# 11

# Molecular Structure

11-1 (a) We add the reactions $K + 4.34\text{ eV} \rightarrow K^+ + e^-$ and $I + e^- \rightarrow I^- + 3.06\text{ eV}$ to obtain $K + I \rightarrow K^+ + I^- + (4.34 - 3.06)\text{ eV}$. The activation energy is $1.28\text{ eV}$.

(b) $$\frac{dU}{dr} = \frac{4\epsilon}{\sigma}\left[-12\left(\frac{\sigma}{r}\right)^{13} + 6\left(\frac{\sigma}{r}\right)^{7}\right]$$

At $r = r_0$ we have $\frac{dU}{dr} = 0$. Here $\left(\frac{\sigma}{r_0}\right)^{13} = \frac{1}{2}\left(\frac{\sigma}{r_0}\right)^{7}$, $\frac{\sigma}{r_0} = 2^{-1/6}$,

$$\sigma = 2^{-1/6}(0.305)\text{ nm} = \boxed{0.272\text{ nm} = \sigma}.$$

Then also

$$U(r_0) = 4\epsilon\left[\left(\frac{2^{-1/6}r_0}{r_0}\right)^{12} - \left(\frac{2^{-1/6}r_0}{r_0}\right)^{6}\right] + E_a = 4\epsilon\left[\frac{1}{4} - \frac{1}{2}\right] + E_a = -\epsilon + E_a$$
$$\epsilon = E_a - U(r_0) = 1.28\text{ eV} + 3.37\text{ eV} = \boxed{4.65\text{ eV} = \epsilon}.$$

(c) $$F(r) = -\frac{dU}{dr} = \frac{4\epsilon}{\sigma}\left[12\left(\frac{\sigma}{r}\right)^{13} - 6\left(\frac{\sigma}{r}\right)^{7}\right]$$

To find the maximum force we calculate $\frac{dF}{dr} = \frac{4\epsilon}{\sigma^2}\left[-156\left(\frac{\sigma}{r}\right)^{14} + 42\left(\frac{\sigma}{r}\right)^{8}\right] = 0$ when

$$\frac{\sigma}{r_{\text{rupture}}} = \left(\frac{42}{156}\right)^{1/6}$$

$$F_{\max} = \frac{4(4.65\text{ eV})}{0.272\text{ nm}}\left[12\left(\frac{42}{156}\right)^{13/6} - 6\left(\frac{42}{156}\right)^{7/6}\right] = -41.0\text{ eV/nm}$$
$$= -41.0\frac{1.6\times10^{-19}\text{ Nm}}{10^{-9}\text{ m}} = -6.55\text{ nN}$$

Therefore the applied force required to rupture the molecule is $\boxed{+6.55\text{ nN}}$ away from the center.

# 11
# Molecular Structure

11-1 (a) We add the reactions $K+4.34\text{ eV}\to K^{+}+e^{-}$ and $I+e^{-}\to I^{-}+3.06\text{ eV}$ to obtain $K+I\to K^{+}+I^{-}+(4.34-3.06)\text{ eV}$. The activation energy is $1.28\text{ eV}$.

(b) $$\frac{dU}{dr}=\frac{4\epsilon}{\sigma}\left[-12\left(\frac{\sigma}{r}\right)^{13}+6\left(\frac{\sigma}{r}\right)^{7}\right]$$

At $r=r_0$ we have $\frac{dU}{dr}=0$. Here $\left(\frac{\sigma}{r_0}\right)^{13}=\frac{1}{2}\left(\frac{\sigma}{r_0}\right)^{7}$, $\frac{\sigma}{r_0}=2^{-1/6}$,

$$\sigma=2^{-1/6}(0.305)\text{ nm}=\boxed{0.272\text{ nm}=\sigma}.$$

Then also

$$U(r_0)=4\epsilon\left[\left(\frac{2^{-1/6}r_0}{r_0}\right)^{12}-\left(\frac{2^{-1/6}r_0}{r_0}\right)^{6}\right]+E_a=4\epsilon\left[\frac{1}{4}-\frac{1}{2}\right]+E_a=-\epsilon+E_a$$

$$\epsilon=E_a-U(r_0)=1.28\text{ eV}+3.37\text{ eV}=\boxed{4.65\text{ eV}=\epsilon}.$$

(c) $$F(r)=-\frac{dU}{dr}=\frac{4\epsilon}{\sigma}\left[12\left(\frac{\sigma}{r}\right)^{13}-6\left(\frac{\sigma}{r}\right)^{7}\right]$$

To find the maximum force we calculate $\frac{dF}{dr}=\frac{4\epsilon}{\sigma^2}\left[-156\left(\frac{\sigma}{r}\right)^{14}+42\left(\frac{\sigma}{r}\right)^{8}\right]=0$ when

$$\frac{\sigma}{r_{\text{rupture}}}=\left(\frac{42}{156}\right)^{1/6}$$

$$F_{\max}=\frac{4(4.65\text{ eV})}{0.272\text{ nm}}\left[12\left(\frac{42}{156}\right)^{13/6}-6\left(\frac{42}{156}\right)^{7/6}\right]=-41.0\text{ eV/nm}$$

$$=-41.0\frac{1.6\times10^{-19}\text{ Nm}}{10^{-9}\text{ m}}=-6.55\text{ nN}$$

Therefore the applied force required to rupture the molecule is $\boxed{+6.55\text{ nN}}$ away from the center.

11-3 For the $l=1$ to $l=2$ transition, $\Delta E = hf = \frac{[2(2+1)-1(1+1)]\hbar^2}{2I}$ or $hf = \frac{2\hbar^2}{I}$. Solving for $I$ gives

$$I = \frac{2\hbar^2}{hf} = \frac{h}{2\pi^2 f} = \frac{6.626\times10^{-34}\text{ J s}}{(2\pi^2)(2.30\times10^{11}\text{ Hz})} = 1.46\times10^{-46}\text{ kg}\cdot\text{m}^2;\ \mu = \frac{m_1 m_2}{m_1+m_2} = 1.14\times10^{-26}\text{ kg},$$

$$R_0 = \left(\frac{I}{\mu}\right)^{1/2} = 0.113\text{ nm, same as Example 11.1.}$$

11-5 (a) The separation between two adjacent rotationally levels is given by $\Delta E = \left(\frac{\hbar^2}{I}\right)l$, where $l$ is the quantum number of the higher level. Therefore

$$\Delta E_{10} = \frac{\Delta E_{65}}{6}$$

$$\lambda_{10} = 6\lambda_{65} = 6(1.35\text{ cm}) = 8.10\text{ cm}$$

$$f_{10} = \frac{c}{\lambda_{10}} = \frac{3.00\times10^{10}\text{ cm/s}}{8.10\text{ cm}} = 3.70\text{ GHz}$$

(b) $\Delta E_{10} = hf_{10} = \frac{\hbar^2}{I};$

$$I = \frac{\hbar}{2\pi f_{10}} = \frac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{(2\pi)(3.70\times10^{9}\text{ Hz})}$$

$$I = 4.53\times10^{-45}\text{ kg}\cdot\text{m}^2$$

11-7 HCl molecule in the $l=1$ rotational energy level: $R_0 = 1.275$ Å, $E_{\text{rot}} = \left(\frac{\hbar^2}{2I}\right)l(l+1)$. For $l=1$,

$$E_{\text{rot}} = \frac{\hbar^2}{I} = \frac{I\omega^2}{2},\ \omega = \left(\frac{2\hbar^2}{I^2}\right)^{1/2} = \left(\frac{\hbar}{I}\right)\sqrt{2}$$

$$I = \left[\frac{m_1 m_2}{m_1+m_2}\right]R_0^2 = \left[\frac{(1\text{ u})(35\text{ u})}{1\text{ u}+35\text{ u}}\right]R_0^2 = \left[0.972\,2\text{ u}\times1.66\times10^{-27}\text{ kg/u}\right]\times\left(1.275\times10^{-10}\text{ m}\right)^2$$

$$= 2.62\times10^{-47}\text{ kg}\cdot\text{m}^2$$

Therefore, $\omega = \left(\frac{\hbar}{I}\right)\sqrt{2} = \left[\frac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{2.62\times10^{-47}\text{ kg}\cdot\text{m}^2}\right]\sqrt{2} = 5.69\times10^{12}\text{ rad/s}.$

11-9 $\mu = \frac{m_1 m_2}{m_1+m_2} = \frac{(1\text{ u})(35\text{ u})}{(1\text{ u}+35\text{ u})} = \left(\frac{35}{36}\right)\text{u} = 1.62\times10^{-27}\text{ kg}$

(a) $I=\mu R_0^2=\left(1.62\times10^{-27}\ \text{kg}\right)\left(1.28\times10^{-10}\ \text{m}\right)^2=2.65\times10^{-47}\ \text{kg}\cdot\text{m}^2$

$$E_{\text{rot}}=\left(\frac{\hbar}{2I}\right)l(l+1)$$

$$\frac{\hbar^2}{2I}=\frac{\left(1.054\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{2\times2.65\times10^{-47}\ \text{kg}\cdot\text{m}^2}=2.1\times10^{-22}\ \text{J}=1.31\times10^{-3}\ \text{eV}$$

$$E_{\text{rot}}=\left(1.31\times10^{-3}\ \text{eV}\right)l(l+1)$$

$l=0$ $\quad E_{\text{rot}}=0$

$l=1$ $\quad E_{\text{rot}}=2.62\times10^{-3}\ \text{eV}$

$l=2$ $\quad E_{\text{rot}}=7.86\times10^{-3}\ \text{eV}$

$l=3$ $\quad E_{\text{rot}}=1.57\times10^{-2}\ \text{eV}$

(b) $U=\dfrac{Kx^2}{2}$, $U=0.15$ eV when $x=0.01$ nm

$$(0.15\ \text{eV})\left(1.6\times10^{-19}\ \text{J/eV}\right)=\frac{K\left(10^{-11}\ \text{m}\right)^2}{2}$$

$$K=480\ \text{N/m}$$

$$f=\frac{1}{2\pi}\left(\frac{K}{\mu}\right)^{1/2}=\frac{1}{2\pi}\left[\frac{480}{1.62\times10^{-27}}\right]^{1/2}=8.66\times10^{13}\ \text{Hz}$$

(c) $E_{\text{vib}}=\left(v+\dfrac{1}{2}\right)hf$

$$hf=\left(6.63\times10^{-34}\ \text{J s}\right)\left(8.66\times10^{13}\ \text{Hz}\right)=5.74\times10^{-20}\ \text{J}=0.359\ \text{eV}$$

$$E_0=\frac{hf}{2}=2.87\times10^{-20}\ \text{J}=0.179\ \text{eV}$$

$$E=\frac{KA^2}{2};\qquad 2.87\times10^{-20}\ \text{J}=\frac{(480\ \text{N/m})A_0^2}{2}$$

$$A_0=\left(\frac{2E}{K}\right)^{1/2}=1.09\times10^{-11}\ \text{m}=0.109\text{Å}=0.010\,9\ \text{nm}$$

$$E_1=\frac{3}{2}hf=8.61\times10^{-20}\ \text{J}=0.538\ \text{eV}$$

$$A_1=\left(\frac{2E}{K}\right)^{1/2}=1.89\times10^{-11}\ \text{m}=0.189\text{Å}=0.018\,9\ \text{nm}$$

(d) $\dfrac{hc}{\lambda_{\max}}=\Delta E_{\min}$ or $\lambda_{\max}=\dfrac{hc}{\Delta E_{\min}}$

<u>Rotational</u>

$$\Delta E_{\min}=E_{l=1}-E_{l=0}=2.62\times10^{-3}\ \text{eV}$$

$$hc=12\,400\ \text{eV}\cdot\text{Å}$$

$$\lambda_{\max}=\frac{12\,400}{2.62\times10^{-3}}=4.73\times10^{6}\ \text{Å}=4.73\times10^{-4}\ \text{m (microwave range).}$$

Vibrational
$\Delta E_{\min} = hf$

$$\lambda_{\max} = \frac{hc}{\Delta E_{\min}} = \frac{hc}{hf} = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{8.66\times10^{13}\ \text{Hz}} = 3.46\times10^{-6}\ \text{m} = 3.46\ \mu\text{m (infrared range)}.$$

11-11 The angular momentum of this system is $L = \frac{mvR_0}{2} + \frac{mvR_0}{2} = mvR_0$. According to Bohr theory, $L$ must be a multiple of $\hbar$, $L = mvR_0 = n\hbar$, or $v = \frac{n\hbar}{mR_0}$ with $n = 1, 2, \ldots$. The energy of rotation is then

$$E_{\text{rot}} = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = m\left(\frac{n\hbar}{mR_0}\right)^2 = \frac{n^2\hbar^2}{mR_0^2}, \qquad n = 1, 2, \ldots .$$

From Equation 11.5 the allowed energies of rotation are

$$E_{\text{rot}} = \frac{\hbar^2}{2I_{\text{cm}}}\{l(l+1)\}, \qquad l = 0, 1, 2, \ldots$$

where $I_{\text{cm}}$ is the moment of inertia about the center of mass. In the present case, we have

$$I_{\text{cm}} = m\left(\frac{R_0}{2}\right)^2 + m\left(\frac{R_0}{2}\right)^2 = \frac{mR_0^2}{2}.$$

Thus,

$$E_{\text{rot}} = \frac{\hbar^2}{mR_0^2}\{l(l+1)\} \qquad l = 0, 1, 2, \ldots .$$

We see that $l(l+1)$ replaces $n^2$ in the Bohr result. The two are indistinguishable for large quantum numbers (Correspondence Principle), but disagree markedly when $n$ (or $l$) is small. In particular, $E_{\text{rot}}$ can be zero according to Quantum Mechanics, while the minimum rotational energy in the Bohr theory is $\frac{\hbar^2}{mR_0^2}$ for $n = 1$.

