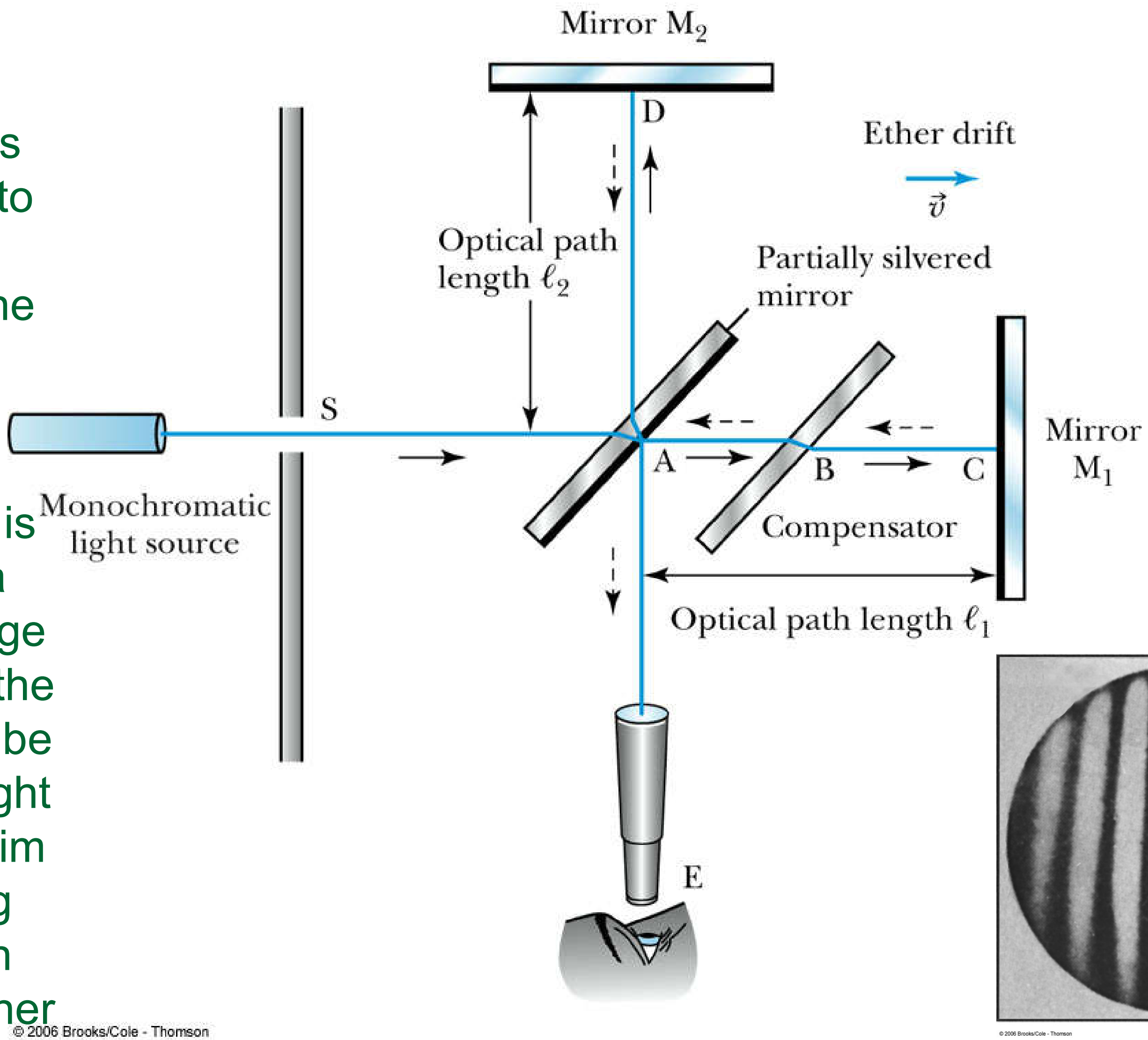


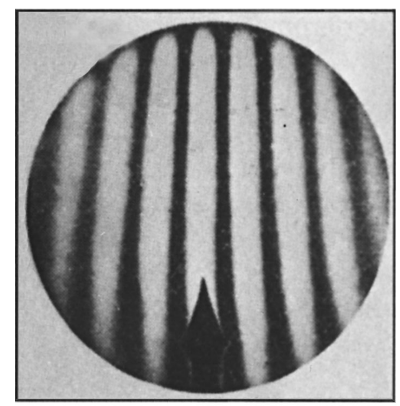
Figure 1-7 Drawing of Michelson-Morley apparatus used in their 1887 experiment. The optical parts were mounted on a 5 ft square sandstone slab, which was floated in mercury, thereby reducing the strains and vibrations during rotation that had affected the earlier experiments. Observations could be made in all directions by rotating the apparatus in the horizontal plane. [*From R. S. Shankland, "The Michelson-Morley Experiment," Copyright © November 1964 by Scientific American, Inc. All rights reserved.*]



The Michelson Interferometer

Whole apparatus is considered to move with respect to the ether





light source is originally a selected range of rays from the sun, but can be any sort of light as the real aim is detecting motion with respect to ether rather then sun

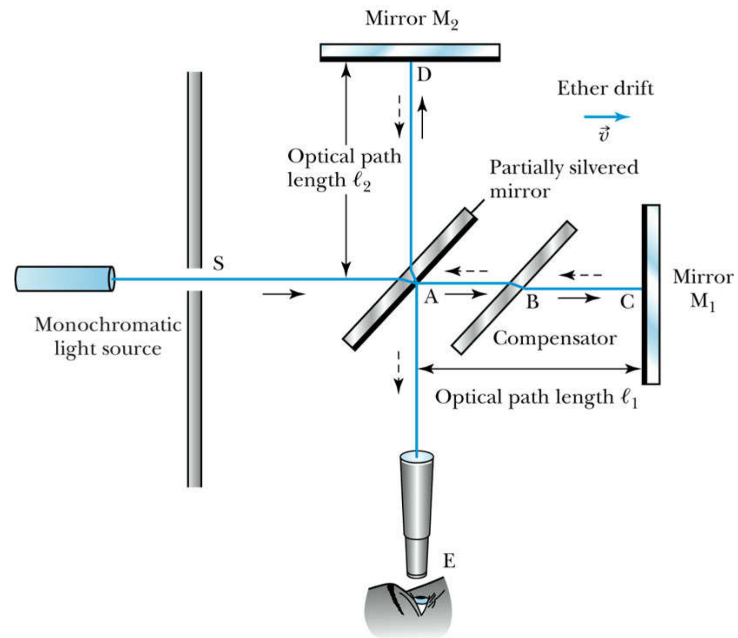
Near mono-chromaticity of light is required for sharp interference pattern

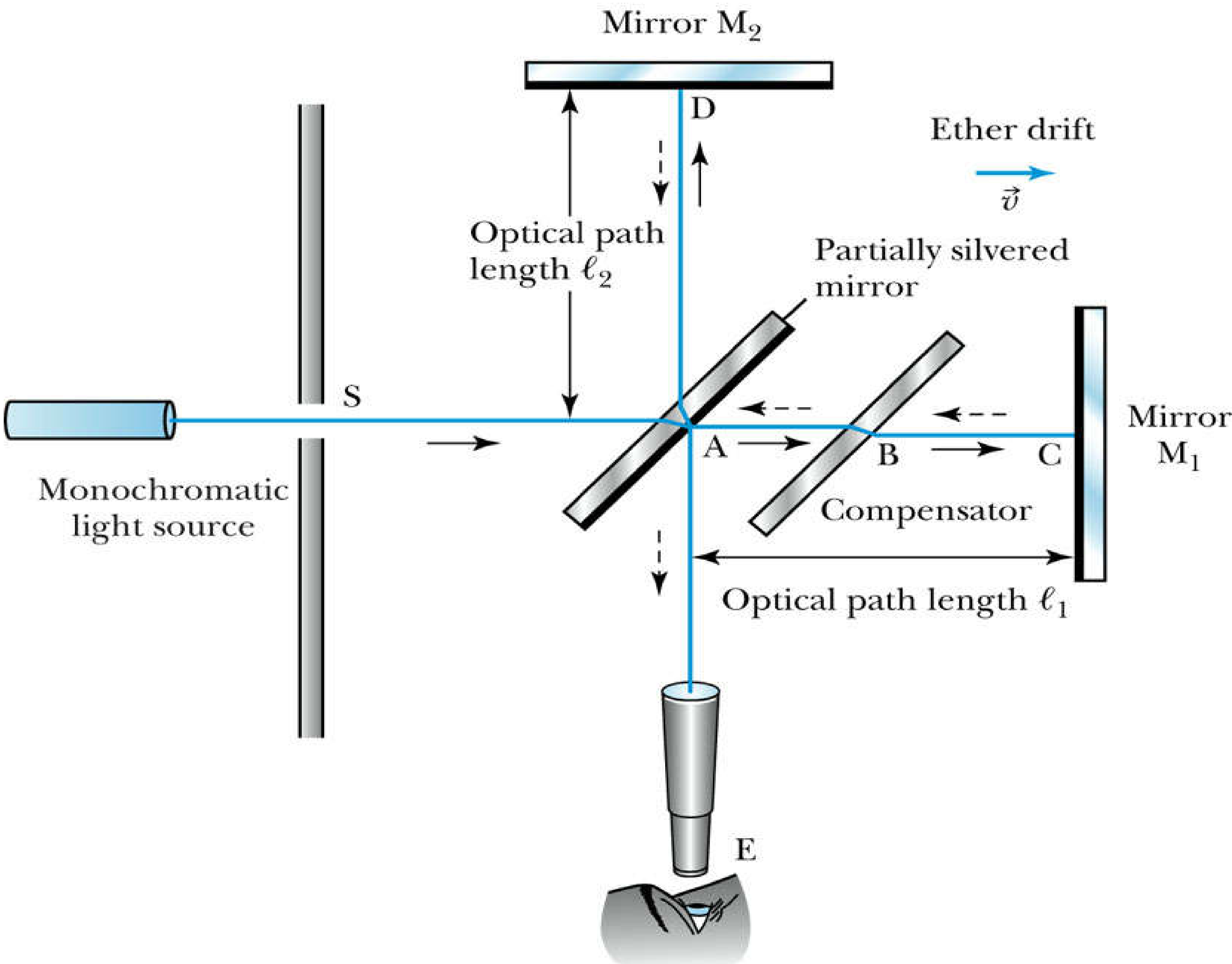
The Michelson Interferometer

1. AC is parallel to the motion of the Earth inducing an "ether wind"

2. Light from source S is split by mirror A and travels to mirrors C and D in mutually perpendicular directions

3. After reflection the beams recombine at A slightly out of phase due to the "ether wind" as viewed by telescope E.




Mirror M_2
D
Ether drift
$\vec{v}$
Optical path length ℓ_2
Partially silvered mirror
S
Mirror M_1
A
B
C
Monochromatic light source
Compensator
Optical path length ℓ_1
E

The Analysis

Time t_1 from A to C and back:

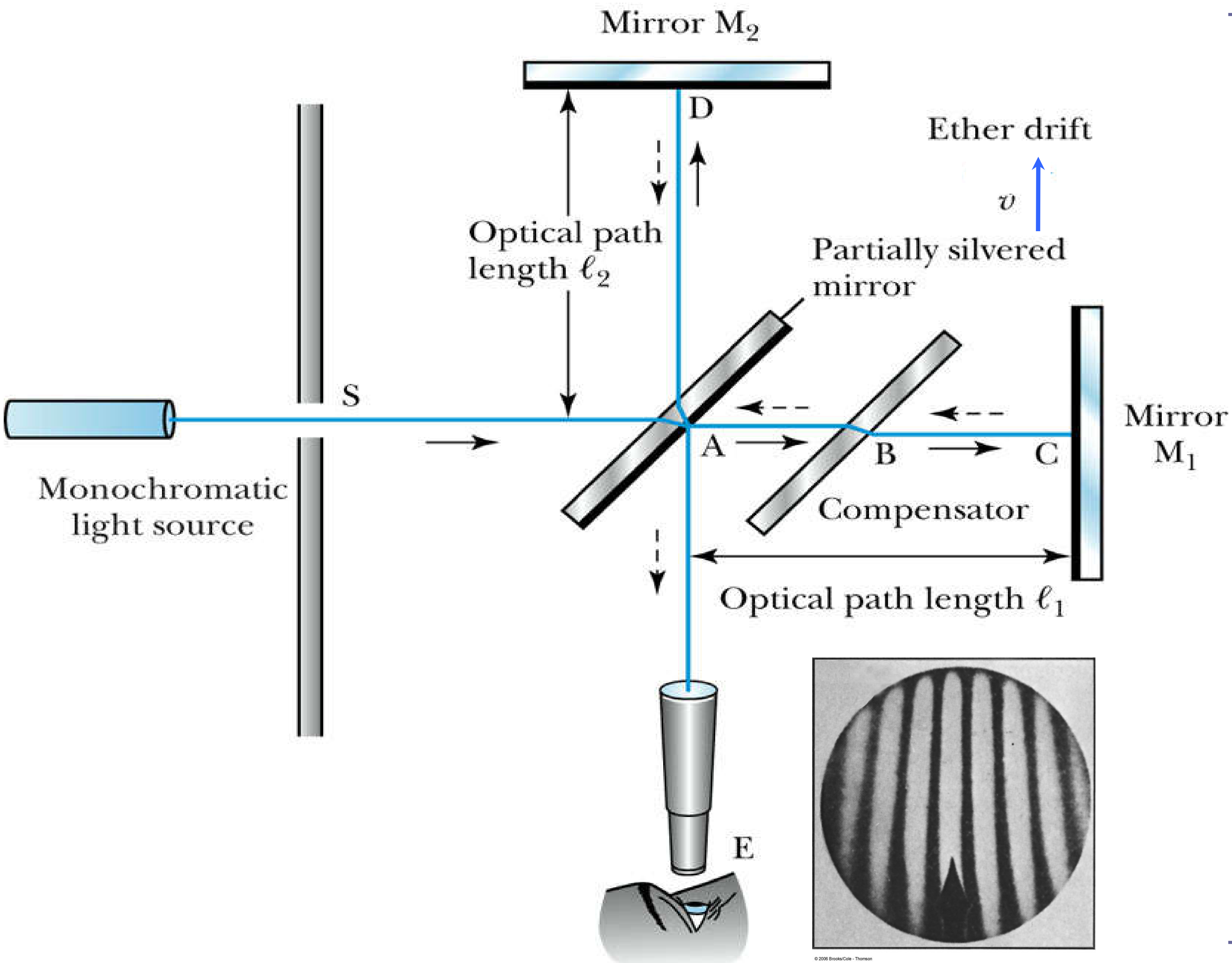
$$t_1 = \frac{\ell_1}{c+v} + \frac{\ell_1}{c-v} = \frac{2c\ell_1}{c^2 - v^2} = \frac{2\ell_1}{c}\left(\frac{1}{1 - v^2/c^2}\right)$$

Time t_2 from A to D and back:

$$t_2 = \frac{2\ell_2}{\sqrt{c^2 - v^2}} = \frac{2\ell_2}{c} \cdot \frac{1}{\sqrt{1 - v^2/c^2}}$$

So that the change in time is:

$$\Delta t = t_2 - t_1 = \frac{2}{c}\left(\frac{\ell_2}{\sqrt{1 - v^2/c^2}} - \frac{\ell_1}{1 - v^2/c^2}\right)$$


Mirror M_2
D
Ether drift
v
Optical path length ℓ_2
Partially silvered mirror
S
Monochromatic light source
A
B
C
Mirror M_1
Compensator
Optical path length ℓ_1
E

The Analysis (continued)

Upon rotating the apparatus (we use (black) primes to mark the rotation), the optical path lengths ℓ_1 and ℓ_2 are interchanged in order to produce a different change in time: (note the change in denominators)

$$\Delta t' = t'_2 - t'_1 = \frac{2}{c}\left(\frac{\ell_2}{1 - v^2/c^2} - \frac{\ell_1}{\sqrt{1 - v^2/c^2}}\right)$$

$\Delta t' - \Delta t$ = ? Should be something small, but just measurable

The Analysis (continued)

1st part of first Homework is the derivation of the classical analysis for this experiment, justifying all the steps from $t_1 = \ldots$ from slide 16 onwards, show all of your intermediate steps and end in the result below, also convince yourself and the teaching assistant that the expressions for t_1, t_2, t_1' and t_2' are all correct

and upon a binomial expansion, assuming *v*/*c* << 1, this reduces to

$$\Delta t' - \Delta t \approx v^2(\ell_1 + \ell_2)/c^3$$

But was measured to be zero !!!!!!

Results

- Using the Earth's orbital speed as:

$$V = 3 \times 10^4 \text{ m/s}$$

together with

$\ell_1 \approx \ell_2 = 1.2$ m (the longer the better for experiment)

So that the time difference becomes

$$\Delta t' - \Delta t \approx v^2(\ell_1 + \ell_2)/c^3 = 8 \times 10^{-17} \text{ s}$$

- Although a very small number, it was within the experimental range of measurement for light waves.

But was measured to be zero !!!!!!

“interpretation”

With characteristic reserve, Michelson described the results thus: 1887

> The actual displacement [of the fringes] was certainly less than the twentieth part [of 0.4 fringe], and probably less than the fortieth part. But since the displacement is proportional to the square of the velocity, the relative velocity of the earth and the ether is probably less than one-sixth the earth’s orbital velocity and certainly less than one-fourth.

R. P. Feynman: The first principle is that you must not *fool* yourself, and you are the easiest person to *fool*.

Michelson's and almost all other's Conclusions

- Michelson should have been able to detect a phase shift of light interference fringes due to the time difference between path lengths but found none. (Speed of Earth in orbit 30 km/s would be sufficiently fast for these kinds of measurements if classical physics were applicable)
- After several repeats and refinements with assistance from Edward Morley (1893-1923), again *a null result.*
- ***Thus, ether does not seem to exist – we then have a problem, there needs to be something wrong with Maxwell's equations, the wave theory of light seems to be wrong, but all of wave optics depends on it …***

Possible Explanations

- Many explanations were proposed but the most popular was the *ether drag* hypothesis.
  - This hypothesis suggested that the Earth somehow “dragged” the ether along as it rotates on its axis and revolves about the sun. Earth would then be the only place in the universe where Maxwell’s equations would be valid without further modifications (correction factors)
  - This was contradicted by *stellar aberration* wherein telescopes had to be tilted to observe starlight due to the Earth’s motion. If ether was dragged along, this tilting would not exist.

The FitzGerald Contraction

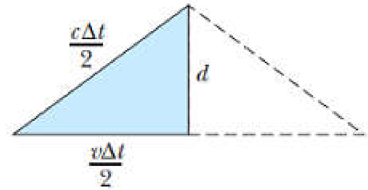
- Another hypothesis proposed independently by both G. F. FitzGerald (just an assumption) and H. A. Lorentz (as part of his transformations) suggested that the length ℓ_1, in the direction of the motion was *contracted* by a factor of

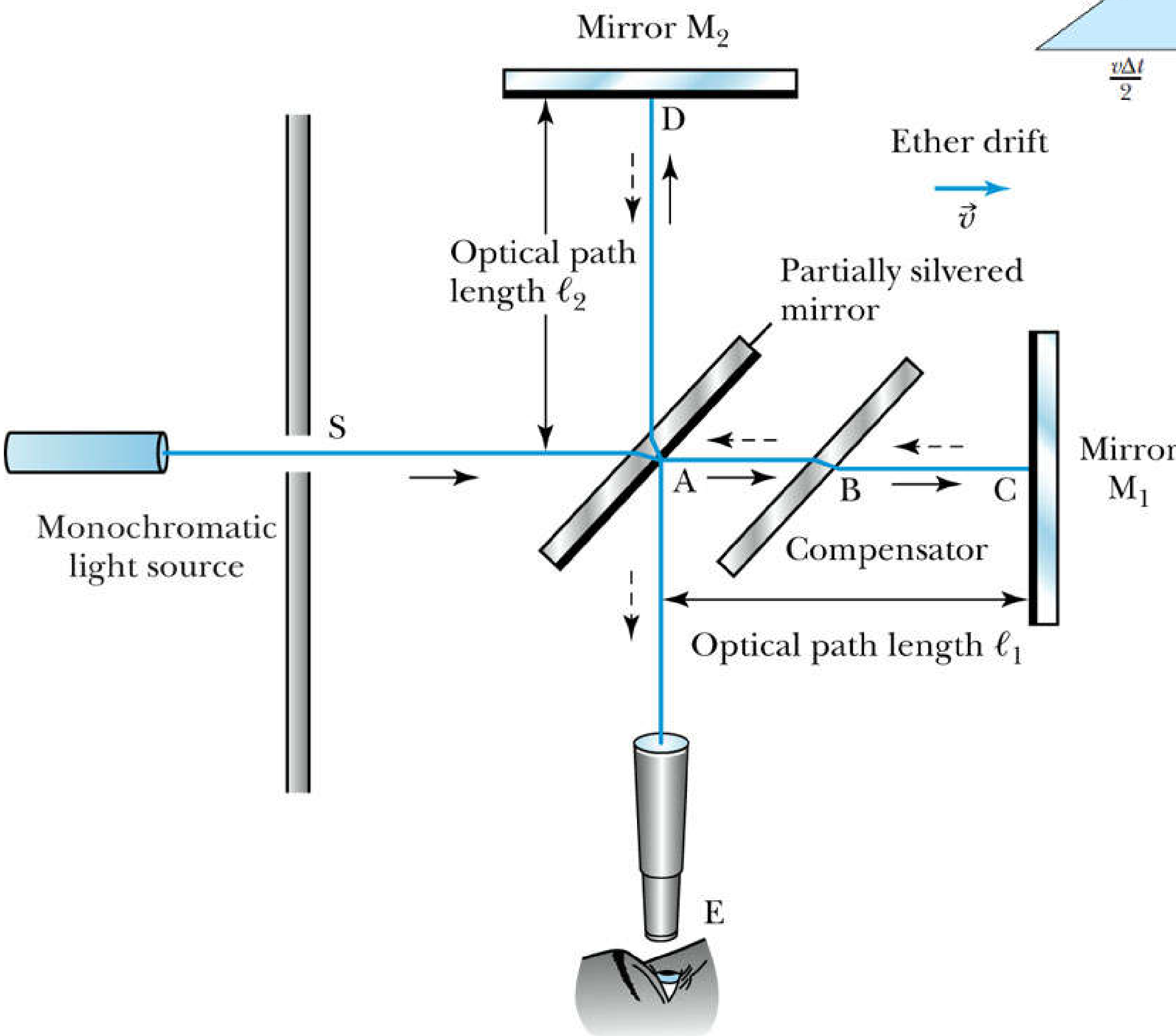
$$\sqrt{1 - v^2 / c^2}$$

On turning by 90 degrees, it will be ℓ_2 that gets contracted …thus making the path lengths equal to account for the zero phase shift.