11-13 At equilibrium separation $R$, $U_{\text{eff}}$ is a minimum: $0 = \left.\frac{dU_{\text{eff}}}{dr}\right|_{R_l} = \mu\omega_0^2(R_l - R_0) - \frac{l(l+1)\hbar^2}{\mu R^3}$ or $R_l = R_0 + \frac{l(l+1)\hbar^2}{\mu^2\omega_0^2}\left(\frac{1}{R_l^3}\right)$. For $l \ll \frac{\mu\omega_0 R_0^2}{\hbar}$, the second term on the right represents a small correction, and may be approximated by substituting for $R$ its approximate value $R_0$ to get the next approximation $R_l \approx R_0 + \frac{l(l+1)\hbar^2}{\mu^2\omega_0^2}\left(\frac{1}{R_0^3}\right)$. The value of $U_{\text{eff}}$ at $R_l$ is the energy offset $U_0$:

$$U_0 = U_{\text{eff}}(R) = \frac{1}{2}\mu\omega_0^2\left[\frac{l(l+1)\hbar^2}{\mu^2\omega_0^2R_l^3}\right]^2 + \frac{l(l+1)\hbar^2}{2\mu R_l^2} = \left[\frac{l(l+1)\hbar^2}{2\mu R_l^2}\right]\left[\frac{l(l+1)\hbar^2}{\mu^2\omega_0^2R_l^4} + 1\right]$$

$$\approx \frac{l(l+1)\hbar^2}{2\mu R_0^2}.$$

The curvature at the new equilibrium point is

$$\left.\frac{d^2U_{\text{eff}}}{dr^2}\right|_{R_l} = \mu\omega_0^2 + \frac{3l(l+1)\hbar^2}{\mu R_l^4}$$

and is identified with $\mu\omega_l^2$ to get the corrected oscillator frequency

$$\omega_l^2 = \omega_0^2 + \frac{3l(l+1)\hbar^2}{\mu^2 R_l^4} \approx \omega_0^2 + \frac{3l(l+1)\hbar^2}{\mu^2 R_0^4}$$

Since the second term on the right is small by assumption, $\omega_l$ differs little from $\omega_0$, so that we may write $\omega_l^2 - \omega_0^2 = (\omega_l - \omega_0)(\omega_l + \omega_0) \approx 2\omega_0\Delta\omega$. The fractional change in frequency is then $\frac{\Delta\omega}{\omega_0} \approx \frac{3l(l+1)\hbar^2}{2\mu^2\omega_0^2 R_0^4}$.

11-15 The Morse levels are given by $E_{\text{vib}} = \left(v+\frac{1}{2}\right)\hbar\omega - \left(v+\frac{1}{2}\right)^2\frac{(\hbar\omega)^2}{4U_0}$. The excitation energy from level $v$ to level $v+1$ is

$$\begin{aligned}\Delta E_{\text{vib}} &= \left(v+\frac{3}{2}\right)\hbar\omega - \left(v+\frac{3}{2}\right)^2\frac{(\hbar\omega)^2}{4U_0} - \left(v+\frac{1}{2}\right)\hbar\omega + \left(v+\frac{1}{2}\right)^2\frac{(\hbar\omega)^2}{4U_0} \\ &= \hbar\omega - \left\{\left(v+\frac{3}{2}\right)^2 - \left(v+\frac{1}{2}\right)^2\right\}\frac{(\hbar\omega)^2}{4U_0} = \hbar\omega\left[1-(v+1)\left(\frac{\hbar\omega}{2U_0}\right)\right].\end{aligned}$$

It is clear from this expression that $\Delta E_{\text{vib}}$ diminishes steadily as $v$ increases. The excitation energy could never be negative, however, so that $v$ must not exceed the value that makes $\Delta E_{\text{vib}}$ vanish: $1 = \frac{\hbar\omega}{2U_0}(v+1)$ or $v_{\max} = \frac{2U_0}{\hbar\omega} - 1$. With this value for $v$, the vibrational energy is

$$E_{\text{vib}} = 2U_0 - \frac{1}{2}\hbar\omega - \frac{[2U_0 - (1/2)\hbar\omega]^2}{4U_0} = U_0 - \frac{(\hbar\omega)^2}{16U_0}.$$

If $\frac{2U_0}{\hbar\omega}$ is not an integer, then $v_{\max}$ and the corresponding $E_{\text{vib}}$ will be somewhat smaller than the values given. However, the maximum vibrational energy will never exceed $U_0 - \frac{(\hbar\omega)^2}{16U_0}$.

11-19 To the left and right of the barrier site $\psi$ is the waveform of a free particle with wavenumber $k = \left(\frac{2mE}{\hbar^2}\right)^{1/2}$:

$$\psi(x) = A\sin(kx) + B\cos(kx) \qquad 0 \le x \le \frac{L}{2}$$

$$\psi(x) = F\sin(kx) + G\cos(kx) \qquad \frac{L}{2} \le x \le L$$

The infinite walls at the edges of the well require $\psi(0)=\psi(L)=0$, or $B=0$ and $G=-F\tan(kL)$ leaving

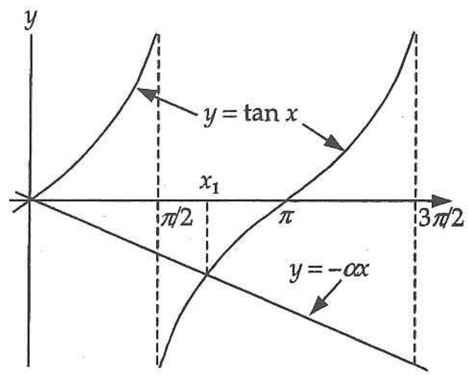
$$\psi(x)=A\sin(kx) \qquad 0\le x\le\frac{L}{2}$$

$$\psi(x)=F\{\sin(kx)-\tan(kL)\cos(kx)\}=C\sin(kx-kL) \qquad \frac{L}{2}\le x\le L$$

For waves antisymmetric about the midpoint of the well, $\psi\left(\frac{L}{2}\right)=0$ and the delta barrier is ineffective: the slope $\frac{d\psi}{dx}$ is continuous at $\frac{L}{2}$, leading to $C=+A$. For this case $\frac{kL}{2}=n\pi$, and

$$E_n=\frac{n^2\pi^2\hbar^2}{2m(L/2)^2} \qquad n=1,\ 2,\ \ldots$$

as befits an infinite well of width $\frac{L}{2}$.




The remaining stationary states are waves symmetric about $\frac{L}{2}$, and require $C=-A$ for continuity of $\psi$. Their energies are found by applying the slope condition with $C=-A$ to get $-Ak\cos\left(\frac{kL}{2}\right)-Ak\cos\left(\frac{kL}{2}\right)=\left(\frac{2mS}{\hbar^2}\right)A\sin\left(\frac{kL}{2}\right)$ or $\tan\left(\frac{kL}{2}\right)=-\left(\frac{2\hbar^2}{mSL}\right)\left(\frac{kL}{2}\right)$. Solutions to this equation may be found graphically as the intersections of the curve $y=\tan x$ with the line $y=-\alpha x$ having slope $-\alpha=-\frac{2\hbar^2}{mSL}$ (see the Figure above). From the points of intersection $x_n$ we find $k_n=\frac{2x_n}{L}$ and $E_n=\frac{\hbar^2k_n^2}{2m}$. Only values of $x_n$ greater than zero need be considered, since the wave function is unchanged when $k$ is replaced by $-k$, and $k=0$ leads to $\psi(x)=0$ everywhere. As $S\to\infty$ we see that $x_n\to n\pi$, giving $E_n=\frac{n^2\pi^2\hbar^2}{2m(L/2)^2}$ for $S\to\infty$ and $n=1,\ 2,\ \ldots$ the same energies found for the antisymmetric waves considered previously. Thus, in this limit the energy levels all are *doubly degenerate*. As $S\to 0$ the roots become $x_n=\frac{\pi}{2},\frac{3\pi}{2},\ldots=\frac{n\pi}{2}$ ($n$ odd), giving $E_n=\frac{n^2\pi^2\hbar^2}{2mL^2}$ $n=1,\ 3,\ \ldots$. These are the energies for the symmetric waves of the infinite well with no barrier, as expected for $S=0$.

The ground state wave is symmetric about $\frac{L}{2}$, and is described by the root $x_1$, which varies anywhere between $\frac{\pi}{2}$ and $\pi$ according to $S$. The ground state energy is

$$E_1 = \frac{\hbar^2(2x_1/L)^2}{2m} = \frac{2x_1^2\hbar^2}{mL^2}.$$

The first excited state wave is antisymmetric, with energy

$$E_2 = \frac{\pi^2\hbar^2}{2m(L/2)^2} = \frac{2\pi^2\hbar^2}{mL^2},$$

which coincides with $E_1$ in the limit $S \to \infty$.

11-21 By trial and error, we discover that the choice $R = 1.44$ (bohr) minimizes the expression for $E_{\text{tot}}$, so that this is the equilibrium separation $R_0$.

The effective spring constant $K$ is the curvature of $E_{\text{tot}}(R)$ evaluated at the equilibrium point $R_0 = 1.44$. Using the given approximation to the second derivative with an increment $\Delta R = 0.01$, we find

$$K = \left.\frac{d^2E_{\text{tot}}}{dR^2}\right|_{R_0} \approx 1.03.$$

(An increment ten times as large changes the result by less than one unit in the last decimal place.) This value for $K$ is in $(\text{Ry}/\text{bohr}^2)$. The conversion to SI units is accomplished with the help of the relations $1\ \text{Ry} = 13.6\ \text{eV} = 2.176\times10^{-18}\ \text{J}$, and $1\ \text{bohr} = 0.529\ \text{Å} = 5.29\times10^{-11}\ \text{m}$. Then $K = 1.03\ \text{Ry}/\text{bohr}^2 = 801\ \text{J}/\text{m}^2 = 801\ \text{N}/\text{m}$. The result is larger than the experimental value because our neglect of electron-electron repulsion leads to a potential well much deeper than the actual one, producing a larger curvature.

# 12

# The Solid State

12-1 $U_{\text{Total}} = U_{\text{attractive}} + U_{\text{repulsive}} = -\dfrac{\alpha ke^2}{r} + \dfrac{B}{r^m}$. At equilibrium, $U_{\text{Total}}$ reaches its minimum value.

$\dfrac{dU_{\text{Total}}}{dr} = 0 = +\dfrac{\alpha ke^2}{r^2} - \dfrac{mB}{r^{m+1}}$. Calling the equilibrium separation $r_0$, we may solve for $B$

$$\frac{mB}{r_0^{m+1}} = \frac{\alpha ke^2}{r_0^2}$$

$$B = \frac{\alpha ke^2}{mr_0^{m-1}}.$$

Substituting into the expression for $U_{\text{Total}}$ we find

$$U_0 = -\frac{\alpha ke^2}{r_0} + \frac{(\alpha ke^2/m)r_0^{m-1}}{r_0^m} = -\left(\frac{\alpha ke^2}{r_0}\right)\left(1 - \frac{1}{m}\right).$$

12-3 $U = -\alpha k\left(\dfrac{e^2}{r_0}\right)\left(1 - \dfrac{1}{m}\right)$

$U = -(1.747\,6)(9 \times 10^9 \text{ Nm}^2/\text{C}^2)\left[\dfrac{(1.6 \times 10^{-19}\text{ C})^2}{0.281 \times 10^{-9}\text{ m}}\right]\left(1 - \dfrac{1}{8}\right)$

$U = -1.25 \times 10^{-18}\text{ J} = -7.84\text{ eV}$. The ionic cohesive energy is $U = 7.84\text{ eV/Na}^+\text{-Cl}^-$ pair.

12-5 $U = -\dfrac{ke^2}{r} - \dfrac{ke^2}{r} + \dfrac{ke^2}{2r} + \dfrac{ke^2}{2r} - \dfrac{ke^2}{3r} - \dfrac{ke^2}{3r} + \dfrac{ke^2}{4r} + \dfrac{ke^2}{4r} - \ldots = -2k\left(\dfrac{e^2}{r}\right)\left[1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \ldots\right]$

but $\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \ldots$ so $U = -\dfrac{(2\ln 2)ke^2}{r}$.

12-7 (a) $|U_0| = \left(\dfrac{\alpha ke^2}{r_0}\right)\left(1 - \dfrac{1}{m}\right) = \dfrac{(1.747\,6)(9.00 \times 10^9\text{ Nm}^2/\text{C}^2)(1.60 \times 10^{-19})^2}{0.314 \times 10^{-19}}$

$\left(1 - \dfrac{1}{9}\right) = 1.14 \times 10^{-18}\text{ J} = 7.12\text{ eV/K}^+\text{-Cl}^-$

(b) Atomic cohesive energy = ionic cohesive energy + energy needed to remove an electron from $\text{Cl}^-$ – energy gained by adding the electron to $\text{K}^+ = 7.12\text{ eV} + 3.61\text{ eV} - 4.34\text{ eV} = 6.39\text{ eV/KCl}$.