  - This, however, was an ad hoc assumption that could not be experimentally tested.
  - ***But it could be derived from Lorentz (Vogt) transformation in which Maxwell's equations are invariant,1895, but no deeper understanding of what these relations mean at that time for modern relativity***

The Michelson Interferometer





l_2 can be considered to be "set equal" to l_1 by nature of fringe shift measurement, but there is also an additional (Fitzgerald) contraction – marked by two green primes - in direction of motion of the apparatus with respect to light source

Time t_1 from A to C and back:

$$t_1 = \frac{\ell_1''}{c+v} + \frac{\ell_1''}{c-v} = \frac{2c\ell_1''}{c^2 - v^2} = \frac{2\ell_1''}{c}\left(\frac{1}{1 - v^2/c^2}\right)$$

Time t_2 from A to D and back:

$$t_2 = \frac{2\ell_2}{\sqrt{c^2 - v^2}} = \frac{2\ell_2}{c} \cdot \frac{1}{\sqrt{1 - v^2/c^2}}$$

So that

$$l_1'' = l_2 \cdot \sqrt{1 - v^2/c^2}$$

$$inverse_{Lorentz_{factor}} = \sqrt{1 - v^2/c^2} = \gamma^{-1}$$

So that the change in time is:

$$\Delta t = t_2 - t_1 = \frac{2}{c}\left(\frac{\ell_2}{\sqrt{1 - v^2/c^2}} - \frac{\ell_1''}{1 - v^2/c^2}\right)$$

= 0, a stable setting of the interferometer

Upon rotating the apparatus (we use black primes to mark the rotation), the optical path lengths ℓ_1 and ℓ_2' are interchanged producing a different change in time: (note the change in denominators)

But now $l_2'' = l_1 \cdot \sqrt{1 - v^2 c^2}$

$$\Delta t' = t'_2 - t'_1 = \frac{2}{c}\left(\frac{\ell_2''}{1 - v^2/c^2} - \frac{\ell_1}{\sqrt{1 - v^2/c^2}}\right)$$

= 0, the same stable setting of the interferometer, so no change on turning apparatus by 90 degrees can be observed and indeed it is not

$\Delta t' - \Delta t$ = 0 – 0 = 0

What if we have two observers, one stationary at the telescope, one far away looking with another telescope while moving with a constant speed in a straight line parallel (or anti-parallel) to the direction of the light that goes into the apparatus and eventually produce the interference pattern

Will they obtain different results? No, just the spacing of the fringes will be ever so slightly reduced (or expanded depending on the direction), but the interference pattern will not change

there is nothing for the experiment to measure according to Einstein, **because there is no electrodynamics experiment that could detect if earth was moving with respect to the sun or ether**, it would not matter how fast the apparatus is moving with respect to the source of light (or the sun), length contraction would just compensate for that, so conceptually that velocity can also be set zero and we also get an analytical result that agrees with the experimental outcome !!!

The Analysis for v = 0 as it does not have any effect on the result of physical experiments

Time t_1 from A to C and back:

$$t_1 = \frac{\ell_1}{c+v} + \frac{\ell_1}{c-v} = \frac{2c\ell_1}{c^2-v^2} = \frac{2\ell_1}{c}\left(\frac{1}{1-v^2/c^2}\right)$$

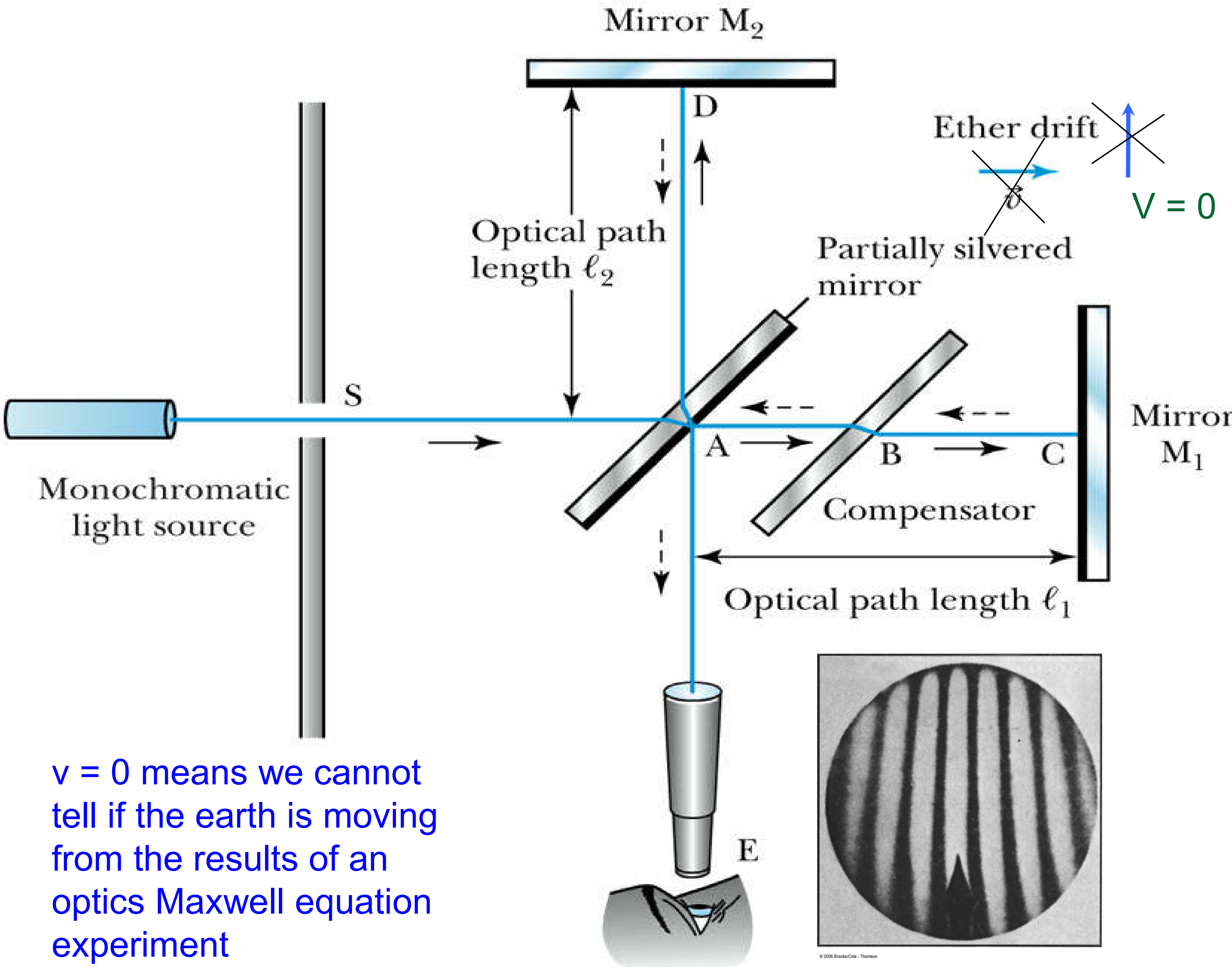
Time t_2 from A to D and back:

$$t_2 = \frac{2\ell_2}{\sqrt{c^2-v^2}} = \frac{2\ell_2}{c} \cdot \frac{1}{\sqrt{1-v^2/c^2}}$$

So that the change in time is:

$$\Delta t = t_2 - t_1 = \frac{2}{c}\left(\frac{\ell_2}{\sqrt{1-v^2/c^2}} - \frac{\ell_1}{1-v^2/c^2}\right) = 0$$

Regardless of the actual lengths l_1 and l_2, we won't see a change in the interference pattern on turning the apparatus by 90 degrees


Mirror M_2
D
Ether drift
$\vec{v}$
V = 0
Optical path length ℓ_2
Partially silvered mirror
S
Monochromatic light source
A
B
C
Compensator
Mirror M_1
Optical path length ℓ_1
E
v = 0 means we cannot tell if the earth is moving from the results of an optics Maxwell equation experiment

The Analysis for v = 0 (continued)

Upon rotating the apparatus (we use (black) primes to mark the rotation), the optical path lengths ℓ_1 and ℓ_2 are interchanged in order to produce a different change in time: (note the change in denominators)

$$\Delta t' = t'_2 - t'_1 = \frac{2}{c}\left(\frac{\ell_2}{1 - v^2/c^2} - \frac{\ell_1}{\sqrt{1 - v^2/c^2}}\right)$$

= 0

$\Delta t' - \Delta t$ = 0, we just would not have any length contraction in the direction of motion in this case

Fitzgerald’s ad hoc idea – is correct, we see it later again

$$l_{contracted} = l_{propper_{length_{when_{at_{rest}}}}} \cdot \sqrt{1 - v^2/c^2}$$

In order to get a length contraction of 10 % of a meter stick flying past you, it would need to have a speed v of about 43.6 % the speed of light

What if v = c ? Length would contract to absolutely nothing, cannot happen as we will see in the wider framework of special relativity, nothing with mass can be accelerated all the way up to the speed of light however much energy would be dedicated to that attempt

(*Bottom*) Clock tower and electric trolley in Bern on Kramstrasse, the street on which Einstein lived. If you are on the trolley moving away from the clock and look back at it, the light you see must catch up with you. If you move at nearly the speed of light, the clock you see will be slow. In this, Einstein saw a clue to the variability of time itself. [*Underwood & Underwood/CORBIS.*]

"What lead me more or less directly to the special theory of relativity was the conviction that the electromagnetic force acting on a body in motion in a magnetic field was due to nothing else but an electric field."

there will be time dilation in addition to length contraction … as there is a 4D space time continuum …

2.3: Einstein's Postulates

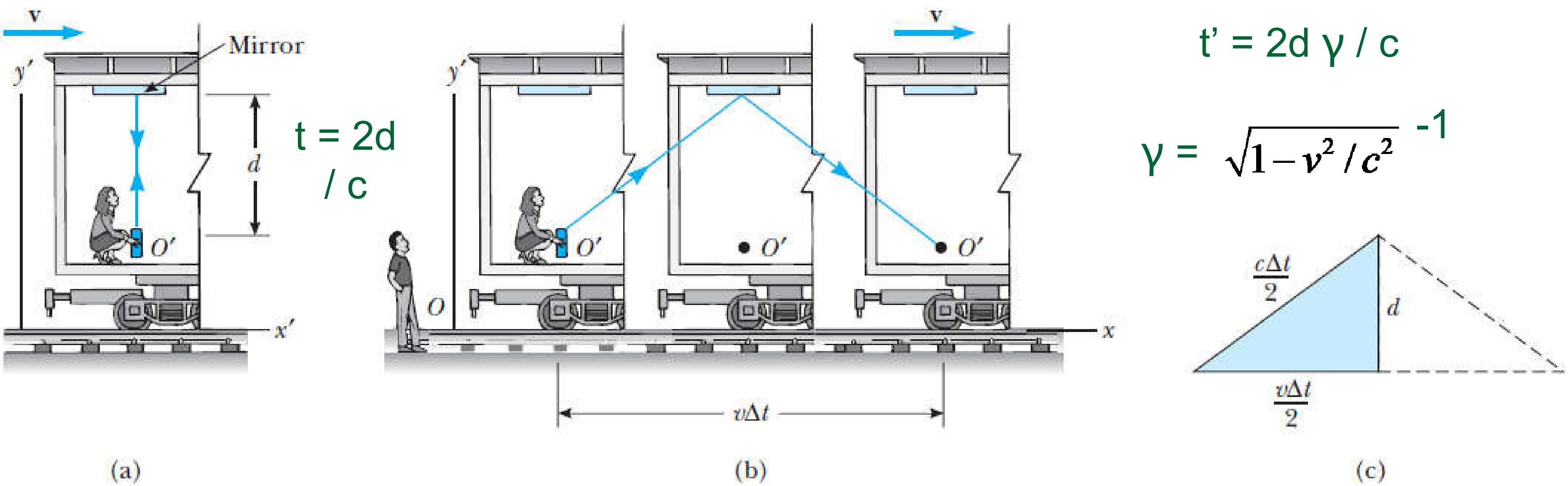
- Albert Einstein (1879–1955) began thinking seriously at the age of 16 about the nature of light and later on about the deep connections between electric and magnetic effects
- In 1905, at the age of 26, he published his startling proposal about the **principle of relativity of inertial frames of reference (special relativity)**
- no reference to Michelson's NULL result, no reference to any other work, just the work of a genius in his spare time all by himself (which nobody asked him to do and paid for …)

Einstein's Two Postulates

With the conviction that Maxwell's equations must be valid in all inertial frames, Einstein proposed in 1905 the following postulates:

1) **The principle of (special) relativity**: The laws of all of physics (not only mechanics) are the same in all inertial frames of reference. There is no way to detect absolute motion (along a straight line without acceleration) by any kind of physical experiment, and no preferred inertial system exists.
2) **The constancy of the speed of light**: Observers in all inertial frames of reference must measure the same value for the speed of light in a vacuum.

By 1915, accelerated motion of any kind cannot be detected either, it's just equivalent to a gravitational field = general relativity



$t = 2d / c$

$t' = 2d\,\gamma / c$

$\gamma = \left(\sqrt{1 - v^2/c^2}\right)^{-1}$

Figure 1.10 (a) A mirror is fixed to a moving vehicle, and a light pulse leaves O' at rest in the vehicle. (b) Relative to a stationary observer on Earth, the mirror and O' move with a speed v. Note that the distance the pulse travels measured by the stationary observer on Earth is greater than $2d$. (c) The right triangle for calculating the relationship between Δt and $\Delta t'$.

Basically the same as A to D and back on slide 15

The girl has a "light clock", all processes in the frame in which she is at rest obey this clock,

The boy watches her light clock, for him her clock runs slower (as she is moving past him), because one of its tick-tocks takes longer than an identical tick-tock on an identical light clock he might use that is stationary to him

The situation is symmetric, the girl can claim that his clock is delayed (and for her it will really be), if something moves past you, its time slows down (really!)

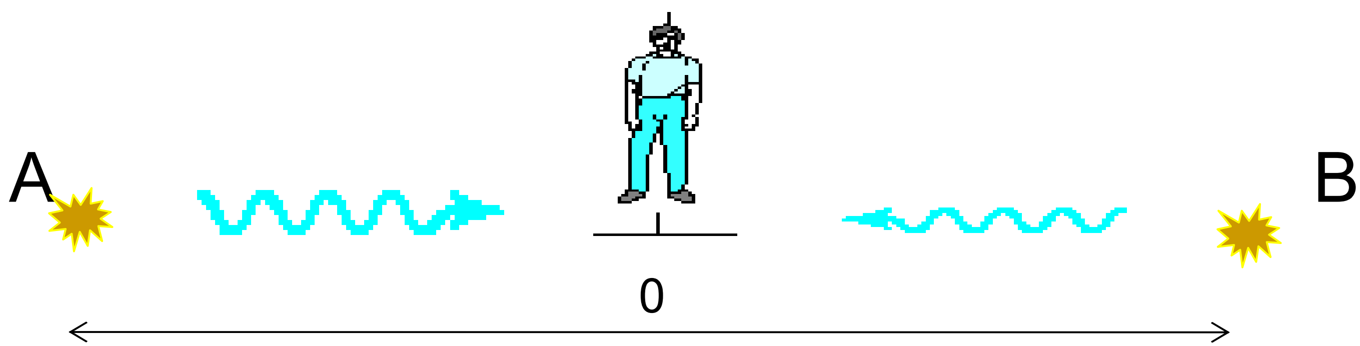
Re-evaluation of Time

- Newtonian physics assumed that $t = t'$

- Einstein does not, as he realized that each inertial frame has its own observers with their own clocks and meter sticks
  - Events considered simultaneous in K are not in K' since it is moving, but we know how to transfer between both frames so that the two types of observers agree on their measurement

The Simultaneity in one inertial frame

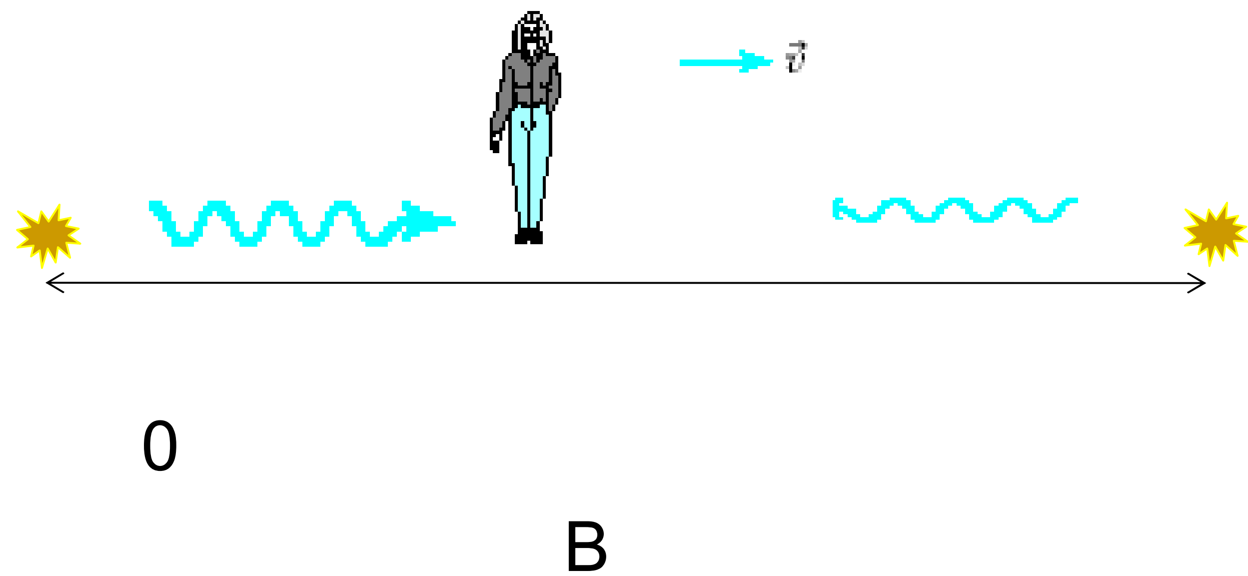
Frank at rest is equidistant from events A and B, say at the middle of an exceedingly fast moving train:



Frank "sees" both flashbulbs go off simultaneously.

The Problem of Simultaneity

Mary standing on the trains station sees the train moving to the right with speed *v*, and observes events A before event B



Frank and Mary are both right, they just need to use the Lorentz transformations instead of relying on Galilean relativity,

We thus observe…

- *Two events that are simultaneous in one reference frame (i.e. K or K') are not simultaneous in another reference frame (K' or K) moving in a straight line with respect to the first frame.*

- *As far as physics is concerned the train may as well stand still while the train station and with it the rest of the town/village moves away, so the situation is completely symmetric as far a physics is concerned, all motion is relative, one cannot prove by any physics experiment that one is moving or at rest*

- *For constant linear motion in a straight line – special relativity*
- *For all types of motion = general relativity (includes special relativity as limiting case)*

The Lorentz Transformations

The special set of linear transformations that had been found earlier which:

preserve the constancy of the speed of light (c) between all inertial observers; as this is a prediction of Maxwell's equation, all the rest of Maxwell's electrodynamics is also invariant to these transformations; and sure enough,

also account for the apparent problem of simultaneity of events as observed from different inertial frames of reference

Lorentz Transformation Equations

$$x' = \frac{x - vt}{\sqrt{1 - v^2 / c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - (vx / c^2)}{\sqrt{1 - v^2 / c^2}}$$

Lorentz Transformation Equations

Short form:

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$x' = \gamma(x - \beta ct)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma(t - \beta x/c)$$

Gamma always larger than one (for some observer) for anything with mass that cannot move as fast as an electromagnetic wave, beta always smaller than one, often very much smaller

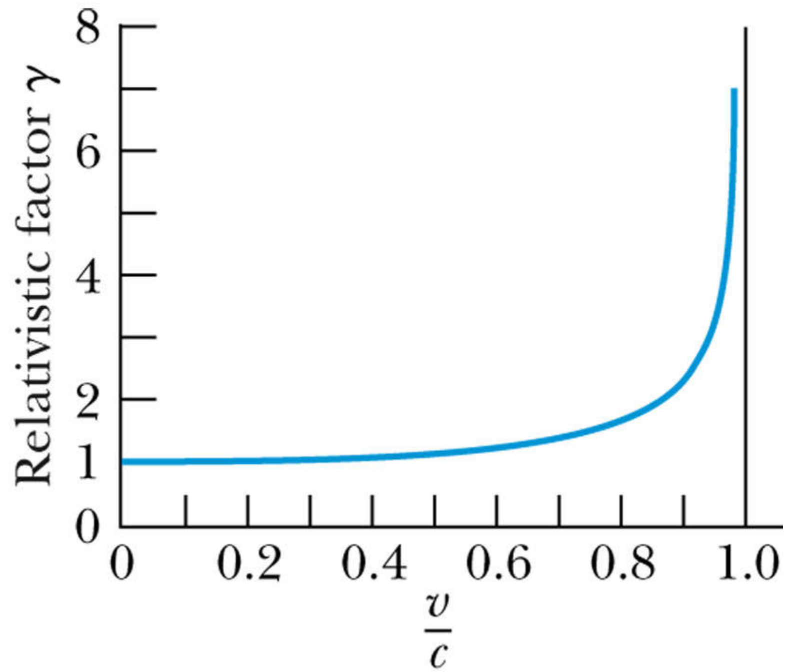
Space and time mix in these transformation, very loosely speaking they are *kind of* the same thing, $x_4 = i\,c\,t$

Properties of γ

Recall $\beta = v/c < 1$ for all observers (with mass).

1) $\gamma \geq 1$ equals 1 only when $v = 0$ for one observer.

1) Graph of gamma vs. β: (note $v < c$)



γ sufficiently close to 1 for most purposes, e.g. Newton's laws can be used, when speeds < 1% of c

Maxwell's equations are invariant with respect to Lorentz Transformation, take the same form in moving frames and frames at rest, are valid on all planets

$$x' = \frac{x - vt}{\sqrt{1-\beta^2}} \qquad x = \frac{x' + vt'}{\sqrt{1-\beta^2}}$$

$$y' = y \qquad y = y'$$

$$z' = z \qquad z = z'$$

$$t' = \frac{t - (vx/c^2)}{\sqrt{1-\beta^2}} \qquad t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1-\beta^2}}$$

Remarks

1) If $v << c$, i.e., $\beta \approx 0$ and $\gamma \approx 1$, we see that these equations reduce to the familiar Galilean transformation.

1) Space and time are no longer separated, formally multiply time with the speed of light (and with the imaginary unit *i*, the square root of 1) and you get physical dimension meter, just like the physical dimensions of the other three spatial dimensions, so there really is a 4D space-time continuum

1) velocity in any reference frame and between frames (where there are masses) cannot exceed *c*.

1) Lorentz transformations correspond to a rotation in 4D space time (Galilean transformations are only a shift in 3D space)

2.5: Time Dilation and Length Contraction

Consequences of the Lorentz Transformation:

- **Time Dilation**:
  Clocks in K' run slow with respect to stationary clocks in K.

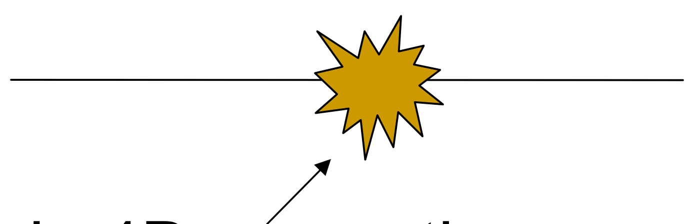
- **Length Contraction**:
  Lengths in K' (meter sticks in the direction of motion, the space itself in that direction) are contracted with respect to the same meter sticks and lengths stationary in K.

Note that we are free to interpret what is K and what is K', so we need concepts of proper time and length

Time Dilation

To understand *time dilation* the idea of **proper time** must be understood:

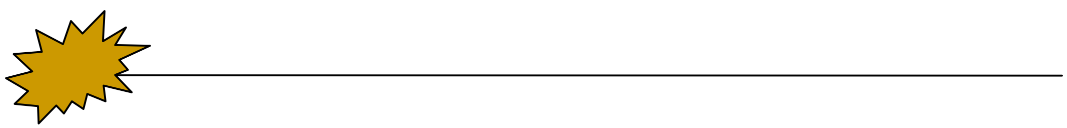
- The term **proper time**, T_0, is the time difference between two events occurring at the same position in an inertial frame as measured by a clock at that position.



Same location in 4D space-time, proper time is not delayed

Time Dilation

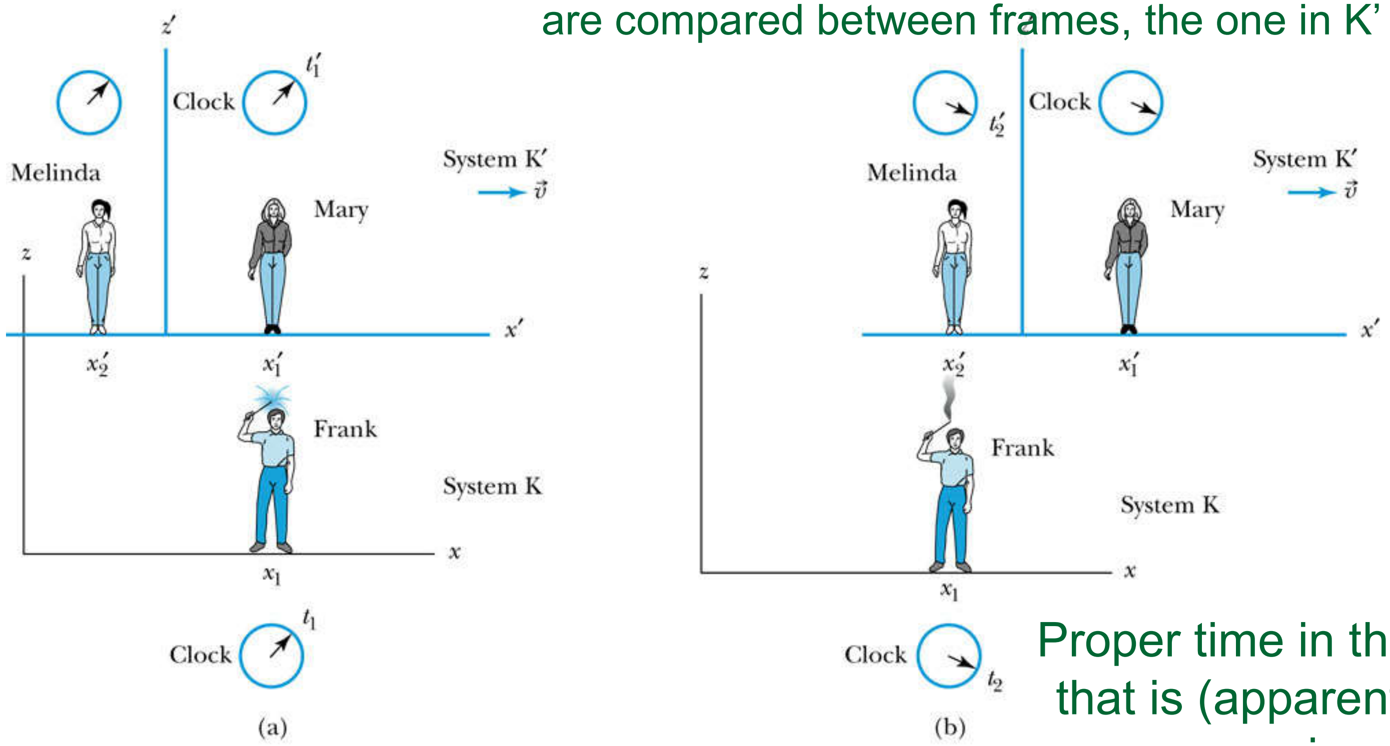
Not Proper Time



Beginning and ending of the event occur at different spatial positions

Time Dilation

we don’t see time delay on the clock in the moving frame, for them all is fine, it is just if the time intervals are compared between frames, the one in K’ is longer



Proper time in the frame that is (apparently) not moving

Frank’s clock is at the same position in system K when the sparkler is lit in (a) and when it goes out in (b). Mary, in the moving system K’, is beside the sparkler at (a). Melinda then moves into the position where and when the sparkler extinguishes at (b). Thus, Melinda, at the new position, measures the time in system K’ when the sparkler goes out in (b).

Time Dilation

1) $T' > T_0$ or the time measured between two events at *different positions* is greater than the time between the same events at *one position:* ***time dilation.***

2) The events do not occur at the same space and time coordinates in the two inertial frames

To transform time and space coordinates between inertial frames, one needs to use the Lorentz transformations (instead of the Galilean transformations)

There is no physical difference between K and K', proper time is not delayed, we just assigned proper time to Frank, we could as well have analyzed the problem from the frame of the two girls, then they would have the proper time

According to Mary and Melinda…

- Mary and Melinda measure the two times for the sparkler to be lit and to go out in system K' as times t'_1 and t'_2 so that by the Lorentz transformation:

$$t'_2 - t'_1 = \frac{(t_2 - t_1) - (v/c^2)(x_2 - x_1)}{\sqrt{1 - v^2/c^2}}$$

  - Note here that Frank records $x_2 - x_1 = 0$ in K with a proper time: $T_0 = t_2 - t_1$ or

$$T' = \frac{T_0}{\sqrt{1 - v^2/c^2}} = \gamma T_0$$

with $T' = t'_2 - t'_1$

Length Contraction

To understand *length contraction* the idea of **proper length** must be understood:

- Let an observer at rest in each system K and K' have a meter stick at rest in their own system such that each measure the same length at rest.
- The length as measured at rest is called the **proper length**. **Proper length is not contracted.**

What Frank and Mary see…

Each observer lays the stick down along his or her respective *x* axis, putting the left end at x_ℓ (or x'_ℓ) and the right end at x_r (or x'_r).

- Thus, in system K, Frank measures his stick to be:

$$L_0 = x_r - x_\ell$$

- Similarly, in system K', Mary measures her stick at rest to be:

$$L'_0 = x'_r - x'_\ell$$

- **Both measure proper lengths in their own frames**

For the ladies, Frank's unit length is contracted, for him the unit length of the ladies is contracted, why because the experiments/theory says so …

- Frank in his rest frame measures the "moving meter stick's length" in Mary's frame (that moves with respect to him).
- Vice versa, Mary measures the same in Frank's frame (that moves with respect to her)
- Thus using the Lorentz transformations Frank measures the length of the stick in K' as:

$$x'_r - x'_\ell = \frac{(x_r - x_\ell) - v(t_r - t_\ell)}{\sqrt{1 - v^2/c^2}}$$

both ends of the stick measured simultaneously, i.e., $t_r = t_\ell$ for Frank

Frank's proper length is measured $L = x_r - x_\ell$, He measures Mary's stick as shortened by the inverse of the Lorentz factor $L' = (x'_r - x'_\ell)\,\gamma^{-1}$

sure if v = 0, both lengths are the same

$$\Delta x' \cdot \sqrt{1 - vc^2} = \Delta x = x_2 - x_1$$

Situation is again symmetric just as time dilation was

Frank's viewpoint is: Mary is in the moving frame, but she can claim just the same, that Frank is in a moving frame and will be equally right!

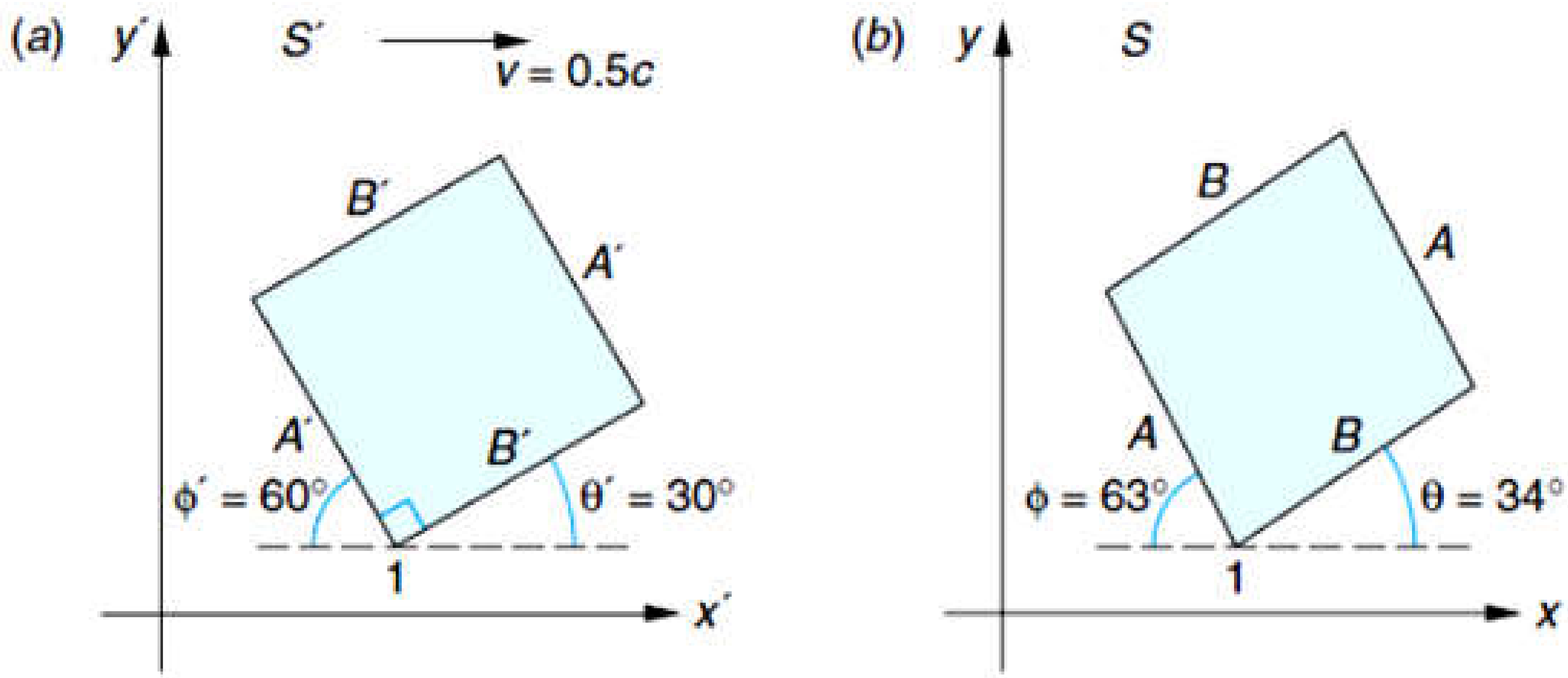


Figure 1-30 Length contraction distorts the shape and orientation of two- and three-dimensional objects. The observer in S measures the square shown in S' as a rotated parallelogram.

Both a moving and a stationary observer are corrected, they just have to relate their observations to each other by taking the Lorentz transformations into account

2.6. Experimental verification special relativity, why are there so many muons detected on earth?

$$N(t) = N_0 e^{(-t/\tau)}$$

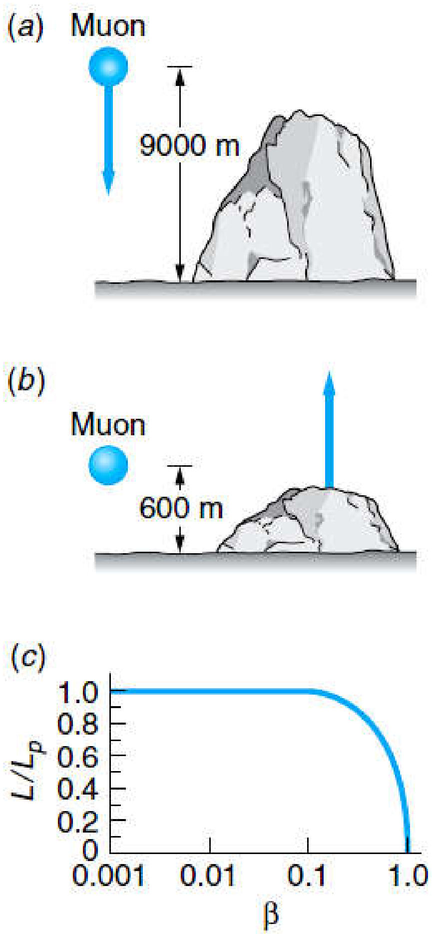


Figure 1-31 Although muons are created high above Earth and their mean lifetime is only about 2 μs when at rest, many appear at Earth's surface. (*a*) In Earth's reference frame, a typical muon moving at 0.998*c* has a mean lifetime of 30 μs and travels 9000 m in this time. (*b*) In the reference frame of the muon, the distance traveled by Earth is only 600 m in the muon's lifetime of 2 μs. (*c*) L varies only slightly from L_p until v is of the order of 0.1c. $L \to 0$ as $v \to c$.

γ ≈ 15, pretty significant

In (b) the muon is considered at rest, earth moves upwards with respect to it, length is contracted in that direction

With L as length and Lp as proper length, β = v/c

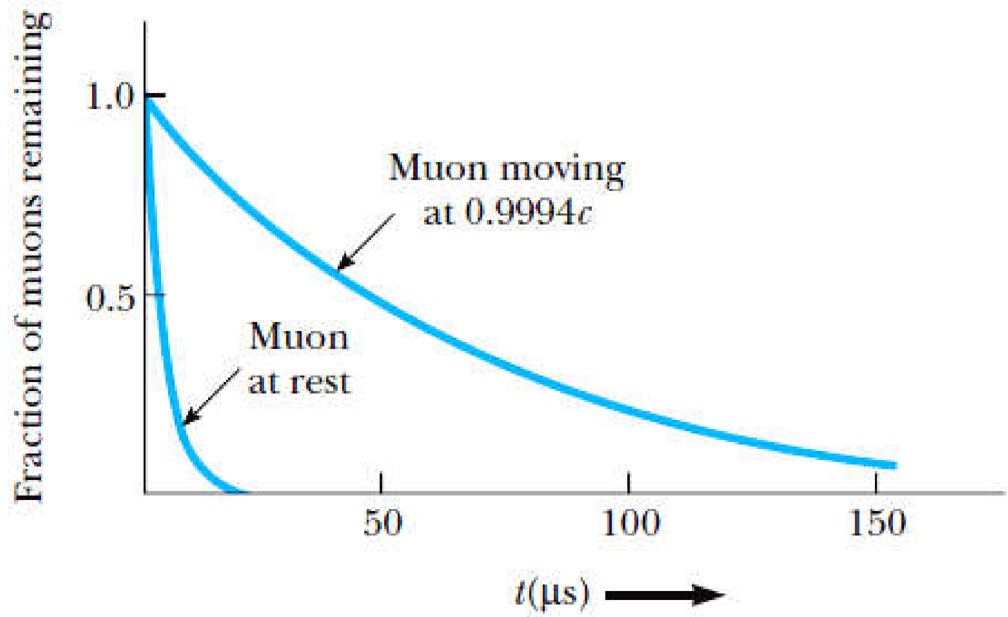


Figure 1.12 Decay curves for muons traveling at a speed of $0.9994c$ and for muons at rest.

Length contraction was symmetric, how about the Twin Paradox

The Set-up

Twins Mary and Frank at age 30 decide on two career paths: Mary decides to become an astronaut and to leave on a trip 8 light years (ly) from the Earth at a great speed and to return; Frank decides to reside on the Earth.

The Problem

Upon Mary's return, Frank reasons that her clocks measuring her age must run slow. As such, she will return younger. However, Mary claims that it is Frank who is moving and consequently his clocks must run slow.

The Paradox

Who is younger upon Mary's return?

The "paradox" is only a conflict between reality and your feeling of what reality "ought to be." R. P. Feynman

The Resolution

Frank's clock is in an **inertial system** during the entire trip; however, Mary's clock is not. So this paradox has nothing to do with special relativity

as long as Mary is traveling at constant speed away from and towards Frank, both of them can argue that the other twin is aging less rapidly; but that is only part of the story, **acceleration and deceleration are required for such a trip, so this all becomes a problem in general relativity (where gravity effects time !!!)**

When all effects are taken care off (in general relativity) Mary is indeed somewhat younger (less aged) than Frank

If we placed a living organism in a box . . . one could arrange that the organism, after an arbitrary lengthy flight, could be returned to its original spot in a scarcely altered condition, while corresponding organisms which had remained in their original positions had long since given way to new generations. **(Einstein's original statement of the twins paradox in 1911)**

Just not a practical proposition, unfortunately he doesn't state here clearly that this is not a special relativity problem, so many people have misunderstood him.

The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a union of the two will preserve an independent reality. (Hermann Minkowski, 1908, in an address to the Assembly of German Natural Scientists and Physicians)

Summary Michelson-Morley experiment

1. If the apparatus is not moving with respect to "special" inertial reference frames (ether or sun), there is no shift of the interference pattern, v = 0

2. If the apparatus is moving with respect to the "special" inertial reference frames (ether or sun) at any velocity, there is no shift of the interference pattern, just length contraction and time delay (consequences of Lorentz transformations) for any v ǂ 0

3. If we move with respect of the apparatus with u_x (watching the interference pattern from afar with a telescope), there is also no shift in the interference pattern (just length contraction of the fringes since we can claim that the apparatus moves with respect to us) regardless if v = 0 or v ǂ 0 in that "special" inertial frame, (but we would need to apply special relativity velocity additions derived from Lorentz transformations in the latter case)

Conclusion, relative motion cannot be detected by an experiment that involves electrodynamics (constant velocity of light) either, there is no special inertial reference frame at all (and thus no need for the ether), all inertial frames of reference are equally valid and the speed of light is constant $c = 1/\sqrt{\mu_0 \varepsilon_0}$ in all of them !