12-9 (a) $\int_0^\infty \left(\frac{N}{\tau}\right) e^{-t/\tau} dt = -Ne^{-t/\tau}\Big|_0^\infty = -N\left[e^{-\infty} - e^0\right] = N$

(b) $\bar{t} = \left(\frac{1}{N}\right)\int_0^\infty \left(\frac{tN}{\tau}\right) e^{-t/\tau} dt = \tau \int_0^\infty \left(\frac{t}{\tau}\right) e^{-t/\tau} \frac{dt}{\tau} = \tau \int_0^\infty z e^{-z} dz$

$$z = u \qquad dv = e^{-z} dz$$
$$dz = du \qquad v = -e^{-z}$$

so $\int_0^\infty z e^{-z} dz = \left(-z e^{-z}\right)\Big|_0^\infty + \int_0^\infty e^{-z} dz = 0 - e^{-z}\Big|_0^\infty = 1$. Therefore, $\bar{t} = \tau$.

(c) Similarly $\overline{t^2} = \left(\frac{1}{N}\right)\int_0^\infty \left(\frac{t^2 N}{\tau}\right) e^{-t/\tau} dt$. Integrating by parts twice, gives $\overline{t^2} = 2\tau^2$.

12-11 (a) Equation 12.12 was $J = nev_d$. As $v_d = \mu E$, $J = ne\mu E$. Also comparing Equation 12.10, $v_d = \frac{e\tau E}{m_e}$, and $v_d = \mu E$, one has $\mu = \frac{e\tau}{m_e}$.

(b) As $J = \sigma E$ and $J = J_{\text{electrons}} + J_{\text{holes}} = ne\mu_n E + pe\mu_p E$, $\sigma = ne\mu_n + pe\mu_p$

(c) The electron drift velocity is given by

$$v_d = \mu_n E = \left(3\,900 \text{ cm}^2/\text{Vs}\right)(100 \text{ V/cm}) = 3.9 \times 10^5 \text{ cm/s}.$$

(d) An intrinsic semiconductor has $n = p$. Thus

$$\sigma = ne\mu_n + pe\mu_p = pe(\mu_n + \mu_p) = \left(3.0 \times 10^{13} \text{ cm}^{-3}\right)\left(1.6 \times 10^{-19} \text{ C}\right)\left(5\,800 \text{ cm}^2/\text{Vs}\right)$$
$$= 0.028 \text{ A/V cm} = 0.028 \ (\Omega \text{ cm})^{-1} = 2.8(\Omega \text{ m})^{-1}$$
$$\rho = \frac{1}{\sigma} = 0.36 \ \Omega \text{ m}$$

12-13 (a) We assume all expressions still hold with $v_{\text{rms}}$ replaced by $v_{\text{F}}$.

$$\tau = \frac{\sigma m_e}{ne^2}$$
$$\sigma = \frac{1}{\rho} = \left(1.60 \times 10^{-8}\right)^{-1} (\Omega \text{ m})^{-1} = 6.25 \times 10^7 \ (\Omega \text{ m})^{-1}$$
$$n = \frac{\#\text{ of } e^-}{m^3} = \left(\frac{1\, e^-}{\text{atom}}\right)\left(6.02 \times 10^{26} \text{ atoms/k mole}\right)\left(10.5 \times 10^3 \text{ kg/m}^3\right)\left(\frac{1 \text{ kmole}}{108 \text{ g}}\right)$$
$$n = 5.85 \times 10^{28} \ e^-/\text{m}^3$$

so $\tau = \frac{\left(6.25 \times 10^7\right)(\Omega \text{ m})^{-1}\left(9.11 \times 10^{-31} \text{ kg}\right)}{\left(5.85 \times 10^{28} \ e^-/\text{m}^3\right)\left(1.6 \times 10^{-19} \text{ C}\right)^2} = 3.80 \times 10^{-14}$ s (no change of course from Equation 12.10).

(b) Now $L = v_F\tau$ and $v_F = \left(\frac{2E_F}{m}\right)^{1/2}$

$$v_F = \left[\frac{2\times 5.48\text{ eV}\times 1.6\times 10^{-19}\text{ J/eV}}{9.11\times 10^{-31}\text{ kg}}\right]^{1/2} = 1.39\times 10^{6}\text{ m/s}.$$

$$L = (1.39\times 10^{6}\text{ m/s})(3.8\times 10^{-14}\text{ s}) = 5.27\times 10^{-8}\text{ m} = 527\text{ Å} = 52.7\text{ nm}$$

(c) The approximate lattice spacing in silver may be calculated from the density and the molar weight. The calculation is the same as the $n$ calculation. Thus, $(\#\text{ of Ag atoms})/\text{m}^3 = 5.85\times 10^{28}$. Assuming each silver atom fits in a cube of side, $d$,

$$d^3 = (5.85\times 10^{28})^{-1}\text{ m}^3/\text{atom}$$

$$d = 2.57\times 10^{-10}\text{ m}$$

So $\frac{L}{d} = \frac{5.27\times 10^{-8}}{2.57\times 10^{-10}} = 205$.

12-15 (a) $E_g = 1.14\text{ eV}$ for Si

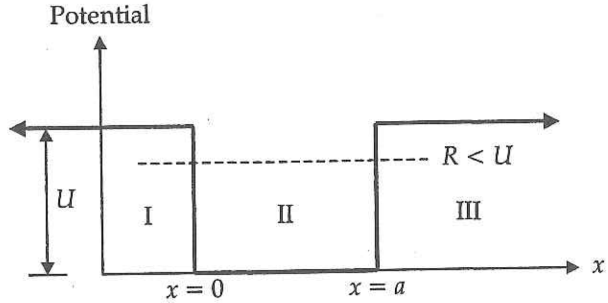
$hf = 1.14\text{ eV} = (1.14\text{ eV})(1.6\times 10^{-19}\text{ J/eV}) = 1.82\times 10^{-19}\text{ J}$

$f = 2.75\times 10^{14}\text{ Hz}$

(b) $c = \lambda f;\ \lambda = \frac{c}{f} = \frac{3\times 10^{8}\text{ m/s}}{2.75\times 10^{14}\text{ Hz}} = 1.09\times 10^{-6}\text{ m}$

$\lambda = 1\,090\text{ nm}$ (in the infrared region)

12-17 (a)




$\psi_{\text{I}} = Ae^{Kx}$ $\qquad K\hbar = [2m(U-E)]^{1/2}$

$\psi_{\text{II}} = B\cos kx + C\sin kx$ $\qquad k\hbar = (2mE)^{1/2}$

$\psi_{\text{III}} = De^{-Kx}$

In region I and III the wave equation has the form $\frac{d^2\psi(x)}{dx^2} = K^2\psi(x)$ with $K = \frac{[2m(U-E)]^{1/2}}{\hbar}$. This equation has solutions of the form

$$\psi_{\text{I}}(x) = Ae^{Kx}\text{ for } x\le 0 \quad \text{(region I)}$$

$$\psi_{\text{III}} = De^{-Kx}\text{ for } x\ge 0 \quad \text{(region III)}$$

In region II where $U(x) = 0$ we have $\frac{d^2\psi(x)}{dx^2} = -k^2\psi(x)$ with $k = \frac{[2mE]^{1/2}}{\hbar}$. This equation has trigonometric solutions

$$\psi_{\text{II}}(x) = B\cos kx + C\sin kx \qquad 0 \le x \le a$$

with $k = \frac{(2mE)^{1/2}}{\hbar}$. The wave function and its slope are continuous everywhere, and in particular at the well edges $x = 0$ and $x = a$. Thus, we must require

$$\begin{array}{ll} A = B & [\text{continuity of } \psi(x) \text{ at } x = 0] \\ KA = kC & \left[\text{continuity of } \frac{d\psi(x)}{dx} \text{ at } x = 0\right] \\ B\cos kx + C\sin kx = De^{-Kx} & [\text{continuity of } \psi(x) \text{ at } x = a] \\ -Bk\sin kx + Ck\cos kx = -DKe^{-Kx} & \left[\text{continuity of } \frac{d\psi(x)}{dx} \text{ at } x = a\right] \end{array}$$

There are four equations in the four coefficients *A, B, C, D*. Use the first equation to eliminate *A*. Then from the second equation we obtain $B = \left(\frac{k}{K}\right)C$. Divide the last two equations to eliminate *D*.

$$\frac{-Bk\sin kx + Ck\cos kx}{B\cos kx + C\sin kx} = -\frac{DKe^{-Kx}}{De^{-Kx}}.$$

Cross multiply, gather terms and write *B* in terms of *C*. Then we have

$$\left(-\frac{k^2}{K}\right)Ck\sin ka + Ck\cos ka = -K\left(\frac{k}{K}\right)C\cos ka - KC\sin ka.$$

Divide out C and gather terms to obtain $\left(K^2 - k^2\right)\sin ka = -2ka\cos k$. Now substitute $k = (2mE)^{1/2}\left[\frac{2m(U-E)}{\hbar^2}\right]^{1/2}\cos ka$. This equation simplifies to:
$U\sin ka = -2[E(U-E)]^{1/2}\cos ka$, which is a transcendental equation for the bound energy states. Rearranging,

$$\tan^2 ka = \tan^2\left[\frac{(2mE)^{1/2}}{\hbar}\right]a = \frac{4E(U-E)}{U^2}.$$

(b) The energy equation is a transcendental equation and can be solved for the roots, $E_n$ by using Newton's root formula as an iterative method employing a computer. If you know the form of $f(x)$ then you can approximate the value of *x* for which $f(x) = 0$. Choose an initial value of *x*. The energy equation can be written as

$$f(E) = \tan^2\left[\frac{(2mE)^{1/2}}{\hbar}\right]a - [4E(U-E)]U^2 = 0.$$

In this problem approximate by using the energy for an electron in a well. The first guess energy is: $E_n = \dfrac{n^2\pi^2\hbar^2}{2m_e a + 2\delta}$ where

$$\delta = \frac{1}{K} \approx \frac{197.3 \text{ eV nm}/c}{2(0.511\times 10^6 \text{ eV}/c^2)(100 \text{ eV})} = 0.0193 \text{ nm}$$

and so

$$E_n = \frac{n^2\pi^2(197.3 \text{ eV nm}/c)}{2(0.511\times 10^6 \text{ eV}/c^2)(0.10 \text{ nm} + 0.039 \text{ nm})^2} = n^2(19.5 \text{ eV})$$
$$E_1 = 19.5 \text{ eV}$$
$$E_2 = (2)^2(19.5 \text{ eV}) = 78.0 \text{ eV}$$
$$E_3 = (3)^2(19.5 \text{ eV}) = 175.5 > U \text{ therefore unbound}$$

These values seem reasonable since there are only two bound states. The next step in Newton's method is to calculate $f(x)$ and $f'(x)$ at the first guess value. Then use the definition of slope and tangent:

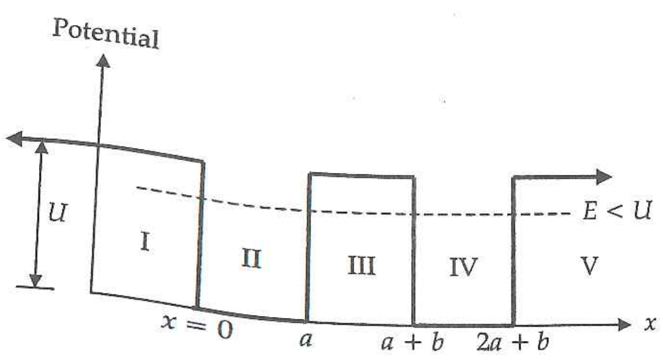
$$x_1 - x_2 = \frac{f(x)}{f'(x)} \text{ or } x_2 = x_1 - \frac{f(x)}{f'(x)}.$$

Use $x_2$ as a new estimate to evaluate $x_3$, etc. Monitor $x$ and $f(x)$ for convergence and divergence. Use the first term of two of the Taylor series for the first guess of $f'(x)$. Thus our first guess would be $x_1 = E = 19.5$ eV and

$$f(E) = \tan^2 ka - \frac{4E(U-E)}{U^2}$$
$$f'(E) = \left(\frac{2a}{\hbar}\right)\left(\frac{2m_e}{E}\right)^{1/2} \frac{\sin\left[(a/\hbar)(2mE)^{1/2}\right]}{\cos^3\left[(a/\hbar)(2m_e E)^{1/2}\right]} + \left(\frac{4}{U^2}\right)(2E - U)$$

where $U = 100$ eV and $E = 19.5$ eV. Calculate $x_2$ and keep repeating, watching for convergence.