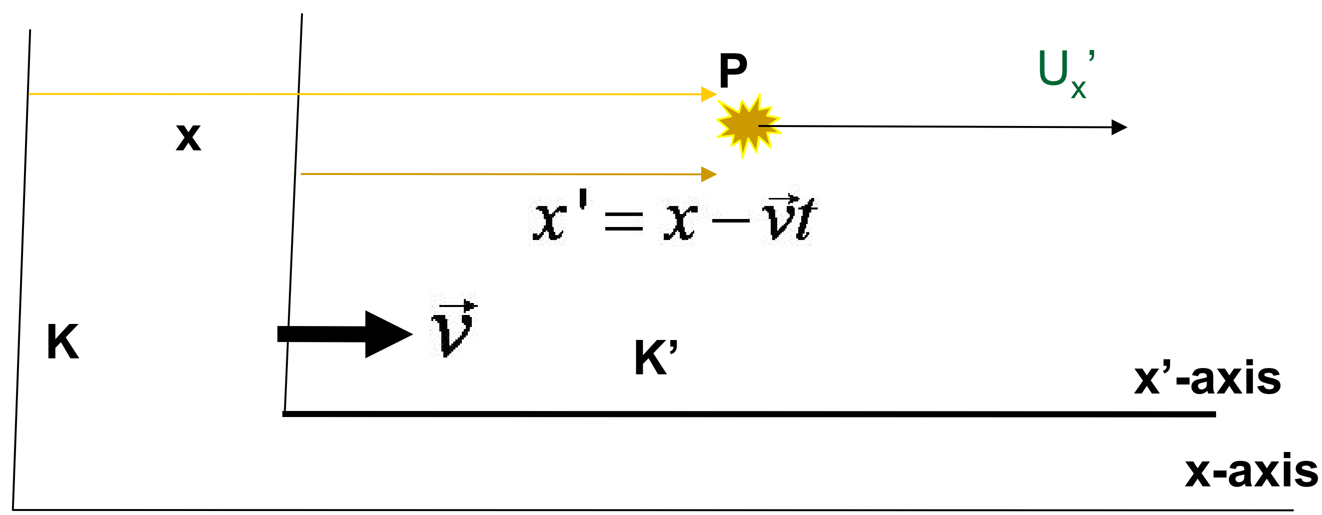
In general relativity, c will also be this constant, consequence is bend space time when masses are around

2.8.1: Addition of Velocities using Galilean transformations

point P is moving along a straight line to the right with constant velocity u_x' (in the moving frame along x)

At any one time, in system K: P = (x, y, z, t)

- In system K': P = (x', y', z', t')



Assume: space and time are separate, both are fundamental frames in which physical processes are happening

2.8.1: Addition of Velocities using Galilean transformations

$$x' = x - \vec{v}\,t$$

$$dx' = dx - \vec{v}\,dt$$

$$t' = t$$

$$dt' = dt$$

$u_y = 0 = u_z = u_y' = u_z'$

$$x = x' + \{\vec{v}\,t\}$$

$$dx = dx' + \{\vec{v}\,dt\}$$

$$\frac{dx'}{dt'} = \frac{dx}{dt} - \vec{v}\,\frac{dt}{dt} = \vec{u}_x' = \{\vec{u}_x - \vec{v}\}$$

To get the velocity of "something" in a moving system, u_x', the velocity between frames, v, need to be taken off the velocity of the "same thing" with respect to a system that is considered at rest

$$\frac{dx}{dt} = \frac{dx'}{dt'} + \vec{v}\,\frac{dt}{dt} = \vec{u}_x = \vec{u}_x' + \{\vec{v}\}$$

To get the velocity of "something" in a frame considered at rest u_x, the velocity between frames v, need to be added the velocity of the "same thing" with respect to a frame that moves with respect to the frame at rest

For simplicity, we just have u_x, v and u_x' all moving along the x axis

2.8.2: Addition of Velocities using Lorentz transformation

Taking differentials of the Lorentz transformation, relative velocities are obtained, further trick d of differential can be expanded to a delta as the velocity is constant

$$dx = \gamma(dx' + v\,dt')$$

$$dy = dy'$$

$$dz = dz'$$

$$dt = \gamma[dt' + (v/c^2)\,dx']$$

We have v parallel to the x-axis for simplicity

So that…

defining velocities as: $u_x = dx/dt$, $u_y = dy/dt$, $u'_x = dx'/dt'$, etc. it is easily shown that:

$$u_x = \frac{dx}{dt} = \frac{\gamma(dx' + v\,dt')}{\gamma[dt' + (v/c^2)\,dx']} = \frac{u'_x + v}{1 + (v/c^2)u'_x}$$

What if $u_x' = c$? u_x is also c, so the speed of light is a constant in all inertial frames of reference !!

With interesting relations for u_y and u_z that result from γ due to relativistic movement between frames (v ǂ 0) along x

$$u_y = \frac{u'_y}{\gamma\left[1 + (v/c^2)u_x'\right]} \qquad u_z = \frac{u'_z}{\gamma\left[1 + (v/c^2)u_x'\right]}$$

In the limit v and u << c, we obtain Galilean velocity addition laws

Lorentz transformations correspond to rotations in 4 dimensional space time

Lorentz Velocity Transformations

In addition to the previous relations, the **Lorentz velocity transformations** for u'_x, u'_y, and u'_z can be obtained by switching primed and unprimed and changing v to $-v$

$$u'_x = \frac{u_x - v}{1-(v/c^2)u_x}$$

What if $u_x = c$? u_x' is also c, so the speed of light is a constant in all inertial frames of reference !!

$$u'_y = \frac{u_y}{\gamma\left[1-(v/c^2)u_x\right]}$$

$$u'_z = \frac{u_z}{\gamma\left[1-(v/c^2)u_x\right]}$$

H. L. Fitzeau's famous 1851 experiment

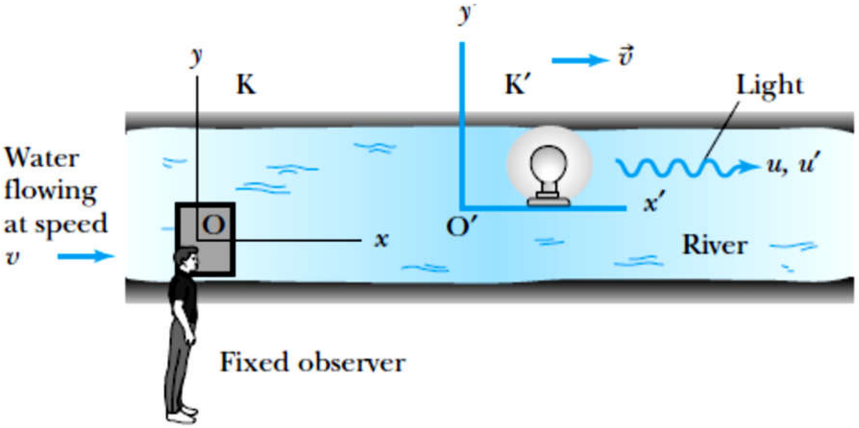


Figure 2.16 A stationary system K is fixed on shore, and a moving system K′ floats down the river at speed v. Light emanating from a source under water in system K′ has speed u, u' in systems K, K′, respectively.

Strategy We note from introductory physics that the velocity of light in a medium of index of refraction n is $u' = c/n$. We use Equation (2.23a) to solve for u.

Solution We have to calculate the speed only in the x-direction, so we dispense with the subscripts. We utilize Equation (2.23a) to determine

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{c/n + v}{1 + v/nc} = \frac{c}{n}\frac{\left(1 + \frac{nv}{c}\right)}{\left(1 + \frac{v}{nc}\right)}$$

Fizeau found experimentally that

$$u = u' + \left(1 - \frac{1}{n^2}\right)v$$

Note that u' and u are both smaller than c !

n = 1.33

Because $v \ll c$ in this case, we can expand the denominator $(1 + x)^{-1} = 1 - x + \cdots$ keeping only the lowest term in $x = v/c$. The above equation becomes

$$u = \frac{c}{n}\left(1 + \frac{nv}{c}\right)\left(1 - \frac{v}{nc} + \cdots\right)$$

$$= \frac{c}{n}\left(1 + \frac{nv}{c} - \frac{v}{nc} + \cdots\right)$$

$$= \frac{c}{n} + v - \frac{v}{n^2} = u' + \left(1 - \frac{1}{n^2}\right)v$$

which is in agreement with Fizeau's experimental result and Fresnel's prediction given earlier. This relativistic calculation is another stunning success of the special theory of relativity. There is no need to consider the existence of the ether.

If Galilean velocity addition was correct, u = u' + v

2.9: The Doppler Effect

- The Doppler effect of sound in introductory physics is represented by an *increased frequency* of sound as a source such as a train (with whistle blowing) approaches a receiver (our eardrum) and a *decreased frequency* as the source recedes.

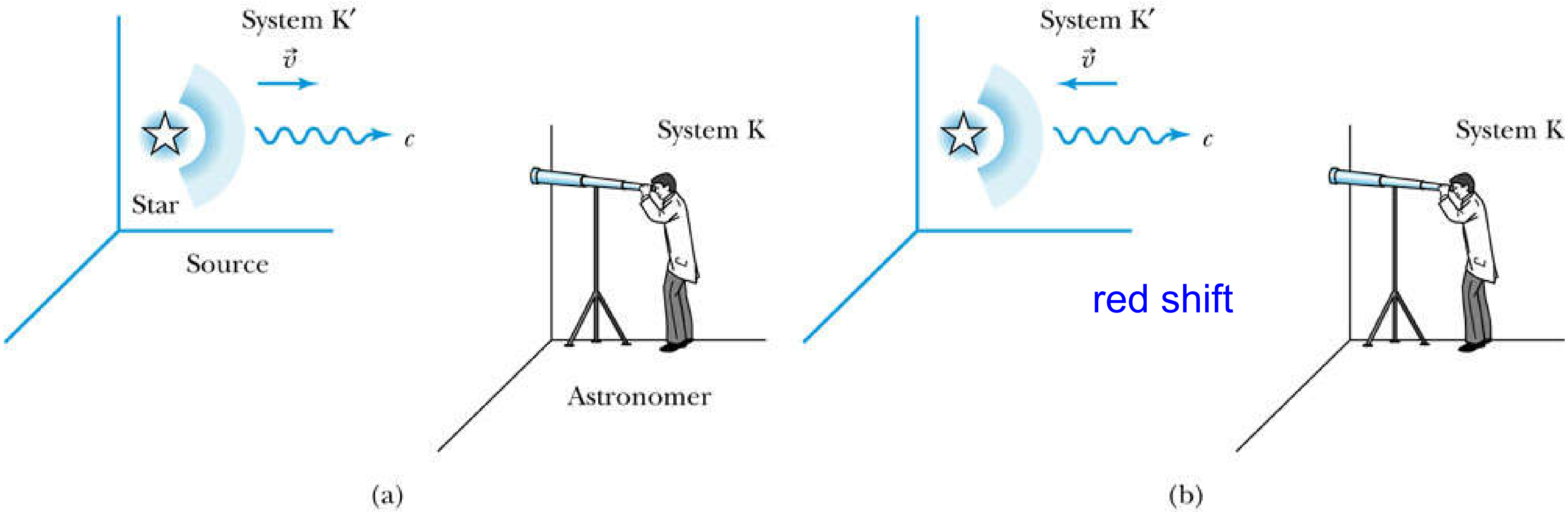
- Also, a change in sound frequency occurs when the source is fixed and the receiver is moving. The change in frequency of the sound wave depends on whether the source or receiver is moving.

- This is, however, a classical physics effect since there is a special frame of reference for sound waves to travel in.

- Well known: pump away the air, a sound wave cannot propagate.

Doppler Effect for light is different

Higher frequency, shorter wavelength, blue shift



red shift

Only difference is whether source and detector are approaching each other or receding from each other along straight lines

Source and Receiver Approaching

With $\beta = v / c$ the resulting frequency from the Doppler effect for electromagnetic radiation is given by:

$$f = \frac{\sqrt{1+\beta}}{\sqrt{1-\beta}} f_0$$

(source and receiver approaching)

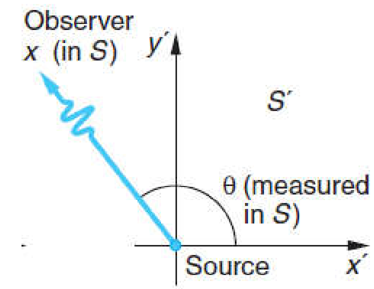
Source and Receiver Receding

In a similar manner, it is found that:

$$f = \frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} f_0$$ (source and receiver receding)

c = λ f, so when f decreases λ must increase, get longer, we call that a red shift, as red light has a larger (longer) wavelength than blue light

Second order (transverse) Doppler effect for light



$$\gamma = \frac{1}{\sqrt{1-u^2/c^2}} \qquad f = \frac{f_0}{\gamma}\frac{1}{1-\beta\cos\theta} \qquad \beta = v/c$$

When $\theta = 0$, this becomes the equation for the source and receiver approaching, and when $\theta = \pi$, the equation becomes that for the source and receiver receding. Equation 1-35*a* also makes the quite surprising prediction that even when viewed perpendicular to the direction of motion, where $\theta = \pi/2$, the observer will still see a frequency shift, the so-called *transverse Doppler effect*, $f = f_0/\gamma$. Note that $f < f_0$ since $\gamma > 1$. It is sometimes referred to as the second-order Doppler effect and is the result of time dilation of the moving source. [The general derivation of Equation 1-35*a* can be found in the French

That's how the police catches cars that go too fast

$$f = \frac{\sqrt{1-\beta^2}}{1-\beta}f_0 = \sqrt{\frac{1+\beta}{1-\beta}}f_0 \quad \text{(approaching)}$$

$$f = \frac{\sqrt{1-\beta^2}}{1+\beta}f_0 = \sqrt{\frac{1-\beta}{1+\beta}}f_0 \quad \text{(receding)}$$

There is no transverse Doppler effect for sound waves

1. Classical relativity		
Galilean transformation	$x' = x - vt \quad y' = y \quad z' = z \quad t' = t$	1-2
Newtonian relativity	Newton's laws are invariant in all systems connected by a Galilean transformation.	
2. Einstein's postulates	The laws of physics are the same in all inertial reference frames. The speed of light is c, independent of the motion of the source.	
3. Relativity of simultaneity	Events simultaneous in one reference frame are not in general simultaneous in any other inertial frame.	
4. Lorentz transformation	$x' = \gamma(x - vt) y' = y\, z' = z$ misleading $t' = \gamma(t - vx/c^2)$ with $\gamma = (1 - v^2/c^2)^{-1/2}$	1-18
5. Time dilation	Proper time is the time interval τ between two events that occur at the same space point. If that interval is $\Delta t' = \tau$, then the time interval in S is $\Delta t = \gamma \Delta t' = \gamma\tau$ where $\gamma = (1 - v^2/c^2)^{-1/2}$	1-26
6. Length contraction	The proper length of a rod is the length L_p measured in the rest system of the rod. In S, moving at speed v with respect to the rod, the length measured is $L = L_p/\gamma$	1-28
7. Spacetime interval	All observers in inertial frames measure the same interval Δs between pairs of events in spacetime, where $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2$ 4D space time consequence	1-31
8. Doppler effect		
Source/ observer approaching	$f = \sqrt{\frac{1 + \beta}{1 - \beta}} f_0$	
Source/ observer receding	$f = \sqrt{\frac{1 - \beta}{1 + \beta}} f_0$	

Since $c = 1/\sqrt{\mu_0 \varepsilon_0}$ in all inertial frames of reference, all of Maxwell's equations will be valid in these frames of reference as well

Maxwell's equations are invariant with respect the Lorentz transformations

Newton's mechanics is invariant with respect to the Galilean transformations
The latter are the low speed approximations of the Lorentz transformations

There is only one world out there and if one needs to consider physics in different inertial frames of reference, one will have to use the Lorentz transformations

So all of Newton's mechanics is only a low speed approximation to special relativity (Einstein) mechanics, which we derive next.

Relativistic Momentum I

Classically for constant v in a straight line

$$u = \frac{\Delta x}{\Delta t} \qquad p = m \cdot u = m \cdot \frac{\Delta x}{\Delta t}$$

Needs to be conserved in collisions, **but we have to include special relativity**

$$p = m \cdot \frac{\Delta x}{\Delta t_0}$$

while Δx is a space distance watched by a stationary (first) observer (not contracted), Δt_0 (proper time), is the time a (second) observer that moves with the particle measures, one can simplify the two observers to one observing movement in his or her own frame while being at rest

With respect to the moving (second) observer, the time of the stationary first observer is delayed

$$\Delta t = \gamma \cdot \Delta t_0$$

$$p = m \cdot \frac{\Delta x}{\Delta t} \cdot \frac{\Delta t}{\Delta t_0} \qquad \gamma = \frac{\Delta t}{\Delta t_0} \qquad p = m \cdot \frac{\Delta x}{\Delta t} \cdot \frac{\Delta t}{\Delta t_0} = \gamma m \cdot u$$

What happens when u -> c, p becomes infinite, i.e. v can come very close to c, but will never reach it

Relativistic Momentum II

- Loosely speaking "leaving u, the movement in the frame alone", we can "blame everything on the mass"

$$p = \gamma m\vec{u} \qquad \gamma = \frac{1}{\sqrt{1-u^2/c^2}}$$

But this kind of Lorentz factor does not include a velocity between frames as it did earlier, just the velocity of something moving with respect to the stationary observer in his own frame

So it ***seams*** like mass were increasing with velocities greater than zero, for movement in its own frame of reference the faster something moves, the larger its momentum already classically, but now there is an extra "Pseudo-Lorentz" factor, the u is in one and the same frame, we do not need to consider two frames moving relative to each other for this effect to occur – tested countless times in particle accelerators !!!

Again this gamma is conceptually different, u is velocity within one frame, v was velocity between frame in former formula for gamma!

$$\frac{m(u = v)}{m(u_0 = 0)} = \frac{u_0}{u_0\sqrt{1 - v^2/c^2}}$$

or

$$m(u) = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

Some books have v for velocity, some u, similarly, K and S, K' and S' for the inertial frames of reference

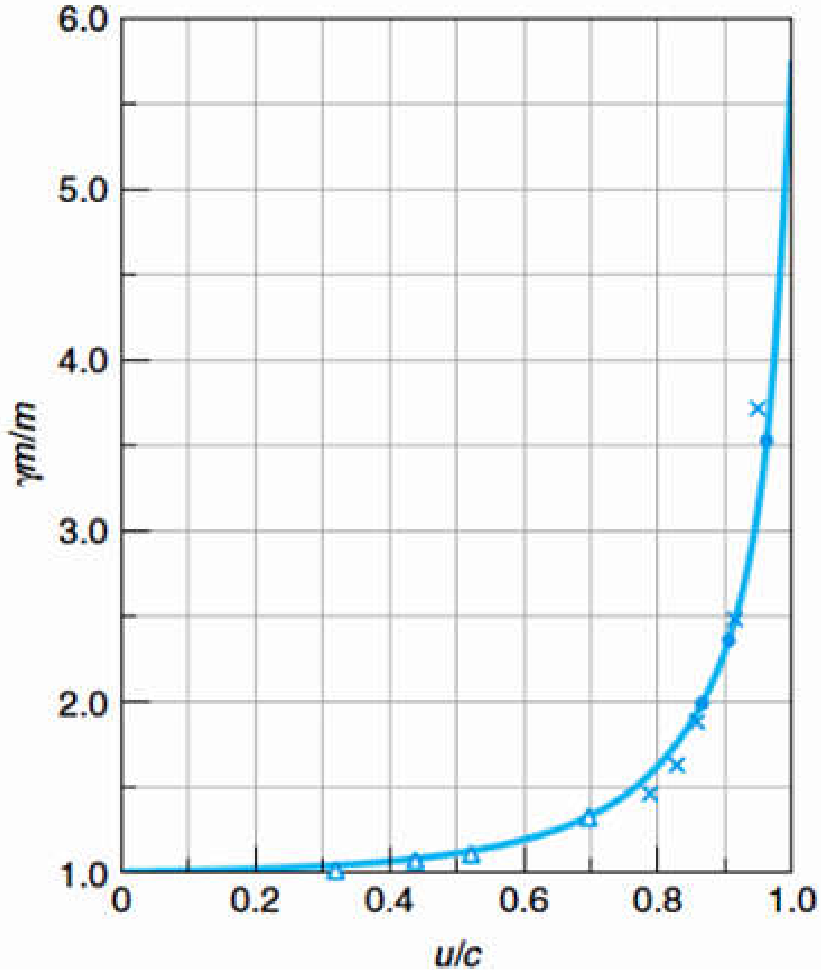


Figure 2-7 A few of the many experimental measurements of the mass of electrons as a function of their speed u/c. The data points are plotted onto Equation 2-5, the solid line. The data points represent the work of W. Kaufmann (×, 1901), A. H. Bucherer (Δ, 1908), and W. Bertozzi (●, 1964). Note that Kaufmann's work preceded the appearance of Einstein's 1905 paper on special relativity. Kaufmann used an incorrect mass for the electron and interpreted his results as support for classical theory. [*Adapted from Figure 3-4 in R. Resnick and D. Halliday, Basic Concepts in Relativity and Early Quantum Theory, 2d ed. (New York: Macmillan, 1992).*]

Mass is not really increasing with velocity, but imagining it were one can keep one's physical intuition

EXAMPLE 2-1 **Measured Values of Moving Mass** For what value of u/c will the measured mass of an object γm exceed the rest mass by a given fraction f?

SOLUTION

From Equation 2-5 we see that

$$f = \frac{\gamma m - m}{m} = \gamma - 1 = \frac{1}{\sqrt{1 - u^2/c^2}} - 1$$

Solving for u/c,

$$1 - u^2/c^2 = \frac{1}{(f + 1)^2} \longrightarrow u^2/c^2 = 1 - \frac{1}{(f + 1)^2}$$

or

$$u/c = \frac{\sqrt{f(f + 2)}}{f + 1}$$

from which we can compute the table of values below or the value of u/c for any other f. Note that the value of u/c that results in a given fractional increase f in the measured value of the mass is independent of m. A diesel locomotive moving at a particular u/c will be observed to have the same f as a proton moving with that u/c.

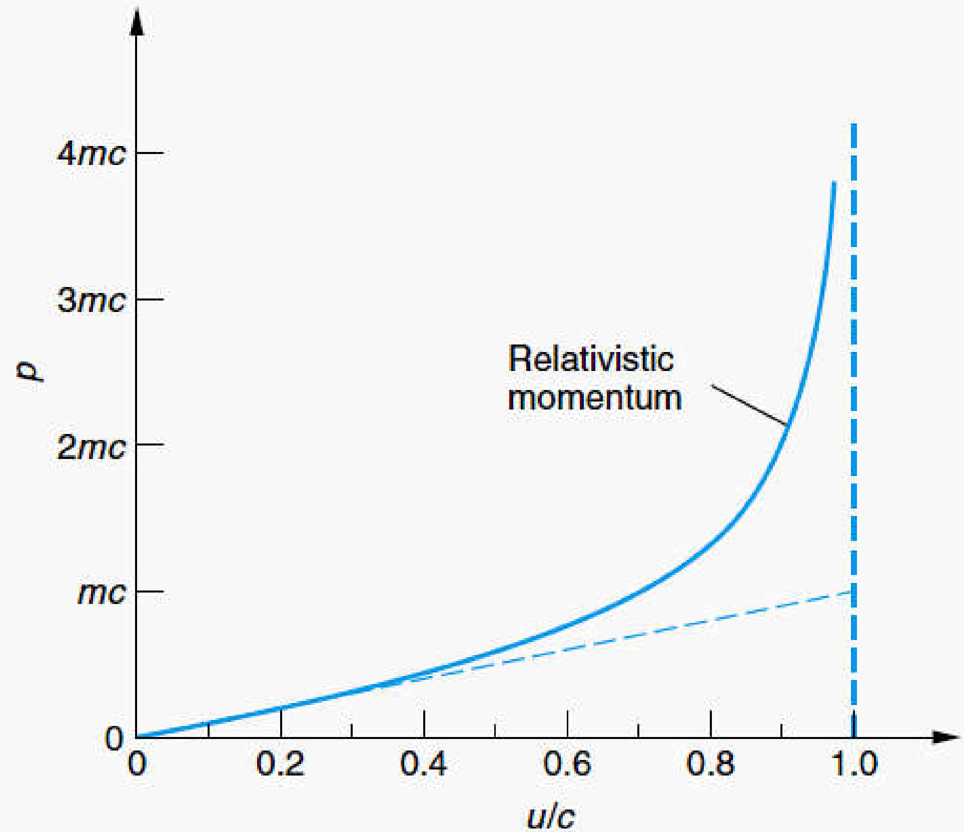
f	u/c	Example
10^{-12}	1.4×10^{-6}	jet fighter aircraft
5×10^{-9}	0.0001	Earth's orbital speed
0.0001	0.014	50-eV electron
0.01 (1%)	0.14	quasar 3C273
1.0 (100%)	0.87	quasar 0Q172
10	0.996	muons from cosmic rays
100	0.99995	some cosmic ray protons

$$p = \gamma m \vec{u}$$

$$\gamma = \frac{1}{\sqrt{1 - u^2 / c^2}}$$

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d(\gamma m \mathbf{u})}{dt}$$

Figure 2-2 Relativistic momentum as given by Equation 2-6 versus u/c, where u = speed of the object relative to an observer. The magnitude of the momentum p is plotted in units of mc. The fainter dashed line shows the classical momentum mu for comparison.



Relativistic Force

- Due to the new idea of "relativistic mass", we must now redefine the concepts of work and energy.
  - Therefore, we modify Newton's second law to include our new definition of linear momentum, and force becomes:

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(\gamma m\vec{u}) = \frac{d}{dt}\left(\frac{m\vec{u}}{\sqrt{1-u^2/c^2}}\right)$$

So a constantly increasing force does no longer produce a constantly increasing acceleration, impossibility to accelerate something with mass to the speed of light

Reason why no particle with mass can move faster than speed of light

It is reasonable to expect that $\mathbf{F} = m\mathbf{a}$ does *not* hold at high speeds, for this equation implies that a constant force will accelerate a particle to unlimited velocity if it acts for a long time. However, if a particle's velocity were greater than c in some reference frame S, we could not transform from S to the rest frame of the particle because γ becomes imaginary when $v > c$. We can show from the velocity transformation that, if a particle's velocity is less than c in some frame S, it is less than c in all frames moving relative to S with $v < c$. This result leads us to expect that particles never have speeds greater than c. Thus, we expect that Newton's second law $\mathbf{F} = m\mathbf{a}$ is not relativistically invariant. We will, therefore, need a new law of motion, but one that reduces to Newton's classical version when $\beta(=v/c) \rightarrow 0$, since $\mathbf{F} = m\mathbf{a}$ is consistent with experimental observations when $\beta \ll 1$.

If a particle has acceleration a_x and velocity u_x in frame S, its acceleration in S', obtained by computing du'_x/dt' from Equation 1-22, is

$$a'_x = \frac{a_x}{\gamma^3(1 - vu_x/c^2)^3}$$

Thus, F/m must transform in a similar way, or else Newton's second law, $\mathbf{F} = m\mathbf{a}$, does not hold.

Reason why no particle with mass can move faster than speed of light

Vice versa, in order to keep on accelerating a particle constant the force on a particle needs to increase beyond bounds, would need to be infinite for u = c

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d(\gamma m\mathbf{u})}{dt}$$

(explicit formula would not give any more insight)

$$W = \Delta K = \int \frac{d}{dt}(\gamma m\vec{u}) \cdot \vec{u}\; dt$$

Another ways of saying essentially the same thing is that an infinite amount of energy would be required to bring u all the way up to c with a particle of mass

Relativistic Energy

The work W_{12} done by a force $\vec{F}$ to move a particle from position 1 to position 2 along a path $\vec{s}$ is defined to be

$$W_{12} = \int_1^2 \vec{F} \cdot d\vec{s} = K_2 - K_1$$

where K_1 is defined to be the kinetic energy of the particle at position 1.

Relativistic Energy

For simplicity, let the particle start from rest under the influence of the force and calculate the kinetic energy *K* after the work is done.

$$W = K = \int \frac{d}{dt}(\gamma m\vec{u}) \cdot \vec{u}\; dt$$

Remember work is the change in kinetic energy

Relativistic Kinetic Energy

$$K = m\int dt\,\frac{d}{dt}(\gamma\vec{u})\cdot\vec{u} = m\int u\,d(\gamma u)$$

The limits of integration are from an initial value of 0 to a final value of γu.

$$K = m\int_0^{\gamma u} u\,d(\gamma u) \qquad (2.57)$$

Calculating this integral is straightforward if done by the method of integration by parts. The result, called the *relativistic kinetic energy*, is

$$K = \gamma mc^2 - mc^2 = mc^2\left(\frac{1}{\sqrt{1-u^2/c^2}} - 1\right) = mc^2(\gamma - 1)$$

Relativistic Kinetic Energy

does not seem to resemble the classical result for kinetic energy, $K = \frac{1}{2}mu^2$. However, if it is correct, we expect it to reduce to the classical result for low speeds. Let's see if it does. For speeds $u \ll c$, we expand γ in a binomial series as follows:

$$K = mc^2\left(1 - \frac{u^2}{c^2}\right)^{-1/2} - mc^2$$

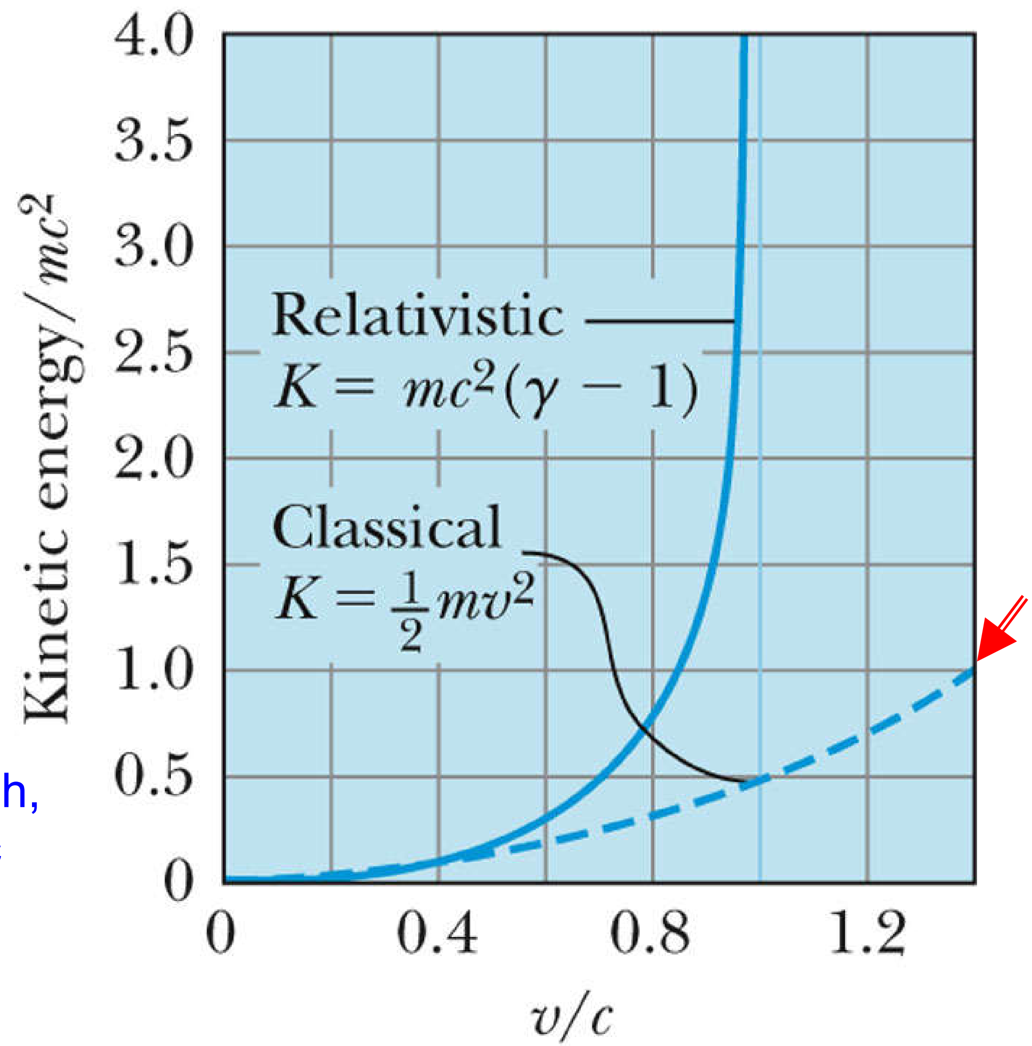
$$= mc^2\left(1 + \frac{1}{2}\frac{u^2}{c^2} + \ldots\right) - mc^2$$

where we have neglected all terms of power $(u/c)^4$ and greater, because $u \ll c$. This gives the following approximation for the relativistic kinetic energy at low speeds:

$$K = mc^2 + \frac{1}{2}mu^2 - mc^2 = \frac{1}{2}mu^2$$

which is the expected classical result. We show both the relativistic and classical kinetic energies in the following Figure. They diverge considerably above a velocity of 0.5 *c, divergence starts at about 10% of c, but for less than 1% of the speed of light one can use the classical formula.*

Relativistic and Classical Kinetic Energies



½ $(1.4)^2$ gives 0.98 on the y-axis

Non-perfect graph, relativistic kinetic energy is always larger than its classical counterpart

Total Energy

is relativistic kinetic energy plus rest energy

$$\gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} = K + mc^2$$

The term *mc^2* is called the rest energy and is denoted by *E_0*.

$$E_0 = mc^2$$

This leaves the sum of the kinetic energy and rest energy to be interpreted as the total energy of the particle. The total energy is denoted by *E* and is given by

$$E = \gamma mc^2 = \frac{mc^2}{\sqrt{1-u^2/c^2}} = \frac{E_0}{\sqrt{1-u^2/c^2}} = K + E_0$$

Momentum and Energy

$$p = \gamma m u = \frac{mu}{\sqrt{1 - u^2 / c^2}}$$

We square this result, multiply by c^2, and rearrange the result.

$$p^2 c^2 = \gamma^2 m^2 u^2 c^2$$

$$= \gamma^2 m^2 c^4 \left(\frac{u^2}{c^2} \right) = \gamma^2 m^2 c^4 \beta^2$$

replace β^2 by its earlier definition

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \beta^2}}$$

v = u

$$p^2 c^2 = \gamma^2 m^2 c^4 \left(1 - \frac{1}{\gamma^2} \right)$$

$$= \gamma^2 m^2 c^4 - m^2 c^4$$

Momentum and Energy (continued)

The first term on the right-hand side is just E^2, and the second term is E_0^2. This equation becomes (the accelerator equation)

$$p^2c^2 = E^2 - E_0^2$$

We rearrange this last equation to find the result we are seeking, a relation between energy and momentum.

or

$$E^2 = p^2c^2 + E_0^2$$

is a useful result to relate $E^2 = p^2c^2 + m^2c^4$ of a particle with its momentum. The quantities $(E^2 - p^2c^2)$ and m are invariant quantities. Note that when a particle's velocity is zero and it has no momentum, this equation correctly gives E_0 as the particle's total energy, but there can also be mass-less particles (e.g. photons) that have momentum and energy

Modified conservation law: Total Energy $E = \gamma m c^2$ is conserved in an isolated system, this includes all energies and masses, no separate conservation law for chemical reactions

$$KE = \gamma mc^2 - mc^2 = totalE - restE$$

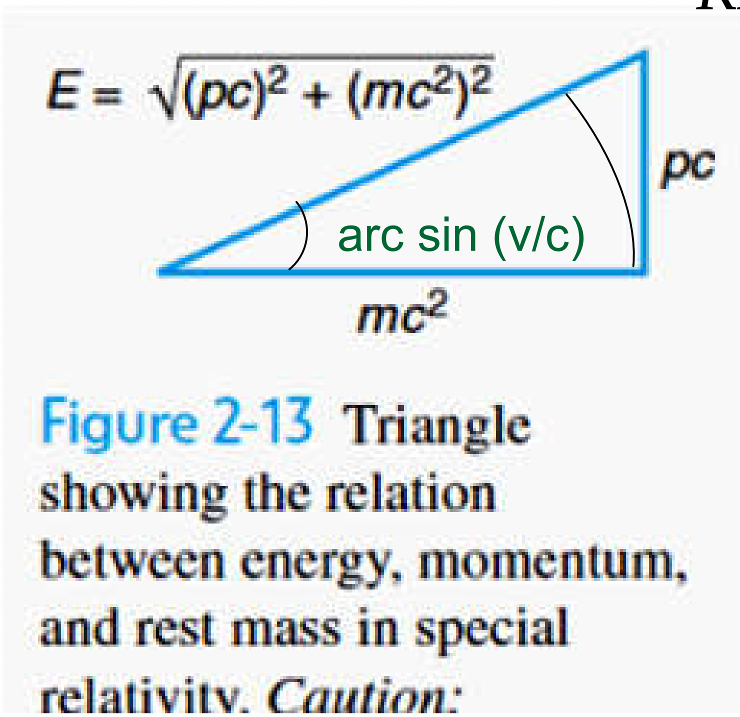


Figure 2-13 Triangle showing the relation between energy, momentum, and rest mass in special relativity. Caution:

$$pc = \gamma m \cdot u \cdot c$$

only if v becomes some significant fraction of c, e.g. 10% angle approx. 5.73º

for v/c = 1%, one gets only approximately 0.573 º degrees for that angle, i.e. not much of a triangle

We need to use relativistic mechanics equations when kinetic energy is on the same order of magnitude than rest energy, i.e. a significant part of total relativistic energy, in other words: when it is not much much smaller than the rest energy that is determined by the rest mass

Very small things can in principle move very fast, light particles are always at c, so we need special relativity for interaction of matter with light, QED Feynman

"… the mass of a body is a measure for its energy content; when the energy changes by L, the mass changes in the same sense by $L / 9\ 10^{20}$ if the energy is given in erg and the mass in gram. It is not inconceivable that the theory can be tested for bodies for which the energy content is highly variable (e.g. the salts of radium). If this theory is correct, radiation transmits inertia between emitting and absorbing bodies"

Albert Einstein, Bern, September 27, 1905

$1\ \text{erg} = 1\ \text{g}\ \text{cm}^2 / \text{s}^2$

Today we simply use $E = m\ c^2$ as such tests have been made a long time ago.

2.13: Computations in Modern Physics

- We were taught in introductory physics that the international system of units is preferable when doing calculations in science and engineering.

- In modern physics (ignoring general relativity, we are dealing with the very fast and very small – typically only very small things are very fast) a somewhat different, more convenient set of units is often used.

Units of Work and Energy

- Recall that the work done in accelerating a charge through a potential difference is given by $W = qV$.

- For a proton, with the charge $e = 1.602 \times 10^{-19}$ C being accelerated across a potential difference of 1 V, the work done is

  $W = (1.602 \times 10^{-19})(1\ \text{V}) = 1.602 \times 10^{-19}\ \text{J}$

The Electron Volt (eV)

- The work done to accelerate the proton across a potential difference of 1 V could also be written as

$$W = (1\ e)(1\ \text{V}) = 1\ \text{eV}$$

- Thus eV, pronounced “electron volt,” is also a unit of energy. It is related to the SI (*Système International*) unit joule by

$$1\ \text{eV} = 1.602 \times 10^{-19}\ \text{J}$$

Other Units

1) Rest energy of a particle:
   Example: E_0 (proton)

$$E_0(\text{proton}) = (1.67\times10^{-27}\ \text{kg})(3\times10^{8}\ \text{m/s})^2 = 1.50\times10^{-10}\ \text{J}$$

$$= 1.50\times10^{-10}\ \text{J}\ \frac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}} = 9.38\times10^{8}\ \text{eV}$$

1) **Atomic mass unit** (amu) approximately:

   Example: carbon-12

$$\text{Mass } (^{12}\text{C atom}) = \frac{12\ \text{g/mol}}{6.02\times10^{23}\ \text{atoms/mol}}$$

$$= 1.99\times10^{-23}\ \text{g/atom}$$

$$\text{Mass } (^{12}\text{C atom}) = 1.99\times10^{-26}\ \text{kg} = 12\ \text{u/atom}$$

Difference to mass of a single proton
is partly due to binding energy

$$1\ \text{u} = 1.66054\times10^{-27}\ \text{kg} = 931.494\ \text{MeV}/c^2$$

Table 2-1 Rest energies of some elementary particles and light nuclei

Particle	Symbol	Rest energy (MeV)
Photon	γ	0
Neutrino (antineutrino)	ν ($\bar{\nu}$)	$<2.8 \times 10^{-6}$
Electron (positron)	e or e^- (e^+)	0.5110
Muon	μ^- μ^+	105.7
Pi mesons (pions)	π^- (π^0) π^+	139.6 (135) 139.6
Proton	p	938.272
Neutron	n	939.565
Deuteron	^{2}H or d	1875.613
Helion	^{3}He or h	2808.391
Alpha	^{4}He or α	3727.379

Mass is just rest energy divided by c^2 as $E_0 = m_0 c^2$

Be aware of the differences, $E_0 = m_0 c^2$ and $E = m c^2$, where $m = \gamma m_0$

Energy becomes a particle / antiparticle pair and vice versa

γ photon with more than 1.022 MeV energy, (more than approx. 6.5 10^{-22} kg m/s)

Before pair creation

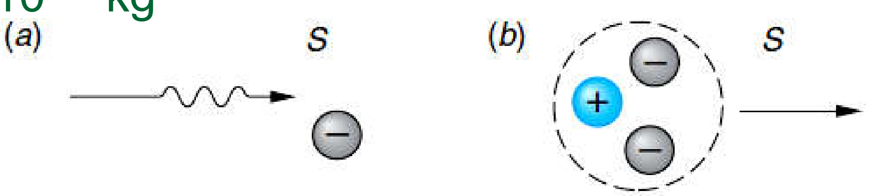
$$E_i = E_\gamma + mc^2$$

$$p_i = \frac{E_\gamma}{c}$$

After pair creation

$$E_f = E_i = E_\gamma + mc^2$$

$$p_f = p_i = \frac{E_\gamma}{c}$$



Mass of both electron and positron approx. 511 keV / c^2 , rest will be kinetic energy, one massive particle is needed for conservation of momentum, but does not need to be an electron, typically it's a whole atom

Figure 2-12 (*a*) A photon of energy E and momentum $p = E/c$ encounters an electron at rest. The photon produces an electron-positron pair (*b*), and the group move off together

Annihilation of particle and antiparticle, one gets all of the energy back as total energy of two photons (which is all kinetic as mass is zero (and associated rest energy is also zero)

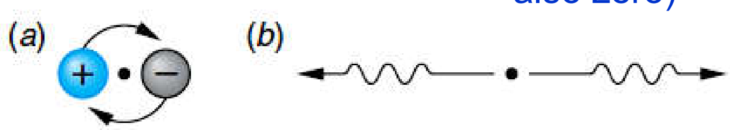


Figure 2-11 (*a*) A positron orbits with an electron about their common center of mass, shown by the dot between them. (*b*) After a short time, typically of the order of 10^{-10} s for the case shown here, the two annihilate, producing two photons. The orbiting electron-positron pair, suggestive of a miniature hydrogen atom, is called *positronium*.