(c)



$$\psi_{\mathrm{I}} = Ae^{Kx} \qquad K\hbar = [2m(U-E)]^{1/2}$$
$$\psi_{\mathrm{II}} = B\cos kx + C\sin kx \qquad k\hbar = (2mE)^{1/2}$$
$$\psi_{\mathrm{III}} = De^{Kx} + E'e^{-Kx}$$
$$\psi_{\mathrm{IV}} = F\cos kx + G\sin kx$$
$$\psi_{\mathrm{V}} = He^{-Kx}$$

At $x = 0$, $\psi_{\mathrm{I}} = \psi_{\mathrm{II}}$. Therefore $A = B$, $\frac{d\psi_{\mathrm{I}}}{dx} = \frac{d\psi_{\mathrm{II}}}{dx}$ yields $KA = kC$. Similarly at $x = a$: $\psi_{\mathrm{II}} = \psi_{\mathrm{III}}$, $B\cos ka + C\sin ka = De^{Ka} + E'e^{-Ka}$ and $\frac{d\psi_{\mathrm{II}}}{dx} = \frac{d\psi_{\mathrm{III}}}{dx}$, $-Bk\cos ka + Ck\cos ka = KDe^{Ka} - KE'e^{-Ka}$. Substitute $C = \frac{KA}{k}$ and $B = A$ to obtain

$$A\cos ka + \left(\frac{KA}{k}\right)\sin ka = De^{Ka} + E'e^{-Ka}$$
$$-Ak\sin ka + \left(\frac{KA}{k}\right)\cos ka = KDe^{Ka} - KE'e^{-Ka}$$

Solve for $A$ in each equation and equate quantities to obtain

$$De^{Ka} + \frac{E'e^{-Ka}}{\cos ka + (K/k)\sin ka} = KDe^{Ka} - \frac{KE'e^{-Ka}}{-k\sin ka + K\cos ka}.$$

Clear denominators and gather terms. After some algebra one obtains

$$\frac{D}{E'} = \frac{-K(\cos ka + (K/a)\sin ka) - (-k\sin ka + K\cos ka)e^{-Ka}}{[-k\sin ka + K\cos ka - K(\cos ka + (K/k)\sin ka)]e^{Ka}}.$$

This can be simplified to obtain

$$\frac{D}{E'} = \frac{2e^{-2Ka}[\cos ka + (1/2)[(K/k) - k/K]\sin ka]}{[(k/K) + K/k]\sin ka}.$$

Impose the continuity conditions at $x = a + b$ and let $\alpha = k(a+b)$ and

$$\beta = K(a+b)$$
$$\psi_{\mathrm{III}} = \psi_{\mathrm{IV}}$$

$$De^{\beta} + E'e^{-\beta} = F\cos\alpha + G\sin\alpha \Rightarrow \frac{F\cos\alpha + G\sin\alpha}{De^{\beta} + E'e^{-\beta}} = 1, \text{ and } \frac{d\psi_{\mathrm{III}}}{dx} = \frac{d\psi_{\mathrm{IV}}}{dx}$$

$$KDe^{\beta} + KE'e^{-\beta} = -Fk\cos\alpha + Gk\sin\alpha \Rightarrow \frac{-Fk\cos\alpha + Gk\sin\alpha}{KDe^{\beta} + KE'e^{-\beta}} = 1.$$

Set quantities equal to 1 equal to each other and clear fractions to obtain

$$(F\cos\alpha + G\sin\alpha)\left(KDe^{\beta} + KE'e^{-\beta}\right) = (-Fk\cos\alpha + Gk\sin\alpha)\left(De^{\beta} + E'e^{-\beta}\right).$$

Divide by $E'$ and gather terms to obtain

$$FK\left[\left(\frac{D}{E'}\right)\cos\alpha+\left(\frac{k}{K}\right)\left(\frac{D}{E'}\right)\sin\alpha\right]e^{\beta}-Fk\left[\cos k\alpha+\left(\frac{k}{K}\right)\sin\alpha\right]e^{-\beta}$$
$$=GK\left[\left(\frac{k}{K}\right)\left(\frac{D}{E'}\right)\cos\alpha-\left(\frac{D}{E'}\right)\sin\alpha\right]e^{\beta}+GK\left[\sin\alpha+\left(\frac{k}{K}\right)\left(\frac{D}{E'}\right)\cos\alpha\right]e^{-\beta}$$

Divide through by $G$ and $\left(\frac{k}{K}\right)$ to obtain

$$\frac{F}{G}=\frac{(D/E')e^{\beta}[\cos\alpha-(K/k)\sin\alpha]+e^{-\beta}[\cos\alpha+(K/k)\sin\alpha]}{(D/E')e^{\beta}[\sin\alpha+(K/k)\cos\alpha]+e^{-\beta}[\sin\alpha-(K/k)\cos\alpha]}$$

at $x=2a$, $\psi_{\mathrm{IV}}=\psi_{\mathrm{V}}$, $F\cos k(2a+b)+G\sin k(2a+b)=He^{-Kx}$ and $\frac{d\psi_{\mathrm{IV}}}{dx}=\frac{d\psi_{\mathrm{V}}}{dx}$ and dividing by (–$K$) we obtain $\frac{1}{K}[Fk\sin k(2a+b)-G\cos k(2a+b)]=He^{-Kx}$. Both equations are equal to the same quantity so set equal to each other.

$$F\cos k(2a+b)+G\sin k(2a+b)=\frac{1}{K}[Fk\sin k(2a+b)-G\cos k(2a+b)].$$

Now gather terms and divide by $G$ and $\left(-\frac{k}{K}\right)$ to obtain

$$\frac{F}{G}=\frac{\cos k(2a+b)+(K/k)\sin k(2a+b)}{\sin k(2a+b)-(K/k)\cos k(2a+b)}.$$

Equating the two expressions for $\frac{F}{G}$

$$\frac{\cos k(2a+b)+(K/k)\sin k(2a+b)}{\sin k(2a+b)-(K/k)\cos k(2a+b)}=\frac{(D/E')e^{\beta}[\cos\alpha-(K/k)\sin\alpha]+e^{-\beta}[\cos\alpha+(K/k)\sin\alpha]}{(D/E')e^{\beta}[\sin\alpha+(K/k)\cos\alpha]+e^{-\beta}[\sin\alpha-(K/k)\cos\alpha]}$$

Bringing all terms to one side gives a transcendental equation in $E$

$$f(E)=\frac{(D/E')e^{\beta}[\cos\alpha-(K/k)\sin\alpha]+e^{-\beta}[\cos\alpha+(K/k)\sin\alpha]}{(D/E')e^{\beta}[\sin\alpha+(K/k)\cos\alpha]+e^{-\beta}[\sin\alpha-(K/k)\cos\alpha]}$$
$$-\frac{\cos k(2a+b)+(K/k)\sin k(a+b)}{\sin k(2a+b)-(K/k)\cos k(2a+b)}=0$$

with $U$, $a$, and $b$ as parameters. This equation can be solved numerically with Newton roots method used in the solution to 12-17(b). The form of the program will depend strongly on the computer language used, including its subroutine (function, module) structure. Assume you can write a module to calculate $f(E)$ where $a=b=1$ and $U=100$. Output tabular values of $E$ and $f(E)$ and/or graph $E$ and $f(E)$. The Newton method requires both function and its derivative to be used. This is algebraically complicated so that it proves more practical to use a more interactive program. Use the computer to calculate $f(E)$ for any $E$ you enter. Use trial and error to converge to the values of $E$ for which $f(E)$ changes sign. Those are the values of $E$, which satisfy the equation and are the bound states of the double square well.

The search procedure is: Guess one value of $E$ and calculate $f(E)$. Guess a second value of E, not very different and calculate $f(E)$. If the sign of $f(E)$ changes, interpolate a new $E$ and calculate its $f(E)$. If the sign of $f(E)$ did not change, extrapolate in a direction toward the smaller $|f(E)|$. Continue until $\Delta E$, which causes $f(E)$ to change spin, is small enough for your needs. That is, less than 1 eV for this problem, since you are looking for other splittings of the single-well energies at 19 eV and 70 eV.

12-19 (a) $\frac{dm}{d\lambda}=\frac{d}{d\lambda}\left\{\frac{2Ln}{\lambda}\right\}=\left(\frac{2L}{\lambda}\right)\left(\frac{dn}{d\lambda}-\frac{n}{\lambda}\right)$

Replacing $dm$ and $d\lambda$ with $\Delta m$ and $\Delta\lambda$ yields

$$\Delta\lambda=\frac{\lambda^2\Delta m}{2L}\left(\frac{\lambda dn}{d\lambda}-n\right)$$

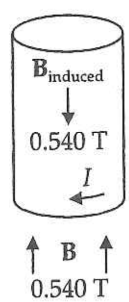
or $|\Delta\lambda|=\frac{\lambda^2}{2L}\left(n-\frac{\lambda dn}{d\lambda}\right)$. Since $\Delta\lambda$ is negative for $\Delta m=+1$.

(b) $|\Delta\lambda|=\frac{(837\times10^{-9}\text{ m})^2}{(0.6\times10^{-3}\text{ m})}\left[3.58-(837\text{ nm})(3.8\times10^{-4}\text{ nm}^{-1})\right]=3.6\times10^{-10}\text{ m}=0.38\text{ nm}$

(c) $|\Delta\lambda|=\frac{(633\times10^{-9}\text{ m})^2}{(0.6\times10^{0}\text{ m})(1)}=6.7\times10^{-13}\text{ m}=0.000\,67\text{ nm}=6.7\times10^{-4}\text{ nm}$

The controlling factor is cavity length, $L$.

12-21 (a) See the figure below.



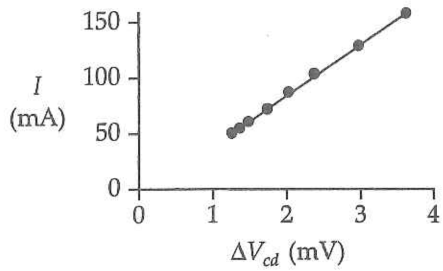

(b) For a surface current around the outside of the cylinder as shown,

$$B=\frac{N\mu_0 I}{\ell}\text{ or }NI=\frac{B\ell}{\mu_0}=\frac{(0.540\text{ T})(2.50\times10^{-2}\text{ m})}{(4\pi\times10^{-7})\text{ T}\cdot\text{m/A}}=\boxed{10.7\text{ kA}}.$$

12-23 (a) $\Delta V=IR$

If $R=0$, then $\Delta V=0$, even when $I\neq0$.

(b) The graph shows a direct proportionality.




$$\text{Slope} = \frac{1}{R} = \frac{\Delta I}{\Delta V} = \frac{(155-57.8)\text{ mA}}{(3.61-1.356)\text{ mV}} = 43.1\ \Omega^{-1}$$

$$R = \boxed{0.023\,2\ \Omega}$$

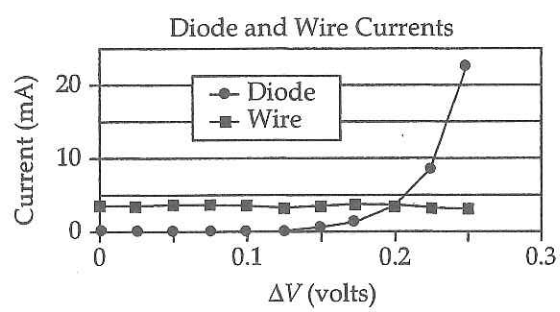
(c) Expulsion of magnetic flux and therefore fewer current-carrying paths could explain the decrease in current.

12-25 (a) The currents to be plotted are

$$I_D = \left(10^{-6}\text{ A}\right)\left(e^{\Delta V/0.025\text{ V}} - 1\right),\ I_W = \frac{2.42\text{ V} - \Delta V}{745\ \Omega}$$

The two graphs intersect at $\Delta V = 0.200$ V. The currents are then

$$I_D = \left(10^{-6}\text{ A}\right)\left(e^{0.200\text{ V}/0.025\text{ V}} - 1\right) = 2.98\text{ mA}$$




$$I_W = \frac{2.42\text{ V} - 0.200\text{ V}}{745\ \Omega} = 2.98\text{ mA}.$$ They agree to three digits. $\therefore I_D = I_W = \boxed{2.98\text{ mA}}$

(b) $$\frac{\Delta V}{I_D} = \frac{0.200\text{ V}}{2.98\times10^{-3}\text{ A}} = \boxed{67.1\ \Omega}$$

(c) $$\frac{d(\Delta V)}{dI_D} = \left[\frac{dI_D}{d(\Delta V)}\right]^{-1} = \left[\frac{10^{-6}\text{ A}}{0.025\text{ V}}e^{0.200\text{ V}/0.025\text{ V}}\right]^{-1} = \boxed{8.39\ \Omega}$$

# 13

# Nuclear Structure

13-1 $R = R_0 A^{1/3}$ where $R_0 = 1.2$ fm;

(a) $A = 4$ so $R_{He} = (1.2)(4)^{1/3}$ fm $= 1.9$ fm

(b) $A = 238$ so $R_U = (1.2)(238)^{1/3}$ fm $= 7.44$ fm

(c) $\frac{R_U}{R_{He}} = \frac{7.44 \text{ fm}}{1.9 \text{ fm}} = 3.92$

13-3 $\frac{\rho_{NUC}}{\rho_{ATOMIC}} = \frac{M_{NUC}/V_{NUC}}{M_{ATOMIC}/V_{ATOMIC}}$ and approximately; $M_{NUC} = M_{ATOMIC}$. Therefore $\frac{\rho_{NUC}}{\rho_{ATOMIC}} = \left(\frac{r_0}{R}\right)^3$ where $r_0 = 0.529$ Å $= 5.29 \times 10^{-11}$ m and $R = 1.2 \times 10^{-15}$ m (Equation 13.1 where $A = 1$). So that $\frac{\rho_{NUC}}{\rho_{ATOMIC}} = \left(\frac{5.29 \times 10^{-11} \text{ m}}{1.2 \times 10^{-15} \text{ m}}\right)^3 = 8.57 \times 10^{13}$.

13-5 (a) The initial kinetic energy of the alpha particle must equal the electrostatic potential energy of the two particle system at the distance of closest approach; $K_\alpha = U = \frac{kqQ}{r_{min}}$

and $r_{min} = \frac{kqQ}{K_\alpha} = \frac{(9 \times 10^9 \text{ N m}^2/\text{C}^2)2(79)(1.6 \times 10^{-19} \text{ C})^2}{[0.5 \text{ MeV}(1.6 \times 10^{-13} \text{ J/MeV})]} = 4.55 \times 10^{-13}$ m.

(b) Note that $K_\alpha = \frac{1}{2}mv^2 = \frac{kqQ}{r_{min}}$, so

$$v = \left[\frac{2kqQ}{mr_{min}}\right]^{1/2} = \left[\frac{2(9 \times 10^9 \text{ N m}^2/\text{C}^2)2(79)(1.6 \times 10^{-19} \text{ C})^2}{4(1.67 \times 10^{-27} \text{ kg})(3 \times 10^{-13} \text{ m})}\right]^{1/2} = 6.03 \times 10^6 \text{ m/s}$$

13-7 $E = -\mu \cdot B$ so the energies are $E_1 = +\mu B$ and $E_2 = -\mu B$. $\mu = 2.7928\mu_n$ and $\mu_n = 5.05 \times 10^{-27}$ J/T
$\Delta E = 2\mu B = 2 \times 2.7928 \times 5.05 \times 10^{-27} \text{ J/T} \times 12.5 \text{ T} = 3.53 \times 10^{-25} \text{ J} = 2.2 \times 10^{-6}$ eV

13-9 We need to use the procedure to calculate a "weighted average." Let the fractional abundances be represented by $f_{63}+f_{65}=1$; then $\dfrac{f_{63}m(^{63}\mathrm{Cu})+f_{65}m(^{65}\mathrm{Cu})}{(f_{63}+f_{65})}=m_{\mathrm{Cu}}$. We find

$$f_{63}=\frac{m(^{65}\mathrm{Cu})-m_{\mathrm{Cu}}}{m(^{65}\mathrm{Cu})-m(^{63}\mathrm{Cu})},\ f_{63}=\frac{64.95\ \mathrm{u}-63.55\ \mathrm{u}}{64.95\ \mathrm{u}-62.95\ \mathrm{u}}=0.30 \text{ or } 30\% \text{ and } f_{65}=1-f_{63}=0.70 \text{ or } 70\%.$$

13-11 $\dfrac{E_b}{A}=\dfrac{1}{3}[1(1.007\,276\ \mathrm{u})+2(1.008\,665\ \mathrm{u})-3.016\,05\ \mathrm{u}](931.5\ \mathrm{MeV/u})=2.657\ \mathrm{MeV/nucleon}$

13-13 (a) The neutron to proton ratio, $\dfrac{A-Z}{Z}$ is greatest for $^{139}_{55}\mathrm{Cs}$ and is equal to 1.53.

(b) Using $E_b=C_1A-C_2A^{2/3}-C_3(Z(Z-1))A^{-1/3}-C_4\dfrac{(N-Z)^2}{A}$ the only variation will be in the coefficients of $C_3$ and $C_4$ since the isotopes have the same $A$ number. For $^{139}_{59}\mathrm{Pr}$

$$E_b=(15.7)(139)-(17.8)(139)^{2/3}-0.71(59)(139)^{1/3}-\frac{23.6(21)^2}{139}=1\,160.8\ \mathrm{MeV}$$

$$\frac{E_b}{A}=\frac{E_b}{139}=8.351\ \mathrm{MeV}$$

For $^{139}_{57}\mathrm{La}$

$$E_b=(15.7)(139)-(17.8)(139)^{2/3}-0.71(55)(54)(139)^{1/3}-\frac{23.6(25)^2}{139}=1\,161.1\ \mathrm{MeV}$$

$$\frac{E_b}{A}=\frac{E_b}{139}=8.353\ \mathrm{MeV}$$

For $^{139}_{55}\mathrm{Cs}$

$$E_b=(15.7)(139)-(17.8)(139)^{2/3}-0.71(55)(54)(139)^{1/3}-\frac{23.6(29)^2}{139}=1\,154.9\ \mathrm{MeV}$$

$$\frac{E_b}{A}=\frac{E_b}{139}=8.308\ \mathrm{MeV}$$

$^{139}\mathrm{La}$ has the largest binding energy per nucleon of 8.353 MeV

(c) The mass of the neutron is greater than the mass of a proton therefore expect the nucleus with the largest $N$ and smallest $Z$ to weigh the most: $^{139}_{55}\mathrm{Cs}$ with a mass of 138.913 u.

13-15 Use Equation 13.4, $E_b=\left[ZM(\mathrm{H})+Nm_{\mathrm{n}}-M(^{A}_{Z}\mathrm{X})\right]$

(a) For $^{20}_{10}\mathrm{Ne}$;

$$E_b=[10(1.007\,825\ \mathrm{u})+10(1.008\,665)-(19.992\,436\ \mathrm{u})](931.494\ \mathrm{MeV/u})=160.650$$

$$\frac{E_b}{A}=8.03\ \mathrm{MeV/nucleon}$$

(b) For $^{40}_{20}\text{Ca}$

$$E_b = [20(1.007\,825\text{ u}) + 20(1.008\,665) - (39.962\,591\text{ u})](931.494\text{ MeV/u}) = 342.053$$
$$\frac{E_b}{A} = 8.55\text{ MeV/nucleon}$$

(c) For $^{93}_{41}\text{N}_b$;

$$E_b = [41(1.007\,825\text{ u}) + 52(1.008\,665) - (92.906\,377\text{ u})](931.494\text{ MeV/u}) = 805.768$$
$$\frac{E_b}{A} = 8.66\text{ MeV/nucleon}$$

(d) For $^{197}_{79}\text{Au}$

$$E_b = [79(1.007\,825\text{ u}) + 118(1.008\,665) - (196.966\,543\,1\text{ u})](931.494\text{ MeV/u}) = 1\,559.416$$
$$\frac{E_b}{A} = 7.92\text{ MeV/nucleon}$$

13-17 $\Delta E = E_{bf} - E_{bi}$

For $A = 200$; $\frac{E_b}{A} = 7.8\text{ MeV}$ so

$$E_{bi} = (A_i)(7.8\text{ MeV}) = (200)(7.8) = 1\,560\text{ MeV}$$

For $A \approx 100$; $\frac{E_b}{A} \approx 8.6\text{ MeV}$ so

$$E_{bf} = (2)(100)(8.6\text{ MeV}) = (200)(8.6) = 1\,720\text{ MeV}$$
$$\Delta E = E_{bf} - E_{bi} = 1\,720\text{ MeV} - 1\,560\text{ MeV} = 160\text{ MeV}$$

13-19 (a) The potential at the surface of a sphere of charge $q$ and radius $r$ is $V = \frac{kq}{r}$. If a thin shell of charge $dq$ (thickness $dr$) is added to the sphere, the increase in electrostatic potential energy will be $dU = Vdq = \left(\frac{kq}{r}\right)dq$. To build up a sphere with final radius $R$, the total energy will be $U = \int_0^R \left(\frac{kq}{r}\right)dq$; where $q = \frac{4}{3}\pi r^3 \rho = \frac{4}{3}\pi r^3\left[\frac{Ze}{4\pi R^3/3}\right] = \left(\frac{Ze}{R^3}\right)r^3$ so that

$$dq = \left(\frac{3Ze}{R^3}\right)r^2 dr$$
$$U = \left(\frac{3kZ^2e^2}{R^6}\right)\int_0^R r^4 dr = \frac{3k(Ze)^2}{5R}$$

(b) When $N = Z = \frac{A}{2}$, $R = R_0 A^{1/3}$ and $R_0 = 1.2 \times 10^{-15}$ m

$$U = \frac{3k(Ze)^2}{5R} = \frac{(3/5)(8.988 \times 10^9 \text{ N m}^2/\text{C}^2)(A/2)^2(1.602 \times 10^{-19} \text{ C})^2}{(1.2 \times 10^{-15} \text{ m})A^{1/3}}$$

$$= 2.88 \times 10^{-14} \left(A^{5/3}\right) \text{ J}$$

(c) For $A = 30$, $U = 8.3 \times 10^{-12}$ J $= 52.1$ MeV.

13-21 (a) Write Equation 13.10 as $\frac{R}{R_0} = e^{-\lambda t}$ so that $\lambda = \frac{1}{t}\ln\left(\frac{R_0}{R}\right)$. In this case $\frac{R_0}{R} = 5$ when $t = 2$ h, so $\lambda = \frac{1}{2\text{ h}}\ln 5 = 0.805 \text{ h}^{-1}$.

(b) $T_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.805 \text{ h}^{-1}} = 0.861$ h

13-23 (a) From $R = R_0 e^{-\lambda t}$, $\lambda = \frac{1}{t}\ln\left(\frac{R_0}{R}\right)$, $\lambda = \frac{1}{4\text{ h}}\ln\left(\frac{10}{8}\right) = 5.58 \times 10^{-2} \text{ h}^{-1} = 1.55 \times 10^{-5} \text{ s}^{-1}$, and $T_{1/2} = \frac{\ln 2}{\lambda} = 12.4$ h.

(b) $R_0 = 10 \text{ mCi} = 10 \times 10^{-3} \times 3.7 \times 10^{10}$ decays/s $= 3.7 \times 10^8$ decays/s and $R = \lambda N$ so $N_0 = \frac{R_0}{\lambda} = \frac{3.7 \times 10^8 \text{ decays s}^{-1}}{1.55 \times 10^{-5} \text{ s}^{-1}} = 2.39 \times 10^{13}$ atoms

(c) $R = R_0 e^{-\lambda t} = (10 \text{ mCi})e^{-(5.58 \times 10^{-2})(30)} = 1.87$ mCi

13-25 Combining Equations 13.8 and 13.11 we have $N = \frac{|dN/dt|}{\lambda} = \frac{|dN/dt|}{0.693/T_{1/2}}$ and since $1 \text{ mCi} = 3.7 \times 10^7$ decays/s.

$$N = \frac{(5 \text{ mCi})(3.7 \times 10^7 \text{ dps/mCi})}{0.693/[(28.8 \text{ yr})(3.16 \times 10^7 \text{ s/yr})]} = 2.43 \times 10^{17} \text{ atoms}$$

Therefore, the mass of strontium in the sample is

$$m = \frac{N}{N_A} M = \frac{2.43 \times 10^{17} \text{ atoms}}{6.022 \times 10^{23} \text{ atoms/mole}}(90 \text{ g/mole}) = 36.3 \times 10^{-6} \text{ g}.$$

13-27 Let $R_0$ equal the total activity withdrawn from the stock solution.

$$R_0 = (2.5 \text{ mCi/ml})(10 \text{ ml}) = 25 \text{ mCi}.$$

Let $R_0'$ equal the initial specific activity of the working solution.

$$R_0' = \frac{25 \text{ mCi}}{250 \text{ ml}} = 0.1 \text{ mCi/ml}$$

After 48 hours the specific activity of the working solution will be

$$R' = R_0' e^{-\lambda t} = (0.1 \text{ mCi/ml}) e^{-(0.693/15 \text{ h})(48 \text{ h})} = 0.011 \text{ mCi/ml}$$

and the activity in the sample will be, $R = (0.011 \text{ mCi/ml})(5 \text{ ml}) = 0.055 \text{ mCi}$.

13-29 The number of nuclei that decay during the interval will be

$$N_1 - N_2 = N_0\left(e^{-\lambda t_1} - e^{-\lambda t_2}\right).$$

First we find $\lambda$;

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{64.8 \text{ h}} = 0.0107 \text{ h}^{-1} = 2.97 \times 10^{-6} \text{ s}^{-1} \text{ and}$$

$$N_0 = \frac{R_0}{\lambda} = \frac{(40\ \mu\text{Ci})(3.7 \times 10^4 \text{ dps}/\mu\text{Ci})}{2.97 \times 10^{-6} \text{ s}^{-1}} = 4.98 \times 10^{11} \text{ nuclei}$$

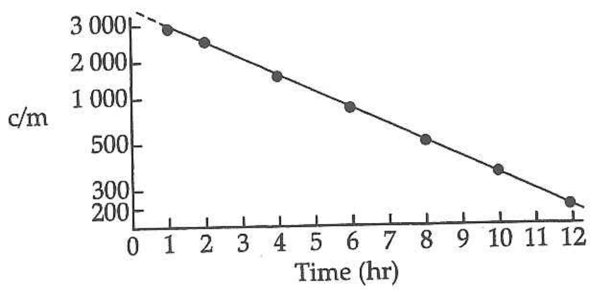
Using these values we find

$$N_1 - N_2 = \left(4.98 \times 10^{11}\right)\left[e^{-(0.0107 \text{ h}^{-1})(10 \text{ h})} - e^{-(0.0107 \text{ h}^{-1})(12 \text{ h})}\right].$$

Hence, the number of nuclei decaying during the interval is

$$N_1 - N_2 = 9.46 \times 10^9 \text{ nuclei}.$$

13-31 (a)




(b) $\lambda = -\text{slope} = -\frac{\ln 200 - \ln 480}{(12-4) \text{ hr}} = 0.25 \text{ hr}^{-1} = 4.17 \times 10^{-3} \text{ min}^{-1}$ and $T_{1/2} = \frac{\ln 2}{\lambda} = 2.77 \text{ hr}$.