3.9: Pair Production and Annihilation

- Antiparticles, such as the positron, had been predicted to exist in 1929 by P. A. M. Dirac when he had derived his special relativity compliant version of standard 3D quantum mechanics (according to Schrödinger and Heisenberg)
- In 1932, C. D. Anderson observed a positively charged electron (e^+) in a nuclear laboratory. If sufficiently energetic in the first place, a photon's energy can be converted entirely into an electron and a positron in a process called pair production ("left over energy" will be kinetic for the created particles and what triggered the pair production in the first place)
- Charge needs to be conserved in pair production as well, i.e. a photon creates an electron and its positively charged antiparticle.
- All four guys mentioned above received Nobel prizes
- We now know that to any particle, there is an antiparticle, there can be anti-atoms (with antiprotons and antineutrons in the core and positrons orbiting), antimatter, …

$$\gamma \rightarrow e^+ + e^-$$

Total energy, momentum and total charge of all particles will be conserved, note that I speak of the γ-ray as a particle already

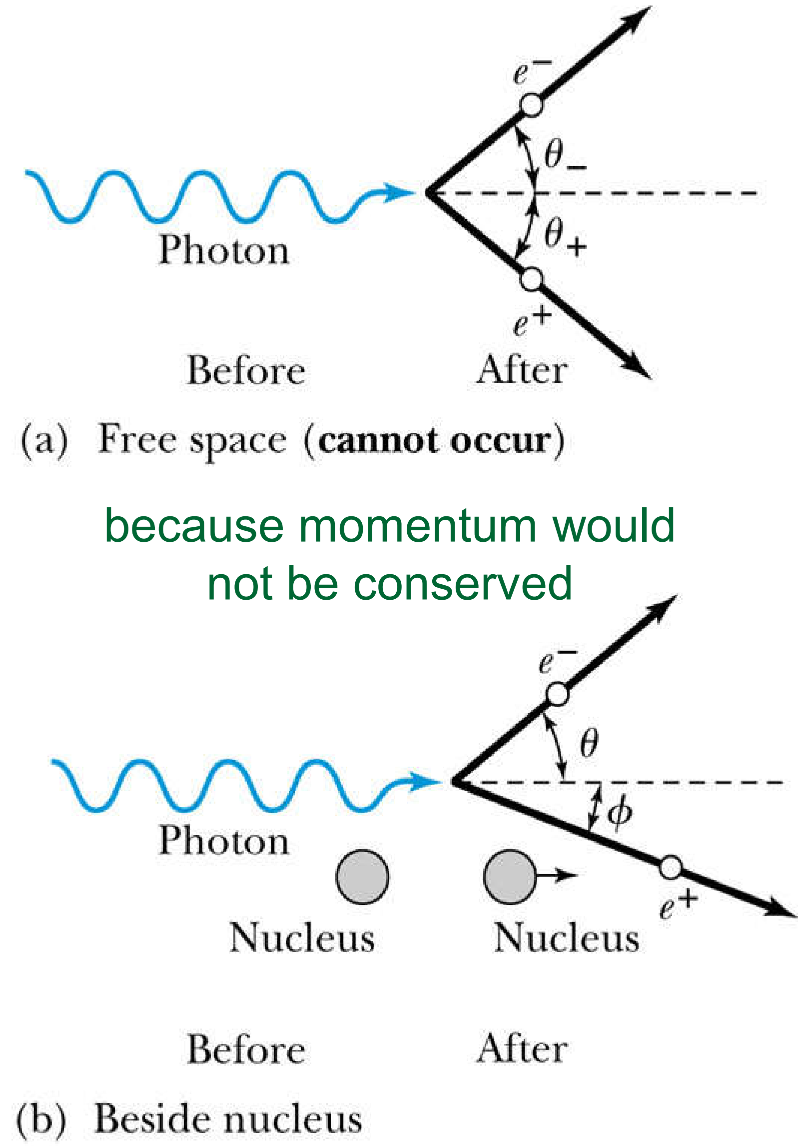
Pair Production in Matter

- In the presence of matter, some other particle absorbs some energy and momentum can be conserved

$$hf = E_+ + E_- + \text{K.E. (nucleus)}$$

- The photon energy required for pair production in the presence of matter is

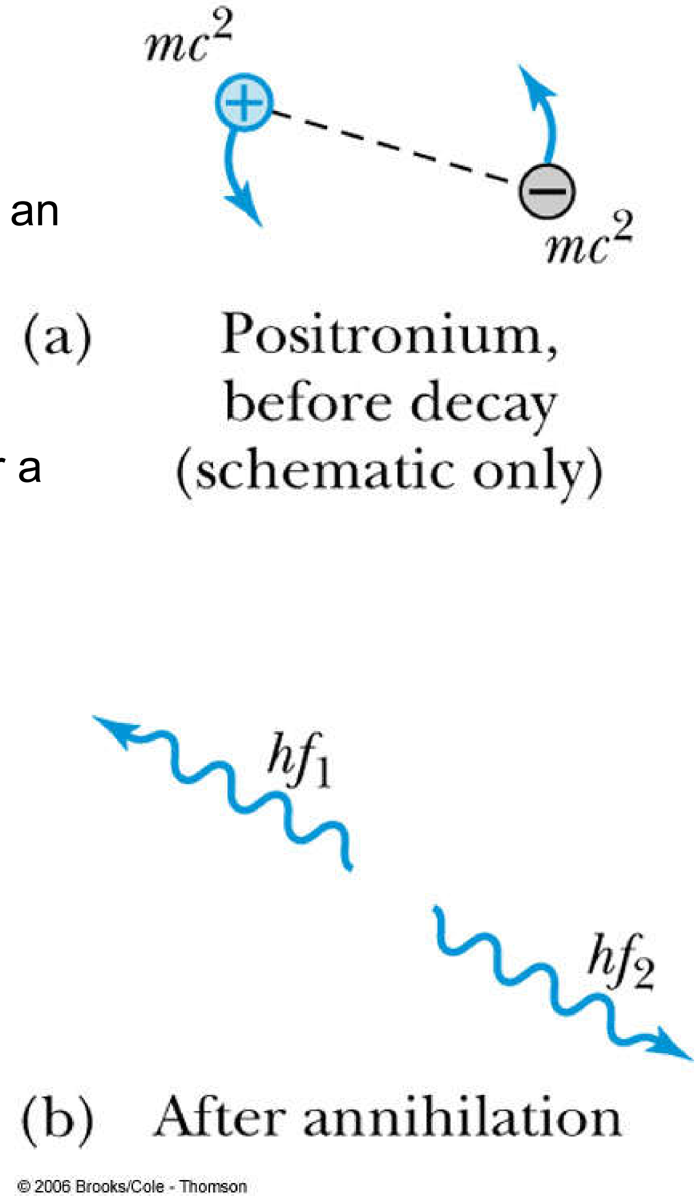
$$hf > 2m_e c^2 = 1.022\ \text{MeV}$$



h is Max Planck's constant: 6.6261 10^{-34} Ws^2 (next chapter, kind of strengths of the "wave-particle duality" coupling)

Pair Annihilation

- A positron passing through matter will likely **annihilate** with an electron. A positron is drawn to an electron by their mutual electric attraction, and the electron and positron then form an "atomlike" configuration called **positronium**.
- Pair annihilation in empty space will produce two photons to conserve momentum. Annihilation near a nucleus can result in a single photon.



- Conservation of energy: $2m_ec^2 \approx hf_1 + hf_2$
- Conservation of momentum: $0 = \dfrac{hf_1}{c} - \dfrac{hf_2}{c}$
- The two photons will be identical, so that

$$f_1 = f_2 = f$$

- The two photons from positronium annihilation will move in opposite directions with an energy:

$$hf = m_ec^2 = 0.511 \text{ MeV}$$

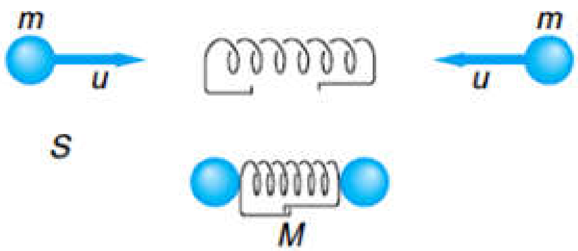
Confirmation of the equivalence of energy and mass of matter

Binding Energy, general

- The equivalence of mass and energy becomes also apparent when we study the binding energy of systems like atoms, molecules and nuclei of atoms that are formed from individual particles.

- The potential energy associated with the force keeping the system together is called the **binding energy** E_B. The force is attractive and positive, the potential energy is negative and needs to be provided by you to break the system up (be always careful with signs)

Binding Energy, concept, simplified



KE changed into mass

The binding energy is *the difference between the sum of the rest energy of the individual particles and the rest energy of the combined bound system*. Example deuteron: the nucleus of deuterium has a binding energy of 2.23 MeV, which is considered to be negative, so the deuteron is not as heavy as the sum of its constituent proton and neutron, (binding energy of electron to nucleus is much smaller (13.6 eV in the hydrogen ground state)

Definition:

$$E_B = \sum_i m_i c^2 - M_{\text{bound system}} c^2$$

(and considered to be negative)

In case of chemical reactions, binding energy changes are only a couple of eV, but in case of nuclear reactions up to approx. 200 MeV per split U atom

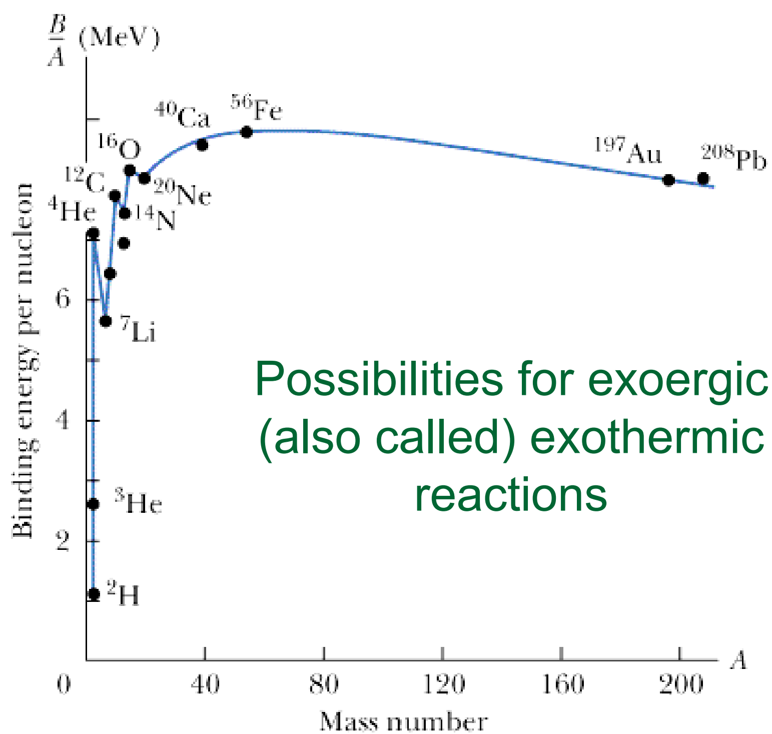
Hiroshima bomb released energy, 20 kT TNT (Nobel's high explosive), corresponds to a total mass loss of approx. 1 gram

Decrease in mass during the reaction means endoergic or endothermic reaction, e.g. fusion of two ^{2}H deuterons to one alpha ^{4}He leads to higher binding energy per nucleon, i.e. a loss of mass (0.0245 u) which results in a release of useful energy

Binding Energy, qualitative more in chapters 12, 13 nuclear physics)

The raw materials to produce energy by nuclear processes are (quite) stable, so one needs energy to trigger a reaction in the first place, …, the overall energy balance is the interesting bit

Thornton/Rex, Modern Physics for Scientists and Engineers, 2/e
Figure 12.6



Energy can be gained by both,

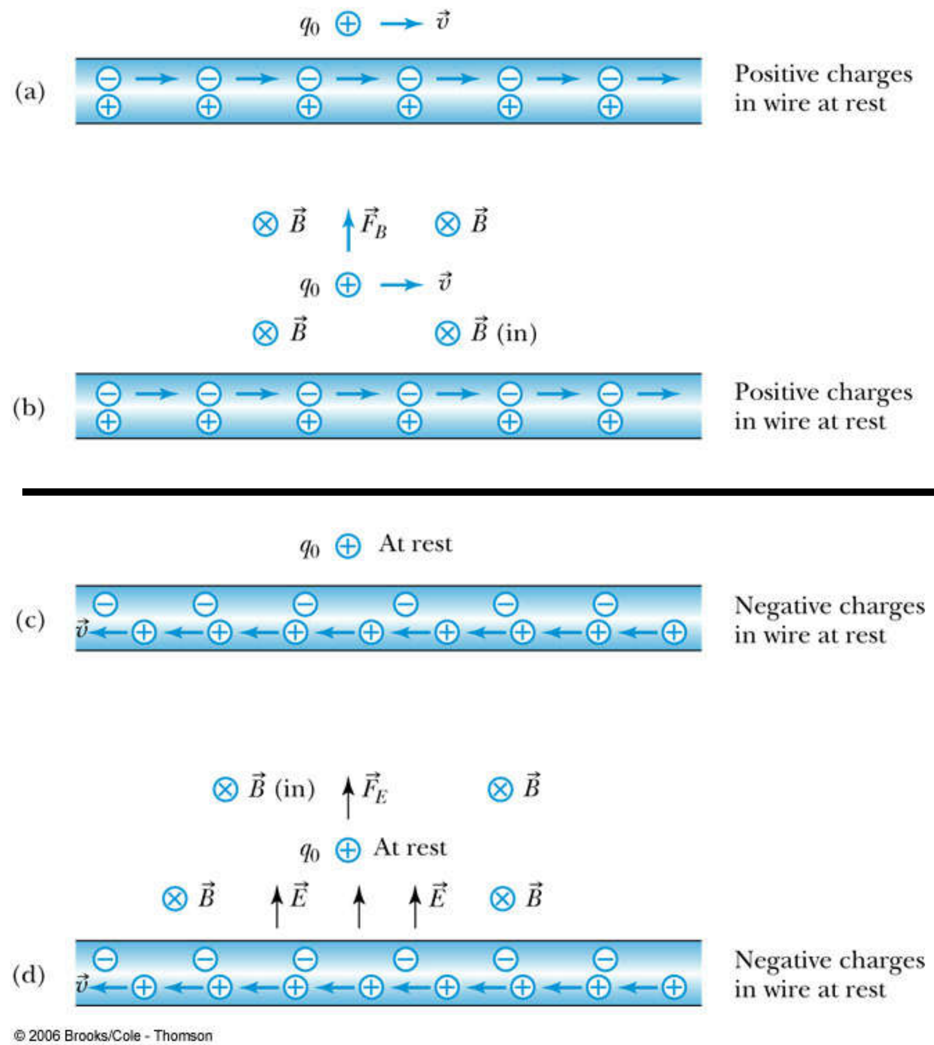
(1) splitting something heavy such as ^{235}U (releasing approx. 200 MeV per event depending on how it splits exactly), the two newly created nuclei will have higher binding energy per nucleon, but smaller total mass when added up

(2) fusing ^{2}H and ^{3}H together to produce ^{4}He + 1 neutron, again the binding energy per nucleon gets larger in He, 17.6 MeV per event are released, the results of the fusion, He and a neutron are lighter than the sum of ^{2}H and ^{3}H

Electromagnetism and Relativity

- Einstein was convinced that magnetic fields appeared as electric fields observed in another inertial frame. relativity unifies electric and magnetic forces, shows them to be identical in nature.
- Einstein established that *Maxwell's equations describe electromagnetism in any inertial frame*
- Maxwell's result that all electromagnetic waves travel at the speed of light and Einstein's postulate that the speed of light is invariant in all inertial frames are intimately connected.
- With the Lorentz transformation for the electric and magnetic field and Einstein's special relativity, one can derived Maxwell's equations.

Conducting Wire



Positive test charge moves at same v ≠ 0 as electrons in wire

magnetic field lines into the paper by right hand rule, thumb opposite to the direction of the moving electrons

Lorentz force on positive test charge, F= q v cross B

Since all movement is relative, positive test charge moves at same v = 0 as electrons in wire

now the positive charges in the wire seem to move to the left, length in that direction is contracted so there is a positive charge imbalance (with respect to the same amount of electrons), which produces a repulsive electrostatic force

1. Relativistic momentum	$\mathbf{p} = \gamma m\mathbf{u}$	2-7
	The relativistic momentum is conserved and approaches $m\mathbf{u}$ for $v \ll c$. $\gamma = (1 - u^2/c^2)^{-1/2}$ in Equation 2-7, where u = particle speed in S.	
2. Relativistic energy	$E = \gamma mc^2$	2-10
Total energy	The relativistic total energy is conserved.	
Kinetic energy	$E_k = \gamma mc^2 - mc^2$	2-9
	mc^2 is the rest energy. $\gamma = (1 - u^2/c^2)^{-1/2}$ in Equations 2-9 and 2-10.	
3. Lorentz transformation for E and $\mathbf{p}$.	$p'_x = \gamma(p_x - vE/c^2)$ $\quad p'_y = p_y$ $E' = \gamma(E - vp_x)$ $\quad p'_z = p_z$	2-16
	where v = relative speed of the systems and $\gamma = (1 - v^2/c^2)^{-1/2}$	
4. Mass/energy conversion	Whenever additional energy ΔE in any form is stored in an object, the rest mass of the object is increased by $\Delta m = \Delta E/c^2$.	
5. Invariant mass	$(mc^2) = E^2 - (pc)^2$	2-32
Accelerator equation	The energy and momentum of any system combine to form an invariant four-vector whose magnitude is the rest energy of the ma˜ss m.	
6. Force in relativity	The force $\mathbf{F} = m\mathbf{a}$ is not invariant in relativity. Relativistic force is defined as	
	$\mathbf{F} = \dfrac{d\mathbf{p}}{dt} = \dfrac{d(\gamma m\mathbf{u})}{dt}$	2-8

4D spacetime also key to quantum mechanical spin, internal degree of freedom of any QM particle (and its antiparticle)

CHAPTER 15
General Relativity

15.0. a "loose end" from classical mechanics

- 15.1 Tenets of General Relativity
- 15.2 Tests of General Relativity
- 15.3 Gravitational Waves
- 15.4 Black Holes
- 15.5 Gravitational wavelength shifts for light

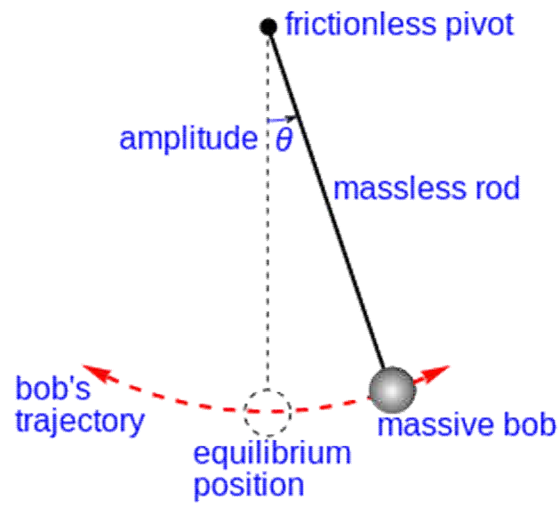
There is nothing in the world except empty, curved space-time. Matter, charge, electromagnetism, and other fields are only manifestations of the curvature. John Archibald Wheeler

There is also some man made joy and misery. Peter Moeck

15.0: a loose end from classical mechanics

Invar pendulum in low pressure tank in Riefler regulator clock, used as the US time standard from 1909 to 1929, ± 15 milliseconds per day if temperature is reasonably constant, (Invar is a very low thermal expansion alloy, temperature variations smaller than 71 °F result in less than 1.3 seconds time error per day

in 1671 a pendulum clock was sent to Cayenne, French Guiana by the French Académie des Sciences, it was determined that the clock was 2½ minutes per day slower than the same clock in Paris ???



a length change of only 0.02%, 0.2 mm in a typical grandfather clock pendulum, will cause an error of a minute per week.

Approx. simple harmonic motion for small swings

$$\theta(t) = \theta_0 \cos(2\pi t/T).$$

$$T = 2\pi\sqrt{\frac{L}{g}}\left(1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4 + \cdots\right)$$

$$T \approx 2\pi\sqrt{\frac{L}{g}} \qquad \theta_0 \ll 1 \text{ rad}$$

L: length of the pendulum

Thompson's approximate equation for period of a pendulum (for small amplitudes neglects all higher orders of Θ_0, but gravitational acceleration (vector) **g (magnitude)**, which varies by as much as 0.5% at different locations on Earth

http://upload.wikimedia.org/wikipedia/commons/thumb/f/f2/Pendulum2secondclock.gif/220px-Pendulum2secondclock.gif

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What is *g* anyway?

$$|\vec{F}| = m \cdot |\vec{g}| = m \cdot \frac{GM}{r^2}$$

Assumption that earth is a perfect sphere with radially symmetric mass density, so that r can be taken as radius of that sphere

G: 6.67 10^{-11} Nm^2/kg^2

$r_{equator}$: 6.378 10^3 km

r_{poles} : 6.357 10^3 km

$M_{(earth)}$: 5.979 10^{24} kg

$$U_2 - U_1 = -\int_1^2 \vec{F} \cdot dx$$

$$U = -G\frac{m_1 M_2}{r} + K$$

With K as an integration constant that we are free to set to zero if we take r as radius of the sphere

So gravitational potential energy is negative (zero at infinity) and we can interpret is as Binding Energy, acceleration due to gravity is equal to the negative gradient of the gravitational potential.