(c) By extrapolation of graph to $t = 0$, we find $(\text{cpm})_0 = 4 \times 10^3$ cpm

(d) $N = \frac{R}{\lambda}$; $N_0 = \frac{R_0}{\lambda} = \frac{(\text{cpm})_0/\text{EFF}}{\lambda}$

$$N_0 = \frac{4 \times 10^4 \text{ dis/min}}{4.17 \times 10^{-3} \text{ min}^{-1}} = 9.59 \times 10^6 \text{ atoms}$$

13-33 (a) Referring to Example 13.11 or using the note in Problem 35 $R = R_0 e^{-\lambda t}$,

$$R_0 = N_0\lambda = 1.3\times10^{-12} N_0(^{12}\mathrm{C})\lambda$$
$$R_0 = \left(\frac{1.3\times10^{-12}\times25\ \mathrm{g}\times6.02\times10^{23}\ \mathrm{atoms/mole}}{12\ \mathrm{g/mole}}\right)\lambda$$

where $\lambda = \dfrac{0.693}{5\,730\times3.15\times10^{7}} = 3.84\times10^{-12}$ decay/s. So $R_0 = 376$ decay/min, and

$$R = (3.76\times10^{2})\exp[(-3.84\times10^{-12}\ \mathrm{s}^{-1})\times(2.3\times10^{4}\ \mathrm{y})\times(3.15\times10^{7}\ \mathrm{s/y})]$$
$$R = 18.3\ \mathrm{counts/min}$$

(b) The observed count rate is slightly less than the average background and would be difficult to measure accurately within reasonable counting times.

13-35 First find the activity per gram at time $t = 0$, $R_0 = N_0(^{14}\mathrm{C})$, where $N_0(^{14}\mathrm{C}) = 1.3\times10^{-12} N_0(^{12}\mathrm{C})$; and $N_0(^{12}\mathrm{C}) = \left(\dfrac{m}{M}\right)N_a$. Therefore $\dfrac{R_0}{m} = \left(\dfrac{\lambda N_a}{M}\right)(1.3\times10^{-12})$ and the activity after decay at time $t$ will be $\dfrac{R}{m} = \left(\dfrac{R_0}{m}\right)e^{-\lambda t} = \left(\dfrac{\lambda N_a}{M}\right)(1.3\times10^{-12})e^{-\lambda t}$ where $\lambda = \dfrac{\ln 2}{T_{1/2}} = 2.3\times10^{-10}\ \mathrm{min}^{-1}$ when $t = 2\,000$ years.

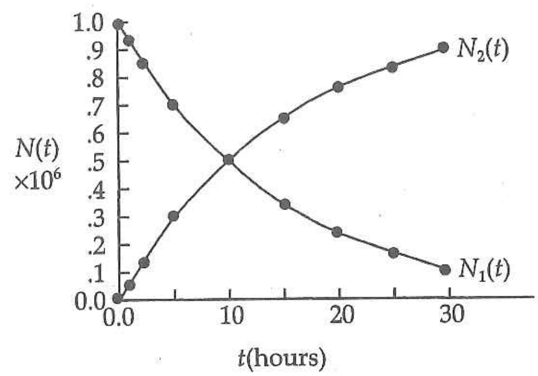
$$\frac{R}{m} = \left(\frac{3.2\times10^{-10}\ \mathrm{min}^{-1}}{12\ \mathrm{g/mole}}\right)(1.3\times10^{-12})(6.03\times10^{23}\ \mathrm{mole}^{-1})\times e^{-(3.2\times10^{-10}\ \mathrm{min}^{-1})(2\,000\ \mathrm{y})(5.26\times10^{5}\ \mathrm{min/y})}$$
$$\frac{R}{m} = 11.8\ \mathrm{decays\ min^{-1}g^{-1}}$$

13-37 (a) Let $N_1$ = number of parent nuclei, and $N_2$ = number of daughter nuclei. The daughter nuclei increase at the rate at which the parent nuclei decrease, or

$$\frac{dN_2}{dt} = \frac{-dN_1}{dt} = \lambda N_1 = \lambda N_{01}e^{-\lambda_1 t}$$
$$dN_2 = \lambda N_{01}e^{-\lambda_1 t}dt$$
$$N_2 = \lambda N_{01}\int e^{-\lambda t}dt = -N_{01}e^{-\lambda t} + \mathrm{Const.}$$

If we require $N_2 = N_{02}$ when $t = 0$ then Const $= N_{02} + N_{01}$. Therefore $N_2 = N_{02} + N_{01} - N_{01}e^{-\lambda t}$. And when $N_{02} = 0$; $N_2 = N_{01}(1 - e^{-\lambda t})$.

(b) Obtain the number of parent nuclei from $N_1 = N_{01}e^{-\lambda t}$ and the daughter nuclei from $N_2 = N_{01}(1 - e^{-\lambda t})$ with $N_{01} = 10^6$, $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{0.693}{10\ \mathrm{h}} = 0.069\,3\ \mathrm{h}^{-1}$. Thus the quantities $N_1 = 10^6 e^{-(0.069\,3\ \mathrm{h}^{-1})t}$ and $N_2 = 10^6\left[1 - e^{-(0.069\,3\ \mathrm{h}^{-1})t}\right]$ are plotted below.




13-39 A number of atoms, $dN = \lambda N dt$, have life times of $t$. Therefore, the average or mean life time will be $\sum(dN)\dfrac{t}{\sum dN}$ or $\int dN\dfrac{t}{N_0}$ so $\tau = \dfrac{1}{N_0}\int_0^\infty \lambda N t dt = \dfrac{1}{N_0}\int_0^\infty \lambda N_0 e^{-\lambda t} t dt = \dfrac{1}{\lambda}$.

13-41 $Q = \left(M_{^{238}\mathrm{U}} - M_{^{234}\mathrm{Th}} - M_{^{4}\mathrm{He}}\right)(931.5\ \mathrm{MeV/u})$
$= (238.048\,608\ \mathrm{u} - 234.043\,583\ \mathrm{u} - 4.002\,603\ \mathrm{u})(931.5\ \mathrm{MeV/u}) = 2.26\ \mathrm{MeV}$

13-43 (a) We will assume the parent nucleus (mass $M_p$) is initially at rest, and we will denote the masses of the daughter nucleus and alpha particle by $M_d$ and $M_\alpha$, respectively. The equations of conservation of momentum and energy for the alpha decay process are

$$M_d v_d = M_\alpha v_\alpha \tag{1}$$

$$M_p c^2 = M_d c^2 + M_\alpha c^2 + \left(\frac{1}{2}\right)M_d v_d^2 + \left(\frac{1}{2}\right)M_\alpha v_\alpha^2 \tag{2}$$

The disintegration energy $Q$ is given by

$$Q = \left(M_p - M_d - M_\alpha\right)c^2 = \left(\frac{1}{2}\right)M_d v_d^2 + \left(\frac{1}{2}\right)M_\alpha v_\alpha^2 \tag{3}$$

Eliminating $v_d$ from Equations (1) and (3) gives

$$Q = \left(\frac{1}{2}\right)M_d\left[\left(\frac{M_\alpha}{M_d}\right)v_\alpha\right]^2 + \left(\frac{1}{2}\right)M_\alpha v_\alpha^2$$

$$Q = \left(\frac{1}{2}\right)\left(\frac{M_\alpha^2}{M_d}\right)v_\alpha^2 + \left(\frac{1}{2}\right)M_\alpha v_\alpha^2$$

$$Q = \left(\frac{1}{2}\right)M_\alpha v_\alpha^2\left(1 + \frac{M_\alpha}{M_d}\right) = K_\alpha\left(1 + \frac{M_\alpha}{M_d}\right)$$

(b) $K_\alpha = \dfrac{Q}{1 + M_\alpha / M} = \dfrac{4.87\ \mathrm{MeV}}{1 + 4/226} = 4.79\ \mathrm{MeV}$

(c) $K_d = (4.87 - 4.97)\ \mathrm{MeV} = 0.08\ \mathrm{MeV}$

(d) For the beta decay of $^{210}\text{Bi}$ we have $Q = K_{e^-}\left(1+\frac{M_{e^-}}{M_Y}\right)$. Solving for $K_{e^-}$ and substituting $M_{e^-} = 5.486\times10^{-4}$ u and $M_Y = 209.982$ u (Po), we find

$$K_{e^-} = \frac{Q}{1+5.486\times10^{-4}\text{ u}/209.982\text{ u}} = \frac{Q}{1+2.61\times10^{-6}}.$$

Setting $2.61\times10^{-6} = \varepsilon$, we get $K_{e^-} = Q(1+\varepsilon)^{-1} \cong Q(1-\varepsilon) = Q\left(1-2.61\times10^{-6}\right)$ for $\varepsilon \ll 1$. This means the daughter Po carries off only about three millionths of the kinetic energy available in the decay. This treatment is only approximately correct since actual beta decay involves another particle (antineutrino) and relativistic effects.

13-45 $Q = (m_{\text{initial}} - m_{\text{final}})(931.5\ \text{MeV/u})$

(a) $Q = m\left({}^{40}_{20}\text{Ca}\right) - m\left(e^+\right) - m\left({}^{40}_{19}\text{K}\right) = (39.962\,59\ \text{u} - 0.000\,548\,6\ \text{u} - 39.964\,00\ \text{u})(931.5\ \text{MeV/u})$
$= -1.82\ \text{MeV}$
$Q < 0$ so the reaction cannot occur.

(b) Using the handbook of Chemistry and Physics
$Q = m\left({}^{98}_{44}\text{Ru}\right) - m\left({}^{4}_{2}\text{He}\right) - m\left({}^{94}_{42}\text{Mo}\right) = (97.905\,5\ \text{u} - 4.002\,6\ \text{u} - 93.904\,7\ \text{u})(931.5\ \text{MeV/u})$
$= -1.68\ \text{MeV}$
$Q < 0$ so the reaction cannot occur.

(c) Using the handbook of *Chemistry and Physics*
$Q = m\left({}^{144}_{60}\text{Nd}\right) - m\left({}^{4}_{2}\text{He}\right) - m\left({}^{140}_{58}\text{Ce}\right) = (143.909\,9\ \text{u} - 4.002\,6\ \text{u} - 139.905\,4\ \text{u})$
$\times(931.5\ \text{MeV/u}) = 1.86\ \text{MeV}$
$Q > 0$ so the reaction can occur.

13-47 We assume an electron in the nucleus with an uncertainty in its position equal to the nuclear diameter. Choose a typical diameter of 10 fm and from the uncertainty principle we have

$$\Delta p \approx \frac{h}{\Delta x} = 6.6\times10^{-34}\ \text{J s}/10^{-14}\ \text{m} = 6.6\times10^{-20}\ \text{N s}.$$

Using the relativistic energy-momentum expression

$$E^2 = (pc)^2 + \left(m_0c^2\right)^2$$

we make the approximation that $pc \approx (\Delta p)c \gg m_0c^2$ so that

$$E \approx pc \approx (\Delta p)c = \left(6.6\times10^{-20}\ \text{N s}\right)\left(3\times10^8\ \text{m/s}\right) = 19.8\times10^{-12}\ \text{J} \approx 124\ \text{MeV}.$$

However, the most energetic electrons emitted by radioactive nuclei have been found to have energies of less than 10% of this value, therefore electrons are not present in the nucleus.

13-49 The disintegration energy, $Q$, is $c^2$ times the mass difference between the parent nucleus and the decay products. In electron emission an electron leaves the system. That is ${}_Z^AX \rightarrow {}_{Z+1}^{A}Y + e^- + \overline{v}$ where $\overline{v}$ has negligible mass and the neutral daughter nucleus has nuclear charge of $Z+1$ and $Z$ electrons. Therefore we need to add the mass of an electron to get the mass of the daughter. The disintegration energy can now be calculated as

$$Q = \{M_Z^AX - M[{}_{Z+1}^{A}Y - m_e] - m_e + 0\}c^2 = [M_Z^AX - M_{Z+1}^AY]c^2.$$

Similar reasoning can be applied to positron emission ${}_Z^AX \rightarrow {}_{Z-1}^{A}Y + e^+ + v$ and so

$$Q = \{M_Z^AX - M[{}_{Z-1}^{A}Y - m_e] - m_e + 0\}c^2 = [M_Z^AX - M_{Z-1}^AY - 2m_e]c^2.$$

For electron capture we have ${}_Z^AX + e^- \rightarrow {}_{Z+1}^{A}Y + v$, which gives

$$Q = \{M_Z^AX + m_e - M[{}_{Z-1}^{A}Y + m_e] + 0\}c^2 = [M_Z^AX - M_{Z-1}^AY]c^2.$$

13-51 In the decay ${}_1^3\text{H} \rightarrow {}_2^3\text{He} + e + \overline{v}$ the energy released is: $E = (\Delta m)c^2 = [M_{1\text{H}} - M_{3\text{He}}]c^2$ since the mass of the antineutrino is negligible and the mass of the electron is accounted for in the atomic masses of ${}_1^3\text{H}$ and ${}_2^3\text{He}$. Thus,

$$E = (3.016\,049\ \text{u} - 3.016\,029\ \text{u}](931.5\ \text{MeV/u}) = 0.018\,6\ \text{MeV} = 18.6\ \text{keV}.$$

13-53

$$N_{\text{Rb}} = 1.82\times10^{10}({}^{87}\text{Rb atoms/g})$$
$$N_{\text{Sr}} = 1.07\times10^{9}({}^{87}\text{Sr atoms/g})$$

$$T_{1/2}\left({}^{87}\text{Rb} \xrightarrow{\beta^-} {}^{87}\text{Sr}\right) = 4.8\times10^{10}\ \text{y}$$

(a) If we assume that all the ${}^{87}$Sr came from ${}^{87}$Rb, then $N_{\text{Rb}} = N_0e^{-\lambda t}$

$$1.82\times10^{10} = (1.82\times10^{10} + 1.07\times10^9)e^{-(\ln 2/4.8\times10^{10})t}$$
$$-\ln(0.944\,47) = \left(\frac{\ln 2}{4.8\times10^{10}}\right)t$$
$$t = 3.96\times10^9\ \text{y}$$

(b) It could be no older. The rock could be younger if some ${}^{87}$Sr were initially present.

13-55 (a) Starting with $N = 0$ radioactive atoms at $t = 0$, the rate of increase is (production-decay)

$$\frac{dN}{dt} = R - \lambda N$$
$$dN = (R - \lambda N)dt$$

Variables are separable

$$\int_{N=0}^{N}\frac{dN}{R-\lambda N}=\int_{t=0}^{t}dt-\left(\frac{1}{\lambda}\right)\ln\left(\frac{R-\lambda N}{R}\right)=t$$

$$\ln\left(\frac{R-\lambda N}{R}\right)=-\lambda t$$

$$\left(\frac{R-\lambda N}{R}\right)=e^{-\lambda t}$$

$$1-\left(\frac{\lambda}{R}\right)N=e^{-\lambda t}$$

$$N=\left(\frac{R}{\lambda}\right)\left(1-e^{-\lambda t}\right)$$

(b) $$\frac{dN}{dt}=R-\lambda N_{\max}$$

$$N_{\max}=\frac{R}{\lambda}$$

13-57 We have all this information: $N_x(0)=2.50N_y(0)$

$$N_x(3\text{d})=4.20N_y(3\text{d})$$

$$N_x(0)e^{-\lambda_x 3\text{d}}=4.20N_y(0)e^{-\lambda_y 3\text{d}}=4.20\frac{N_x(0)}{2.50}e^{-\lambda_y 3\text{d}}$$

$$e^{3\text{d}\lambda_x}=\frac{2.5}{4.2}e^{3\text{d}\lambda_y}$$

$$3\text{d}\lambda_x=\ln\left[\frac{2.5}{4.2}\right]+3\text{d}\lambda_y$$

$$3\text{d}\frac{0.693}{T_{1/2x}}=\ln\left(\frac{2.5}{4.2}\right)+3\text{d}\frac{0.693}{1.60\text{ d}}=0.781$$

$$T_{1/2x}=2.66\text{ d}$$

13-59 $$N=N_0e^{-\lambda t}$$

$$\left|\frac{dN}{dt}\right|=R=\left|-\lambda N_0e^{-\lambda t}\right|=R_0e^{-\lambda t}$$

$$e^{-\lambda t}=\frac{R}{R_0}$$

$$e^{\lambda t}=\frac{R_0}{R}$$

$$\lambda t=\ln\left(\frac{R_0}{R}\right)=\frac{\ln 2}{T_{1/2}}t$$

$$t=T_{1/2}\frac{\ln(R_0/R)}{\ln 2}$$

If $R=0.13\text{ Bq}$, $t=5\,730\text{ yr}\dfrac{\ln(0.25/0.13)}{0.693}=5\,406\text{ yr}$.

If $R=0.11\text{ Bq}$, $t=5\,730\text{ yr}\dfrac{\ln(0.25/0.11)}{0.693}=6\,787\text{ yr}$.

The range is most clearly written as between 5 400 yr and 6 800 yr, without understatement.

13-61 (a) Let $N$ be the number of $^{238}$U nuclei and $N'$ be $^{206}$Pb nuclei. Then $N = N_0 e^{-\lambda t}$ and $N_0 = N + N'$ so $N = (N + N')e^{-\lambda t}$ or $e^{\lambda t} = 1 + \frac{N'}{N}$. Taking logarithms, $\lambda t = \ln\left(1 + \frac{N'}{N}\right)$ where $\lambda = \frac{\ln 2}{T_{1/2}}$. Thus, $t = \left(\frac{T_{1/2}}{\ln 2}\right)\ln\left(1 + \frac{N'}{N}\right)$. If $\frac{N}{N'} = 1.164$ for the $^{238}\text{U} \to {}^{206}\text{Pb}$ chain with $T_{1/2} = 4.47 \times 10^9$ yr, the age is:

$$t = \left(\frac{4.47 \times 10^9 \text{ yr}}{\ln 2}\right)\ln\left(1 + \frac{1}{1.164}\right) = 4.00 \times 10^9 \text{ yr}.$$

(b) From above, $e^{\lambda t} = 1 + \frac{N'}{N}$. Solving for $\frac{N}{N'}$ gives $\frac{N}{N'} = \frac{e^{-\lambda t}}{1 - e^{-\lambda t}}$. With $t = 4.00 \times 10^9$ yr and $T_{1/2} = 7.04 \times 10^8$ yr for the $^{235}\text{U} \to {}^{207}\text{Pb}$ chain,

$$\lambda t = \left(\frac{\ln 2}{T_{1/2}}\right)t = \frac{(\ln 2)(4.00 \times 10^9 \text{ yr})}{7.04 \times 10^8 \text{ yr}} = 3.938 \text{ and } \frac{N}{N'} = 0.019\,9.$$

With $t = 4.00 \times 10^9$ yr and $T_{1/2} = 1.41 \times 10^{10}$ yr for the $^{232}\text{Th} \to {}^{208}\text{Pb}$ chain,

$$\lambda t = \frac{(\ln 2)(4.00 \times 10^9 \text{ yr})}{1.41 \times 10^{10} \text{ yr}} = 0.196\,6 \text{ and } \frac{N}{N'} = 4.60.$$

# 14

# Nuclear Physics Applications

14-1 $^{18}O = 17.999\ 160$ $\quad ^{18}F = 18.000\ 938$

$m_n = 1.008\ 664\ 9$ $\quad ^{1}H = 1.007\ 825$ $\quad$ all in u.

(a) $Q = [M_O + M_H + M_F - m_n]c^2 = [-0.002\ 617\ 9\ \text{u}][931.494\ 3\ \text{MeV/u}] = -2.438\ 6\ \text{MeV}$ compared to $-2.453 \pm 0.000\ 2\ \text{MeV}$.

(b) $$K_{th} = -Q\left[1 + \frac{M_a}{M_X}\right] = (2.438\ 6\ \text{MeV})\left(1 + \frac{1.007\ 825}{17.999\ 160}\right) = 2.575\ 1\ \text{MeV}$$

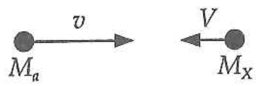
14-3 $$Q = \left(M_\alpha + M_{(^9\text{Be})} - M_{(^{12}\text{C})} - M_n\right)(931.5\ \text{MeV/u})$$
$$= (4.002\ 603\ \text{u} + 9.012\ 182\ \text{u} - 12.000\ 000\ \text{u} - 1.008\ 665\ \text{u})(931.5\ \text{MeV/u})$$
$$Q = 5.70\ \text{MeV}$$

14-5 $$Q = (m_a + m_X - m_Y - m_b)[931.5\ \text{MeV/u}]$$
$$Q = \left[m_{(^1\text{H})} + m_{(^7\text{Li})} - m_{(^4\text{He})} - m_\alpha\right]\text{u}[931.5\ \text{MeV/u}]$$
$$Q = [1.007\ 825\ \text{u} + 7.016\ 004\ \text{u} - 4.002\ 603\ \text{u} - 4.002\ 603\ \text{u}][931.5\ \text{MeV/u}]$$
$$Q = 17.35\ \text{MeV}$$

14-7 (a) $$Q = \left[m(^{14}\text{N}) + m(^4\text{He}) - m(^{17}\text{O}) - m(^1\text{H})\right](931.5\ \text{MeV/u})$$
Using Table 13.6 for the masses.
$$Q = (14.003\ 074\ \text{u} + 4.002\ 603\ \text{u} - 16.999\ 132\ \text{u} - 1.007\ 825\ \text{u})(931.5\ \text{MeV/u})$$
$$Q = -1.19\ \text{MeV}$$
$$K_{th} = -\frac{Q\left[m(^4\text{He}) + m(^{14}\text{N})\right]}{m(^{14}\text{N})} = -(-1.19\ \text{MeV})\left(1 + \frac{4.002\ 603}{14.003\ 074}\right) = 1.53\ \text{MeV}$$

(b) $$Q = \left[m(^7\text{Li}) + m(^1\text{H}) - 2m(^1\text{He})\right](931.5\ \text{MeV/u})$$
$$Q = [(7.016\ 004\ \text{u} + 1.007\ 825\ \text{u}) - (2)(4.002\ 603\ \text{u})](931.5\ \text{MeV/u})$$
$$Q = 17.35\ \text{MeV}$$

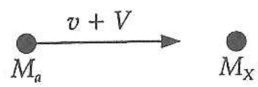
14-9 (a) CM SYSTEM




$$p = M_a v = M_\chi V$$

$$K_{\text{CM}} = \frac{p^2}{2M_a} + \frac{p^2}{2M_\chi} = \frac{p^2}{2}\left[\frac{M_\chi + M_a}{M_a M_\chi}\right]$$

LAB SYSTEM




$$P_{\text{lab}} = M_a(v+V)(\text{Eq. } 1)$$

$$= p\left[\frac{M_\chi + M_a}{M_\chi}\right] \text{ for substituting } v = \frac{p}{M_a} \text{ and } V = \frac{p}{M_\chi} \text{ in Eq. 1.}$$

$$K_{\text{lab}} = \frac{p_{\text{lab}}^2}{2M_a} = \frac{p^2[(M_\chi + M_a)/M_\chi]^2}{2M_a}$$

Comparing to $K_{\text{CM}}$, $K_{\text{lab}} = K_{\text{CM}}\left[\frac{M_\chi + M_a}{M_\chi}\right]$ or $K_{th} = -Q\left(1 + \frac{M_a}{M_\chi}\right)$

(b) First calculate the $Q$-value

$$Q = \left[m\left({}^{14}\text{N}\right) + m\left({}^{4}\text{He}\right) - m\left({}^{17}\text{O}\right) - m\left({}^{1}\text{H}\right)\right](931.5 \text{ MeV/u})$$

$$Q = [14.003\,074 \text{ u} + 4.002\,603 \text{ u} - 16.999\,132 \text{ u} - 1.007\,825 \text{ u}](931.5 \text{ MeV/u})$$

$$Q = -1.19 \text{ MeV}$$

Then

$$K_{th} = -Q\left[1 + \frac{m\left({}^{4}\text{He}\right)}{m\left({}^{14}\text{N}\right)}\right]$$

$$K_{th} = -(-1.19 \text{ MeV})\left[1 + \frac{4.002\,603}{14.003\,074}\right] = 1.53 \text{ MeV}$$

14-11 $R = R_0 e^{-n\sigma x}$, $x = 2$ m, $R = 0.8R_0$, $n = \frac{\rho}{m_{\text{atom}}} = \frac{70 \text{ kg/m}^3}{1.67 \times 10^{-27} \text{ kg}} = 4.19 \times 10^{28} \text{ m}^{-3}$, $0.8R_0 = R_0 e^{-n\sigma x}$,

$0.8 = e^{-n\sigma x}$, $n\sigma x = -\ln 0.8$, $\sigma = \frac{-1}{nx}\ln(0.8) = \frac{0.223}{4.19 \times 10^{28} \text{ m}^{-3} \times 2 \text{ m}} = 2.66 \times 10^{-30} \text{ m}^2 = 0.026\,6 \text{ b}$

14-13 Equation 14.4 gives $R=(R_0 nx)\sigma$. Using values of $E$ and $\sigma$, we have

(a) $\dfrac{R_{10}}{R_1}=\dfrac{\sigma_{10}}{\sigma_1}=0.037\,3$,

(b) $\dfrac{R_1}{R_{0.1}}=0.066\,3$, and

(c) $\dfrac{R_{0.1}}{R_{0.01}}\approx 1$

(d) Therefore we can use cadmium as an energy selector in the range 0.1 eV to 10 eV to detect order of magnitude changes in energy.