What happens to the force of gravity, the gravitational potential energy and the gravitational potential some distance away from Earth's surface?

Gravitational potential $-\frac{GM}{x}$ http://en.wikipedia.org/wiki/Gravitational_potential

$$V(x) = \frac{W}{m} = \frac{1}{m}\int_{\infty}^{x} F\,dx = \frac{1}{m}\int_{\infty}^{x} \frac{GmM}{x^2}dx = -\frac{GM}{x},$$

Earth's surface **(-g times radius earth)**	- 60 MJ/kg
Low Earth orbit	- 57 MJ/kg
Voyager 1 (17,000 million km from Earth)	- 23 J/kg
0.1 light-year from Earth	- 0.4 J/kg

What does this imply about the period of a pendulum (Thompson's law?) How do we need to update it to have a time piece in a space ship? What will this time piece read? So time depends on the height above earth already classically !!!

$$T \approx 2\pi\sqrt{\frac{L}{g}}$$

$\theta_0 \ll$ 180/π

$$T \approx 2\pi\sqrt{\frac{L}{-(-GM)}\cdot x}$$

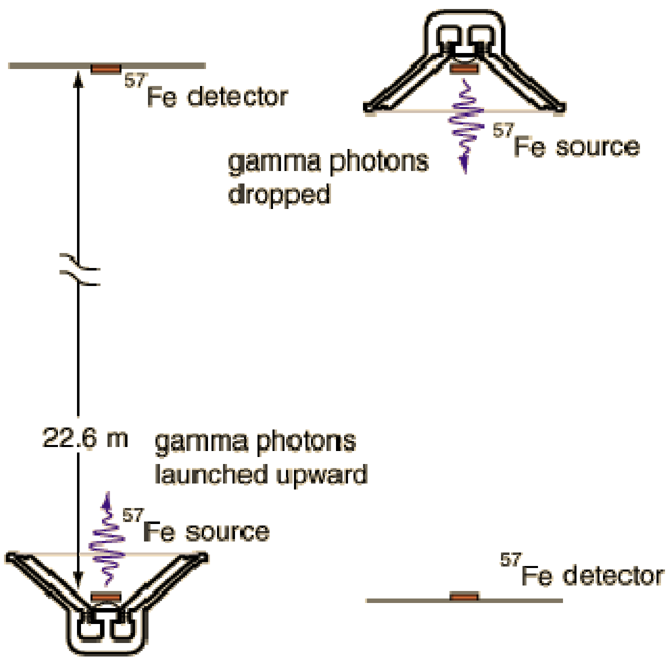
Which means time runs faster because 1/T = f is higher

If mass effects time, with general relativity it must also affect space as there is only 4D space time according to special relativity !!!

The further away from Earth, the higher the gravitational potential (i.e. the smaller is negative value), the larger the time interval T that measures a full cycle of the clock, (the higher the frequency of subsequent tick-tock's), the faster the time - as measured by the pendulum or any other clock - runs

Partying in the basement, having a physics exam on the 99999999 … floor, or the other way around?

Harvard Tower Experiment



In just 22.6 meters, the fractional gravitational red shift given by

$$f = \upsilon = f_0 \cdot \left(1 + \frac{gh}{c^2}\right)$$

is just 4.9 x 10^{-15}, but the Mössbauer effect with the 14.4 keV gamma ray from iron-57 has a high enough resolution to detect that difference. In the early 60's physicists Pound, Rebka, and Snyder at the Jefferson Physical Laboratory at Harvard measured the shift to within 1% of the predicted shift.

$$\Delta E = mgh = \frac{E}{c^2} gh = \frac{14.4keV}{c^2} g \cdot 22.6m$$

$$\Delta E = 3.5x10^{-11} eV$$

$$\left(\frac{\Delta E}{E}\right)_{down} - \left(\frac{\Delta E}{E}\right)_{up} = \frac{2(3.5x10^{-11} eV)}{(14.4keV)} = 4.9x10^{-15}$$

E = h f , Max Planck's "discovery", chapter on particle properties of light waves

Left over from classical physics, why is the heavy mass equal to the dynamic mass within measurement error? Should they be exactly the same or are there really two types of mass?

Inertial Mass and Gravitational Mass

- Recall from Newton's 2nd law that an object accelerates in reaction to a force according to its inertial mass: $\vec{F} = m_I \vec{a}$

- Inertial mass measures how strongly an object resists a change in its motion.

- Gravitational mass measures how strongly it attracts other objects such as the earth. $\vec{F} = m_G \vec{g}$

- For the same force, we get a ratio of masses: $\vec{a} = \left(\frac{m_G}{m_I}\right)\vec{g}$

Then there occurred to me the happiest thought of my life, in the following form. The gravitational field has only a relative existence in a way similar to the electric field generated by electromagnetic induction. *Because for an observer falling freely from the roof of a house there exists*—at least in his immediate surroundings—*no gravitational field* [Einstein's italics]. . . . The observer then has the right to interpret his state as "at rest."

1907, when he embarked on deriving general relativity

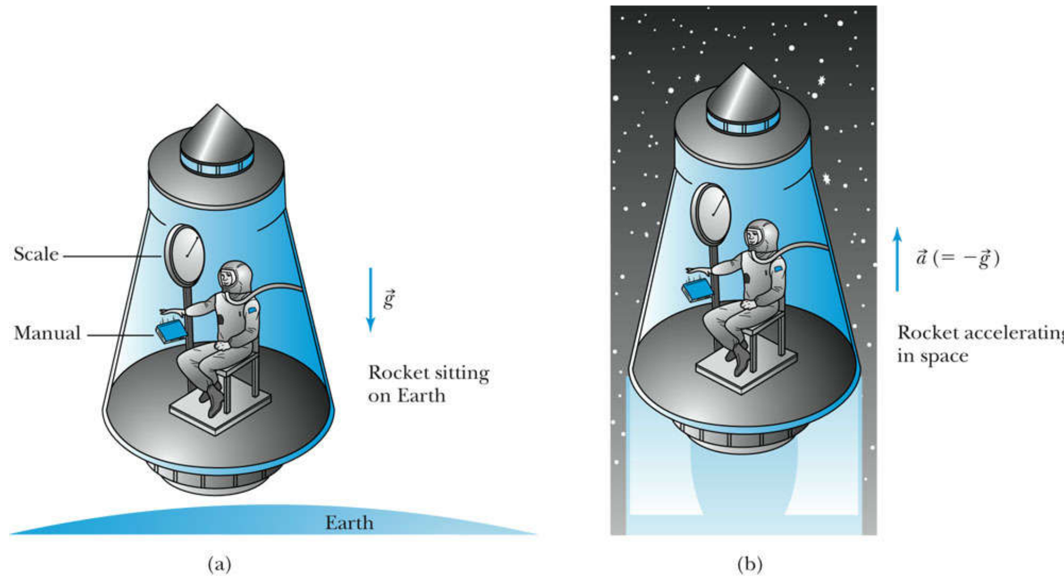
15.1: Tenets of General Relativity

- General relativity is the extension of special relativity. It includes the effects of accelerating objects and their masses on spacetime.
- As a result, the theory is an explanation of gravity.
- It is based on two concepts: (1) the principle of equivalence of the heavy mass and the dynamic mass, i.e. there is only one type of mass, and no way of detecting if one is in non-uniform motion, is “kind of” an extension of Einstein’s first postulate of special relativity and (2) the curvature of spacetime due to gravity.

Principle of Equivalence

- principle of equivalence shown by experiments in one nearly inertial reference frames.

- Consider an astronaut sitting in a confined space on a rocket placed on Earth. The astronaut is strapped into a chair that is mounted on a weighing scale that indicates a mass *M*. The astronaut drops a safety manual that falls to the floor.

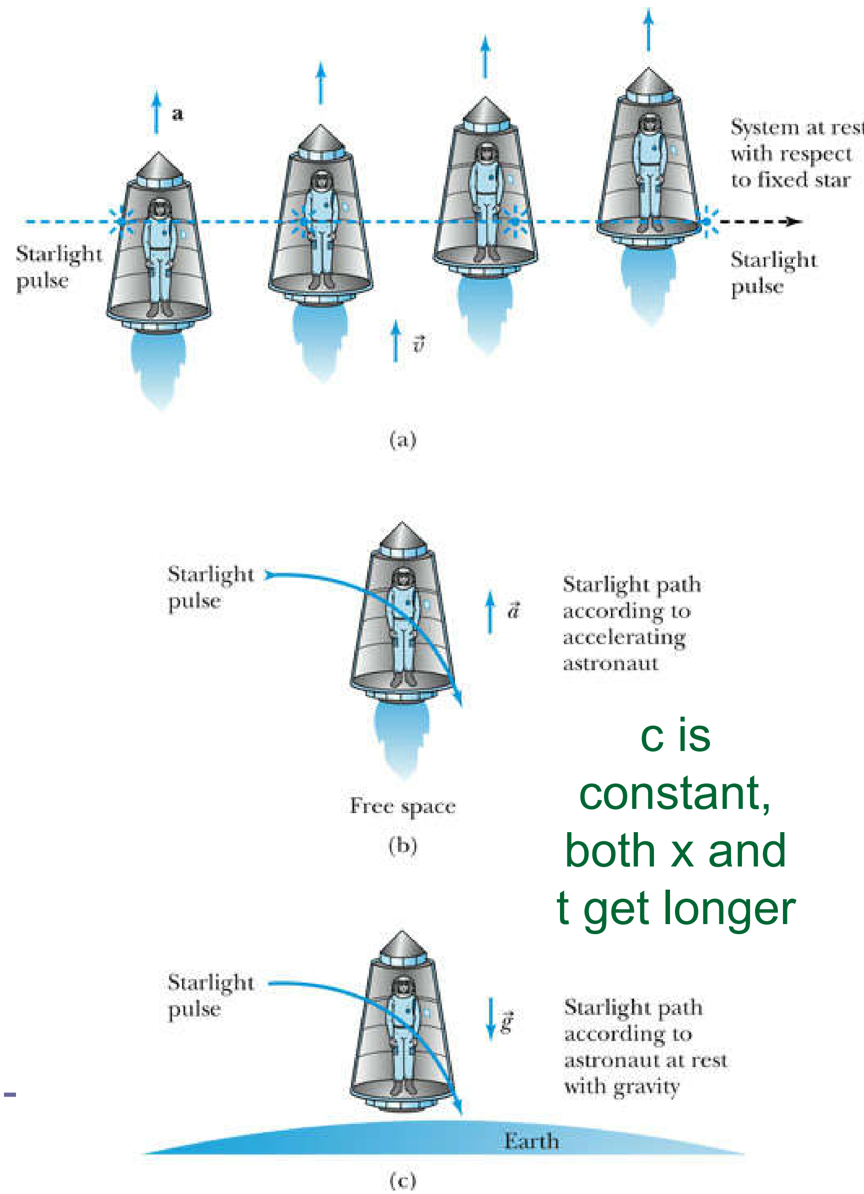


- Now contrast this situation with the rocket accelerating through space. The gravitational force of the Earth is now negligible. If the acceleration has exactly the same magnitude as *g* on Earth, then the weighing scale indicates the same mass *M* that it did on Earth, and the safety manual still falls with the same acceleration as measured by the astronaut. The question is: How can the astronaut tell whether the rocket is on earth or in space?

- **Principle of equivalence**: There is no physical experiment that can detect the difference between a uniform gravitational field and an equivalent uniform acceleration. (but g varies very slightly with height)

Light Deflection

- Consider accelerating through a region of space where the gravitational force is negligible. A small window on the rocket allows a beam of starlight to enter the spacecraft. Since the velocity of light is finite, there is a nonzero amount of time for the light to shine across the opposite wall of the spaceship.
- During this time, the rocket has accelerated upward. From the point of view of a passenger in the rocket, the light path appears to bend down toward the floor.
- The same effect, light is "bending down" could be due to the force of gravity, so gravity affects the path the light takes
- This prediction seems surprising, however the unification of mass and energy from the special theory of relativity hints that the gravitational force of the Earth acts on the "effective mass" of the light beam.

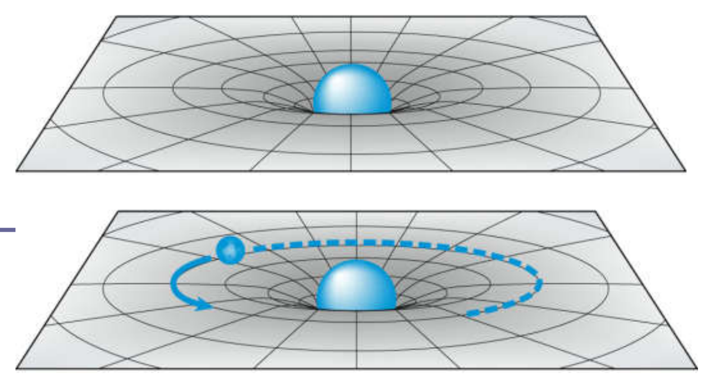


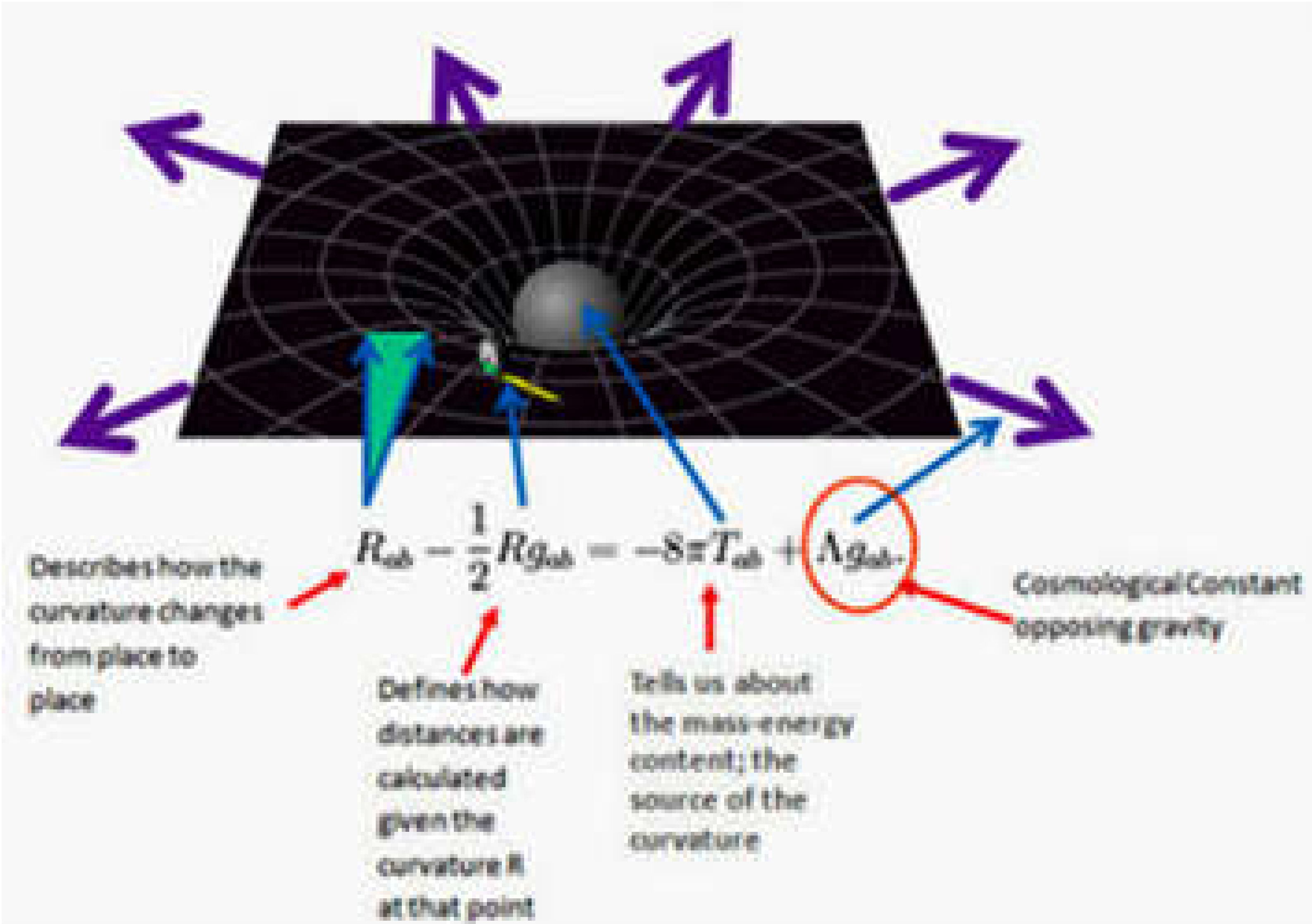
c is constant, both x and t get longer

Spacetime Curvature of Space

- Light bending for the Earth observer seems to violate the premise that the velocity of light is constant from special relativity. Light traveling at a constant velocity implies that it travels in a straight line.
- Einstein recognized that we need to expand our definition of a *straight line.*
- The shortest distance between two points on a flat surface appears different than the same distance between points on a sphere. The path on the sphere appears curved. We shall expand our definition of a *straight line* to include any minimized distance between two points.
- Thus if the spacetime near the Earth is not flat, then the straight line path of light near the Earth will appear curved.

$$R_{\mu\nu} - \frac{1}{2} R \, g_{\mu\nu} + \Lambda \, g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$





https://byjus.com/physics/einstein-field-equation/

The Unification of Mass and Spacetime

- Einstein mandated that the mass of the Earth creates a dimple on the “spacetime surface”. In other words, the mass changes the geometry of the spacetime.
- The geometry of the spacetime then tells matter how to move.
- Einstein’s famous field equations sum up this relationship as:

  * *mass-energy tells spacetime how to curve*

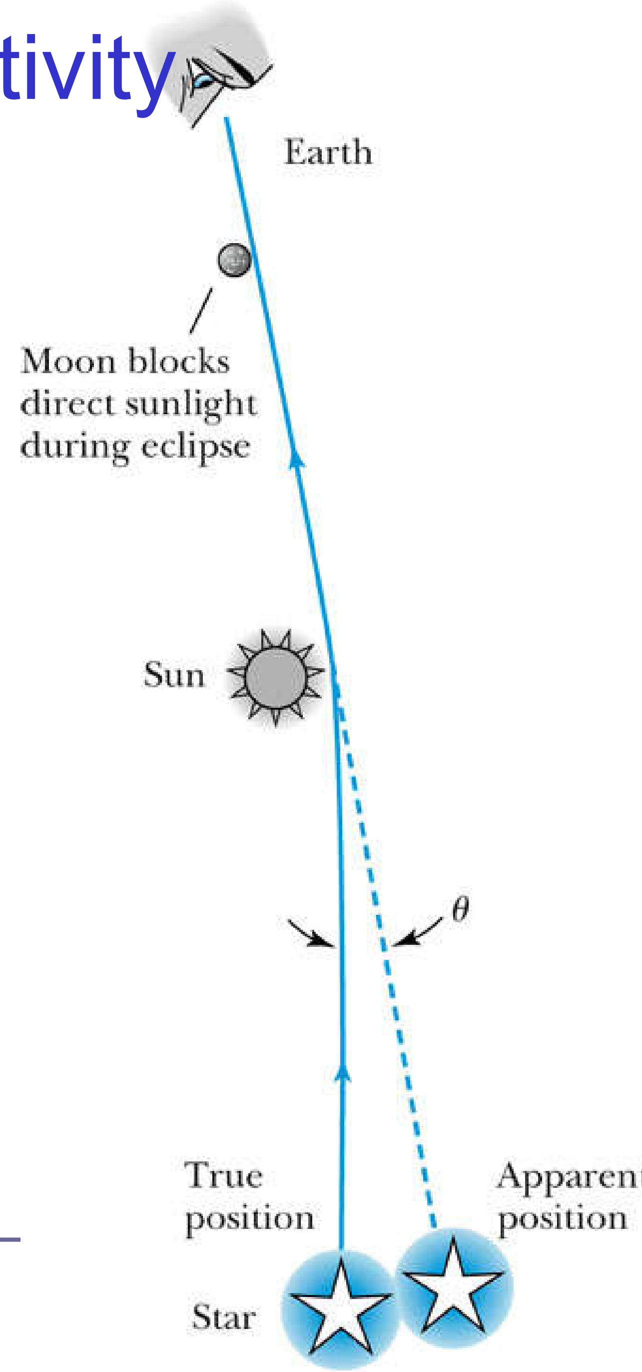
  * *Spacetime curvature tells matter how to move*

- another result is that a standard unit of length such as a meter stick increases in the vicinity of a mass. This is because c is still the same constant and as the time is delayed by the gravitational field of the mass, a longer meter stick is required in order to give the same c

15.2: Tests of General Relativity

Bending of Light

- During a solar eclipse of the sun by the moon, most of the sun’s light is blocked on Earth, which afforded the opportunity to view starlight passing close to the sun in 1919. The starlight was bent as it passed near the sun which caused the star to appear displaced.
- Einstein’s general theory predicted a deflection of 1.75 seconds of arc, and the two measurements found 1.98 ± 0.16 and 1.61 ± 0.40 seconds.
- Since the eclipse of 1919, many experiments, using both starlight and radio waves from quasars, have confirmed Einstein’s predictions about the bending of light with increasingly good accuracy.



LIGHTS ALL ASKEW IN THE HEAVENS

Men of Science More or Less Agog Over Results of Eclipse Observations.

EINSTEIN THEORY TRIUMPHS

Stars Not Where They Seemed or Were Calculated to be, but Nobody Need Worry.

A BOOK FOR 12 WISE MEN

No More in All the World Could Comprehend It, Said Einstein When His Daring Publishers Accepted It.

v / A New York Times headline from November 10, 1919, describing the observations discussed in example 1.

Gravitational Redshift

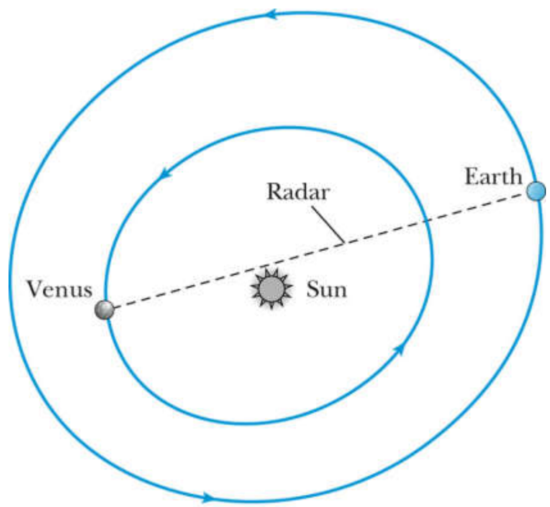
- The second test of general relativity is the predicted frequency change of light near a massive object.
- Imagine a light pulse being emitted from the surface of the Earth to travel vertically upward. The gravitational attraction of the Earth cannot slow down light, but it can do work on the light pulse to lower its energy. This is similar to a rock being thrown straight up. As it goes up, its gravitational potential energy increases while its kinetic energy decreases. A similar thing happens to a light pulse.
- A light pulse's energy depends on its frequency f through Planck's constant, $E = hf$. As the light pulse travels up vertically, it loses kinetic energy and its frequency decreases (which also means its period with dimension time increases). Its wavelength increases, so the wavelengths of visible light are shifted toward the red end of the visible spectrum. (The period (1/frequency) also increases – just as it did with the pendulum clock)
- This phenomenon is called **gravitational red shift**.

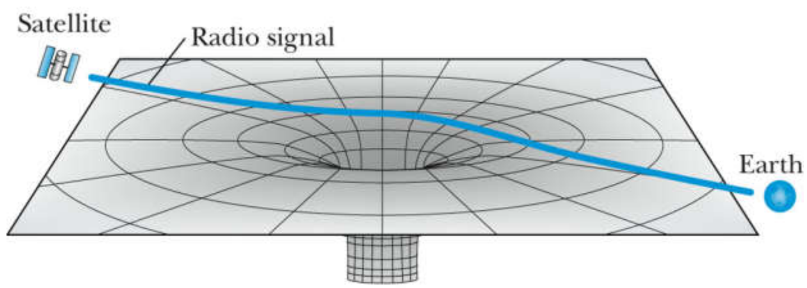
Gravitational Time Dilation

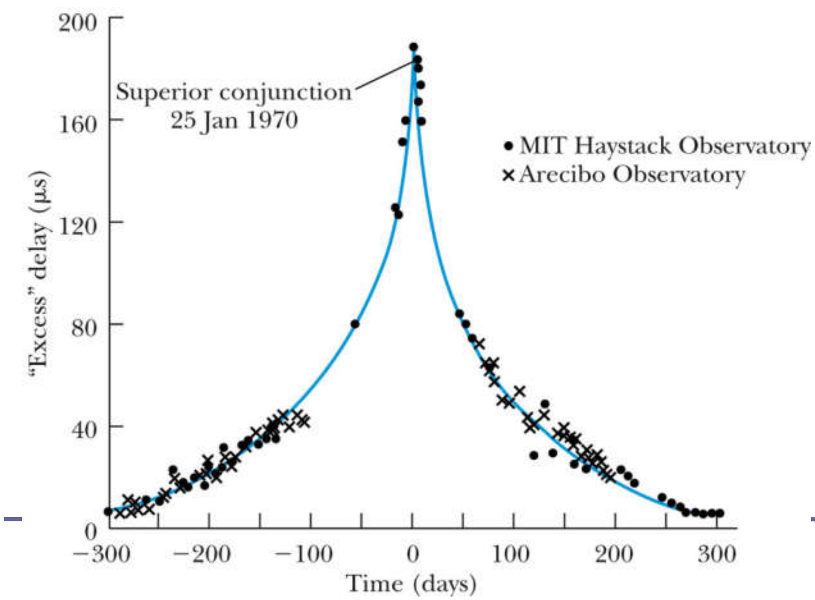
- A very accurate experiment was done by comparing the frequency of an atomic clock flown on a Scout D rocket to an altitude of 10,000 km with the frequency of a similar clock on the ground. The measurement agreed with Einstein's general relativity theory to within 0.02%.

- Since the frequency of the clock increases near the Earth, a clock in a lower (higher negative value) gravitational potential runs more slowly due to gravitational time dilation than a clock at a higher gravitational potential (smaller negative value)

Light Retardation





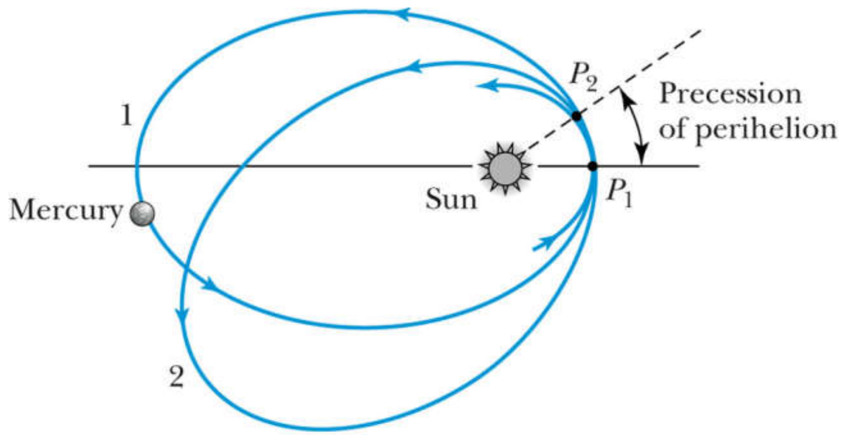


- As light passes by a massive object, the path taken by the light is longer because of the spacetime curvature.
- The longer path causes a time delay for a light pulse traveling close to the sun while c remains constant.
- This effect was measured by sending a radar wave to Venus, where it was reflected back to Earth. The position of Venus had to be in the "superior conjunction" position on the other side of the sun from the Earth. The signal passed near the sun and experienced a time delay of about 200 microseconds. This was in excellent agreement with the general theory of relativity.

See also http://adsabs.harvard.edu/abs/1988IAUS..128..453S

Perihelion Shift of Mercury

- The orbits of the planets are ellipses, and the point closest to the sun in a planetary orbit is called the perihelion. It has been known for hundreds of years that Mercury's orbit precesses about the sun. Accounting for the perturbations of the other planets left 43 seconds of arc per century that was previously unexplained by classical physics.
- The curvature of spacetime explained by general relativity accounted for these 43 seconds of arc shift in the orbit of Mercury.



15.3: Gravitational Waves

- When a charge accelerates, the electric field surrounding the charge redistributes itself. This change in the electric field produces an electromagnetic wave, which is easily detected. In much the same way, an accelerated mass should also produce gravitational waves.
- Gravitational waves carry energy and momentum, travel at the speed of light, and are characterized by frequency and wavelength.
- As gravitational waves pass through spacetime, they cause small ripples. The stretching and shrinking is on the order of 1 part in 10^{21} even for strong gravitational wave sources.
- Due to their small amplitude, gravitational waves are very difficult to detect. Large astronomical events could create measurable spacetime waves such as the collapse of a neutron star, a black hole or the Big Bang.
- This effect has been compared to noticing a single grain of sand added to all the beaches of Long Island, New York.

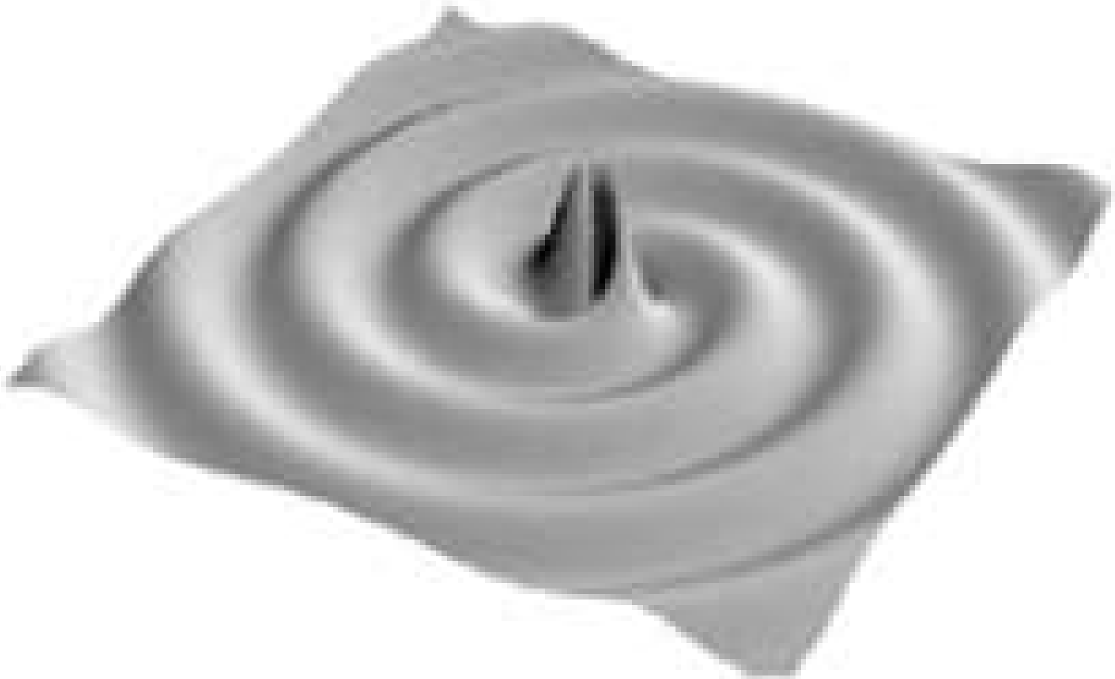
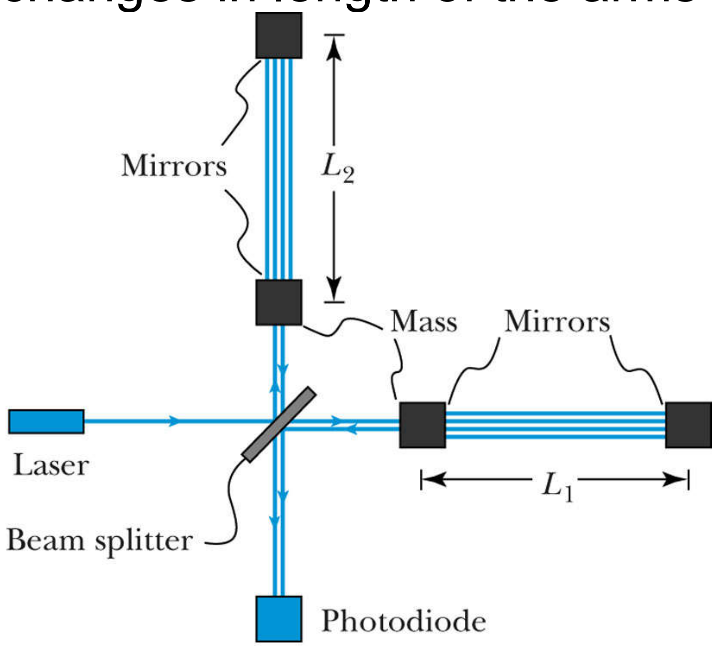


Figure 2-25 Gravitational waves, intense ripples in the fabric of spacetime, are expected to be generated by a merging binary system of neutron stars or black holes. The amplitude decreases with distance due to the $1/R$ falloff and because waves farther from the source were emitted at an earlier time, when the emission was weaker. [*Courtesy of Patrick Brady.*]

Gravitational Wave Experiments

- Taylor and Hulse discovered a binary system of two neutron stars that lose energy due to gravitational waves that agrees with the predictions of general relativity.
- LIGO is a large “Michelson interferometer” device that uses 4 test masses on two arms of the interferometer. The device detected changes in length of the arms due to a passing wave in February 2016



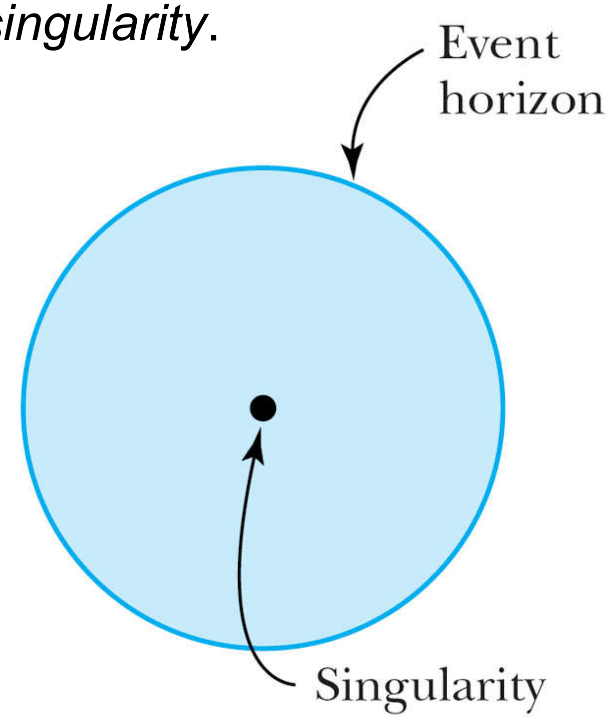
Nobel prize 2017 to Rainer Weiss, Barry C. Barish, and Kip S. Thorne,

"for decisive contributions to the LIGO detector and the observation of gravitational waves."

15.4: Black Holes

- While a star is burning, the heat produced by the thermonuclear reactions pushes out the star's matter and balances the force of gravity. When the star's fuel is depleted, no heat is left to counteract the force of gravity, which becomes dominant. The star's mass can collapse into an incredibly dense ball that could wrap spacetime enough to not allow light to escape. The point at the center is called a *singularity*.
- A collapsing star greater than 3 solar masses will distort spacetime in this way to create a black hole.
- Karl Schwarzschild determined the radius of a black hole known as the *event horizon*.

$$r_S = \frac{2GM}{c^2}$$



15.5: gravitational shifts of the wavelength of light – there sure is no absolute time in the universe

Having moved away from a very heavy object, e.g. sun, light is red-shifted, i.e. longer wavelengths, shorter frequency, larger period, means a clock on the basis of that light runs faster

Having arrived red shifted at a "not so heavy object", e.g. earth, sun light is blue-shifted a bit, i.e. shorter wavelength, higher frequency, shorter period, means "light clocks" tick somewhat faster (frequency), time runs slower (period)

Analyzing wavelength shifts of light from outer space gets somewhat complicated as there are also Doppler shifts, which typically lead to red-shifts, because almost all light sources move away from us

What are the reference wavelength? Spectral lines, characteristic light from exited atoms, to be explained later in the course

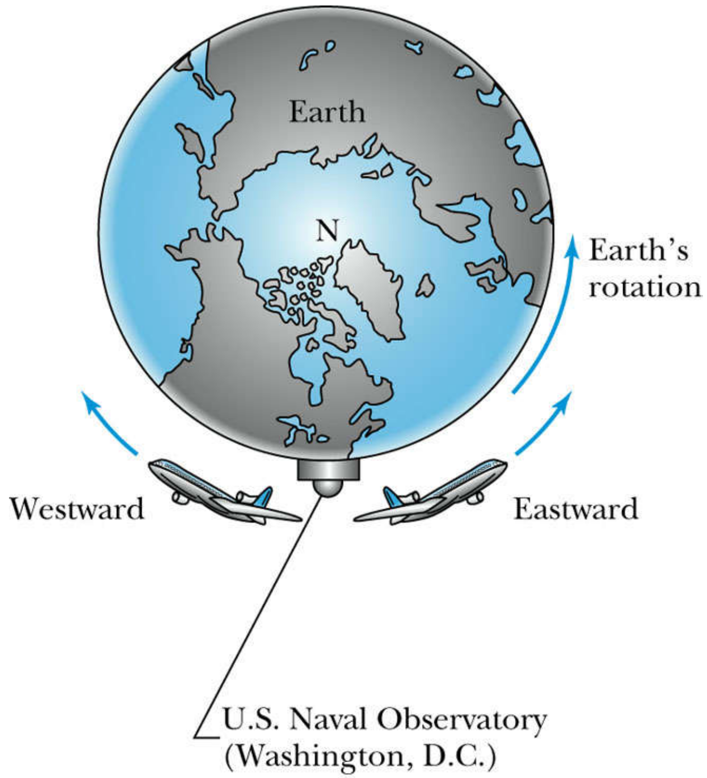
To appreciate Einstein's greatness

Special relativity is correct only in a universe where there are no masses (sure not very interesting to study unless you are a philosopher),

but a very good approximation for small masses, e.g. the mass of the Earth, so makes a lot of sense if you are an applied physicist collaborating with engineers

General relativity describes a universe that contains masses, but because it is a field theory in disagreement with quantum mechanics, it may not be "completely correct" either.

Atomic Clock Measurement



Two airplanes took off (at different times) from Washington, D.C., where the U.S. Naval Observatory is located. The airplanes traveled east and west around Earth as it rotated. Atomic clocks on the airplanes were compared with similar clocks kept at the observatory to show that the moving clocks in the airplanes ran slower. There is a certain height above the earth at which both the stationary and moving clock would be exactly in tune, measure the same time. This is because the time delaying effect of the movement of the plane (as already present in special relativity) would be counteracted exactly in the time speeding up effect due to the height above the earth

Giordano Bruno:

"It is proof of a base and low mind for one to wish to think with the masses or majority, merely because the majority is the majority.

Truth does not change because it is, or is not, believed by a majority of the people."

Included as a quotation in The Great Quotations (1977) by George Seldes, p. 35, this appears to be a paraphrase of a summation of arguments of Bruno's speech in a debate at the College of Cambray (25 May 1588).

https://youtu.be/CYv5GsXEf1o

https://www.youtube.com/watch?v=CYv5GsXEf1o
&feature=youtu.be

7 minutes video that won a major price $400,000 – the referees either didn't know better or the whole exercise (sponsored by the founders of the Kahn Academy among others) is a scam

More at

http://www.usatoday.com/story/tech/2015/11/08/breakthrough-junior-prize-mark-zuckerberg-priscilla-chan/75325460/

http://www.popsci.com/first-ever-breakthrough-prize-junior-winner-made-this-cool-science-video

Going back to Michelson–Morley experiment, it is clear that it could not measure any “ether wind” as it was done with light, see next 2 slides

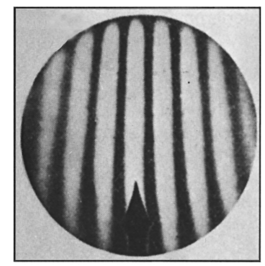
The Analysis for c +- v = c as experiment was done with light, its consequences are the same as v = 0

Time t_1 from A to C and back:

$$t_1 = \frac{\ell_1}{c + \cancel{v_x}} + \frac{\ell_1}{c - \cancel{v_x}} = \frac{2c\ell_1}{c^2 - \cancel{v_x^2}} = \frac{2\ell_1}{c}\left(\frac{1}{1 - \cancel{v_x^2/c^2}}\right)$$

Time t_2 from A to D and back:

$$t_2 = \frac{2\ell_2}{\sqrt{c_x^2 - v_y^2}} = \frac{2\ell_2}{c} \cdot \cancel{\frac{1}{\sqrt{1 - v_x^2/c^2}}}$$



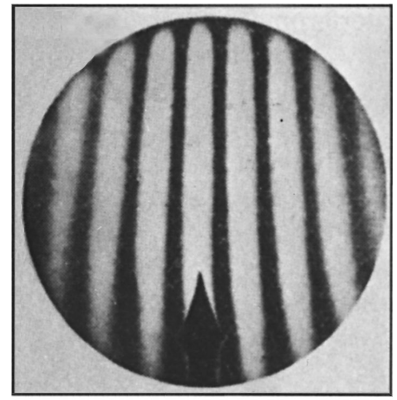
So that the change in time is:

$$\Delta t = t_2 - t_1 = \frac{2}{c}\left(\frac{\ell_2}{\cancel{\sqrt{1 - v^2/c^2}}} - \frac{\ell_1}{\cancel{1 - v^2/c^2}}\right) = 0$$

Regardless of the actual lengths l_1 and l_2, we won’t see a change in the interference pattern on turning the apparatus by 90 degrees

The Analysis for c +- v = c (continued)

Upon rotating the apparatus by +- 90°



$$\Delta t' = t'_2 - t'_1 = \frac{2}{c}\left(\frac{\ell_2}{1-\cancel{v^2/c^2}} - \frac{\ell_1}{\cancel{\sqrt{1-v^2/c^2}}}\right) = 0$$

$\Delta t' - \Delta t$ = 0 – 0 = 0, we observe no change in interference pattern as we and the apparatus can be considered at rest with respect to the non-existing ether, so no length contraction and time dilation (delay) for us, there is no preferred inertial frames of reference, for transformations between frames, we need to use the Lorentz transformations