14-15 (a) $\dfrac{N}{N_0}=e^{-n\sigma x}$, $x=$ thickness in m, $\sigma=$ cross section in m$^2$ and

$$n=\#\text{ gold nuclei/m}^3$$
$$n=\left(6.02\times10^{23}\text{ atoms/mole}\right)\left(1\text{ mole/197 g}\right)\left(19.3\text{ g/cm}^3\right)$$
$$n=5.9\times10^{22}\text{ atoms/cm}^3=5.9\times10^{28}\text{ atoms/m}^3$$

Taking $x=5.1\times10^{-5}$ m, we get

$$\frac{N}{N_0}=\exp\left(-5.9\times10^{28}\text{ atoms/m}^3\times500\times10^{-28}\text{ m}^2\times5.1\times10^{-5}\text{ m}\right)=0.86$$

(b) $N=0.86N_0$ $\quad N_0=\dfrac{0.1\ \mu\text{A}}{1.6\times10^{-19}\text{ C}}$

$N_0=6.3\times10^{11}$ protons/s and $N=6.1\times10^{11}$ protons/s

(c) The number of protons abs. or scat. per sec $0.14N_0=8.7\times10^{10}$ protons/s

14-17 Since $N=N_0e^{-n\sigma x}$, $\dfrac{dN}{dx}=-Nn_c\sigma$, where $N$ = neutron density, $n_c$ = cadmium nuclei density, and $\sigma$ is the absorption cross-section. Thus, $\left(\dfrac{dN}{dt}\right)_a=-Nn_c\sigma v_{th}$ where $v_{th}$ is the neutron thermal velocity given by $v_{th}=\left(\dfrac{1.5k_BT}{m_n}\right)^{1/2}$. The neutron decay rate, $\left(\dfrac{dN}{dt}\right)_D$, comes from differentiating $N=N_0e^{-\lambda t}$: $\left(\dfrac{dN}{dt}\right)_D=-N\lambda$ where $\lambda=\dfrac{0.693}{T_{1/2}}=\dfrac{0.693}{636\text{ s}}=1.09\times10^{-3}\text{ s}^{-1}$. Finally

$$\frac{(dN/dt)_a}{(dN/dt)_D}=\frac{-Nn_c\sigma v_{th}}{-N\lambda}=\frac{n_c\sigma v_{th}}{\lambda}$$

$$\text{As } n_c = (8.65\ \text{g/cm}^3)(6.02\times10^{23}\ \text{nuclei/112 g})$$
$$\sigma = (2\,450\ \text{b})(10^{-24}\ \text{cm}^2/\text{b})$$
$$v_{th} = \left[\frac{(1.5)(1.38\times10^{-23}\ \text{J/K})(300\ \text{K})}{1.67\times10^{-27}\ \text{kg}}\right]^{1/2}$$
$$\lambda = 1.09\times10^{-3}\ \text{s}^{-1}$$
$$\frac{(dN/dt)_a}{(dN/dt)_D} = 2.25\times10^{12}$$

14-19 $E_T \equiv E(\text{thermal}) = \frac{3}{2}k_BT = 0.038\,9\ \text{eV}$. $E_T = \left(\frac{1}{2}\right)^n E$ where $n \equiv$ number of collisions, and $E$ is the initial kinetic energy. $0.0389 = \left(\frac{1}{2}\right)^n (10^6)$. Therefore $n = 24.6$ or 25 collisions.

14-21 $\Delta E = c^2(m_{\text{U}} - m_{\text{Ba}} - m_{\text{Kr}} - m_{\text{n}})$

$$\Delta E = (931.5\ \text{MeV/u})[235.043\,9\ \text{u} - 140.913\,9\ \text{u} - 91.897\,3\ \text{u} - 2(1.008\,7\ \text{u})]$$
$$\Delta E = (931.5\ \text{MeV/u})[0.215\,3\ \text{u}] = 200.6\ \text{MeV}$$

14-23 (a) For a sphere: $V = \frac{4}{3}\pi r^3$ and $r = \left(\frac{3V}{4\pi}\right)^{1/3}$, so $\frac{A}{V} = \frac{4\pi r^2}{(4/3)\pi r^3} = 4.84V^{-1/3}$.

(b) For a cube: $V = l^3$ and $l = V^{1/3}$, so $\frac{A}{V} = \frac{6l^2}{l^3} = 6V^{-1/3}$.

(c) For a parallelepiped: $V = 2a^3$ and $a = (2V)^{1/3}$, so $\frac{A}{V} = \frac{2a^2 + 8a^2}{2a^3} = 6.30V^{-1/3}$.

(d) Therefore for a given volume, the sphere has the least leakage.

(e) The parallelepiped has the greatest leakage.

14.25 (a) $\text{eff} = \frac{P_{\text{delivered}}}{P_{\text{out}}} = 0.3$, $P_{\text{out}} = \frac{1\,000\ \text{MW}}{0.3} = 3\,333\ \text{MW}$

(b) $P_{\text{heat}} = P_{\text{out}} - P_{\text{delivered}} = 3\,333 - 1\,000 = 2\,333\ \text{MW}$

(c) The energy released per fission event is $Q = 200\ \text{MeV}$. Therefore

$$\text{Rate} = \frac{P_{\text{out}}}{Q} = \frac{3.333\,3\times10^9\ \text{W}/200\ \text{MeV}}{1.6\times10^{-13}\ \text{J/MeV}}$$
$$\text{Rate} = 1.04\times10^{20}\ \text{events/s}$$

(d) $M = (\text{Rate})\left[\dfrac{235\times10^{-3}\ \text{kg/mole}}{6.0\times10^{23}\ \text{atoms/mole}}\right](\text{time})$

$M = (1.04\times10^{20}\ \text{events/s})(3.92\times10^{-25}\ \text{kg/atom})(365\ \text{days})(24\ \text{h/day})\times(3\,600\ \text{s/h})$

$= 1.34\times10^{3}\ \text{kg}$

(e) $\dfrac{dM}{dt} = \left(\dfrac{1}{c^2}\right)\left(\dfrac{dE}{dt}\right) = \dfrac{3.333\times10^{9}\ \text{W}}{(3\times10^{8}\ \text{m/s})^2} = 3.7\times10^{-8}\ \text{kg/s}$. To compare with (d) we need the mass for a year.

$$\frac{dM}{dt}(\text{year}) = (3.7\times10^{-8}\ \text{kg/s})(365\ \text{days})(24\ \text{h/day})\times(3\,600\ \text{s/h}) = 1.17\ \text{kg}.$$

This is 8% of the total mass found in (d).

14-27 (a) $r = r_D + r_T = (1.2\times10^{-15}\ \text{m})(2^{1/3}+3^{1/3}) = 2.70\times10^{-15}\ \text{m}$

(b) $U = \dfrac{ke^2}{r} = \dfrac{(9\times10^{9}\ \text{N m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}{2\times10^{-15}\ \text{m}} = 1.15\times10^{-13}\ \text{J} = 720\ \text{keV}$

(c) Conserving momentum: $v_F = \dfrac{v_0 m_D}{m_D + m_T}$ (1)

(d) $\dfrac{1}{2}m_D v_0^2 = \dfrac{1}{2}(m_D+m_T)v_F^2 + U$ (2)

Eliminating $v_F$ from (2) using (1), gives

$$\left(\frac{m_D}{2}\right)v_0^2 - \frac{1}{2}(m_D+m_T)\frac{v_0^2 m_D^2}{(m_D+m_T)^2} = U \text{ or}$$

$$\frac{1}{2}(m_D+m_T)m_D v_0^2 - \frac{1}{2}m_D^2 v_0^2 = (m_D+m_T)U \text{ or}$$

$$\frac{1}{2}m_D^2 v_0^2 = \left(\frac{m_D+m_T}{m_T}\right)U = \frac{5}{3}U = \frac{5}{3}(720\ \text{keV})$$

$$\frac{1}{2}m_D^2 v_0^2 = 1.2\ \text{MeV}.$$

(e) Possibly by tunneling.

14-29 (a) $Q = K_\alpha + K_n = 17.6\ \text{MeV} = (1.2)m_\alpha v_\alpha^2 + \dfrac{1}{2}m_n v_n^2$. Momentum conservation yields $m_n v_n = m_\alpha v_\alpha$. Substituting $v_\alpha = \dfrac{m_n}{m_\alpha}v_n$ into the energy equation gives $K_n = \dfrac{m_\alpha Q}{m_\alpha + m_n}$, $K_\alpha = \dfrac{m_n Q}{m_\alpha + m_n}$ Finally, $K_n = \dfrac{(4.003)(17.6\ \text{MeV})}{4.003+1.009} = 14.1\ \text{MeV}$, $K_\alpha = 3.45\ \text{MeV}$.

(b) Yes, since the neutron is uncharged, it is not confined by the **B** field and only $K_\alpha$ can be used to achieve critical ignition.

14-31 (a) The pellet contains

$$\left(\frac{4\pi R^3}{3}\right)(0.2\ \text{g/cm}^3) = \left(\frac{4\pi(0.5\times10^{-2}\ \text{cm})^3}{3}\right)(0.2\ \text{g/cm}^3) = 1.05\times10^{-7}\ \text{g}$$

of ${}_1^2\text{H} + {}_1^3\text{H}$ "molecules." The number of molecules, $N$, is

$$\left(\frac{1.05\times10^{-7}\ \text{g}}{5.0\ \text{g/mole}}\right)(6.02\times10^{23}\ \text{molecules/mole}) = 1.26\times10^{16}.$$

Since each molecule consists of 4 particles $\left({}_1^2\text{H},\ {}_1^3\text{H},\ 2e^-\right)$, $E = (4N)\frac{3}{2}k_B T$ or

$$T = \frac{E}{6Nk_B} = \frac{0.01(200\times10^3\ \text{J})}{6(1.26\times10^{16})(1.38\times10^{-23}\ \text{J/K})} = 1.9\times10^9\ \text{K}.$$

(b) The energy released $= (17.59\ \text{MeV})(1.26\times10^{16})(1.6\times10^{-13}\ \text{J/MeV}) = 355\ \text{kJ}$.

14-33 (a) Roughly $\frac{7}{2}(15\times10^6\ \text{K})$ or $52\times10^6\ \text{K}$ since 6 times the coulombic barrier must be surmounted.

(b) $Q = \Delta mc^2 = (12.000\,000\ \text{u} + 1.007\,825\ \text{u} - 13.005\,738\ \text{u})(931.5\ \text{MeV/u})$
$Q = 1.943\ \text{MeV}$
The other energies are calculated in a similar manner and the total energy released is

$$(1.943 + 1.709 + 7.551 + 7.297 + 2.242 + 4.966)\ \text{MeV} = 25.75\ \text{MeV}.$$

The net effect is ${}_6^{12}\text{C} + 4p \rightarrow {}_6^{12}\text{C} + {}_2^4\text{He}$.

(c) Most of the energy is lost since $v$'s have such low cross-section (no charge, little mass, etc.)

14-35 Total energy = number of ${}^6\text{Li}$ nuclei (22 MeV)

$$= (0.075)(2\times10^{-13}\ \text{g})\left(\frac{6.02\times10^{23}\ \text{nuclei}}{6.01\ \text{g}}\right)(22\ \text{MeV})(1.60\times10^{13}\ \text{J/MeV}) = 5.3\times10^{23}\ \text{J}$$

About twice as great as total world's fuel supply.

14-37 (a) $N$ = number of ${}_1^3\text{H}$,

$${}_1^2\text{H pairs in 3 mg} = \frac{(3\times10^{-3}\ \text{g})(6.02\times10^{23}\ \text{pairs/mole})}{5.0\ \text{g/mole}}$$
$$= 3.61\times20^{20}\ \text{pairs}.$$

$$\text{Power Output} = (10)(0.3)(3.61\times10^{20})(17.6\ \text{MeV/fusion})(1.60\times10^{-13}\ \text{J/MeV})/\text{s}$$
$$= 3.1\times10^9\ \text{W}$$
$$\text{Power Input} = (10)(5\times10^{14}\ \text{J/s})(10^{-8}\ \text{s})/\text{s} = 5\times10^7\ \text{W}$$
$$\text{Net Power} = (3.1\times10^9 - 5\times10^7)\ \text{W} \simeq 3.0\times10^9\ \text{W} = 3\,000\ \text{MW}$$

(b) 1 day's fusion energy $=(3\,000\text{ MW})(3\,600\text{ s/h})(24\text{ h/day})=2.6\times10^{14}\text{ J}$. This is equivalent to $\dfrac{2.6\times10^{14}\text{ J}}{50\times10^{6}\text{ J/liter}}=5.2\times10^{6}$ liters of oil or 5 million liters of oil!

14-39 (a) $$E=\frac{ke^2Z_1Z_2}{r}=\frac{(9\times10^{9}\text{ N m}^2/\text{C}^2)(1.6\times10^{-19}\text{ C})^2Z_1Z_2}{10^{-14}\text{ m}}=2.3\times10^{-19}Z_1Z_2\text{ J}$$

(b) D-D and D-T: $Z_1=Z_2=1$ and $E=2.3\times10^{-19}\text{ J}=0.14\text{ MeV}$

14-41 (a) $E=(931.5\text{ MeV/u})\Delta m=(931.5\text{ MeV/u})[(2\times2.014\,102)-4.002\,603]\text{ u}$. $E=23.85\text{ MeV}$ for every two $^2$H's.

$$(3.17\times10^{8}\text{ mi}^3)[(5\,280\text{ ft/mi})(12\text{ in/ft})(0.025\,4\text{ m/in})]^3[10^{6}\text{ g(H}_2\text{O)/m}^3]\left[\frac{2\text{ g(H)}}{18\text{ g(H}_2\text{O)}}\right]$$
$$\times[6.02\times10^{23}\text{ protons/g(H)}](0.015\,6\ {}^2\text{H/proton})(23.85\text{ MeV}/{}^2\text{H})(1.6\times10^{-13}\text{ J/MeV})$$
$$=2.63\times10^{33}\text{ J}$$

(b) $$\left(\frac{2.63\times10^{33}\text{ J}}{7\times10^{14}\text{ J/s}}\right)\left(\frac{\text{year}}{3.16\times10^{7}\text{ s}}\right)=119\text{ billion years}$$

14-43 (a) $$n=\frac{10^{14}\text{ s/cm}^3}{1\text{ s}}=10^{14}/\text{cm}^3$$

(b) $$2nk_BT=(2\times10^{14}/\text{cm}^3)(1.38\times10^{-23}\text{ J/K})(8\times10^{7}\text{ K})(10^{6}\text{ cm}^3/\text{m}^3)$$
$$2nk_BT=2.2\times10^{5}\text{ J/m}^3$$

(c) $$\frac{B^2}{2\mu_0}\approx10(2nk_BT)\qquad B=[20\mu_0(2nk_BT)]^{1/2}$$
$$B=[20(4\pi\times10^{-7}\text{ N/A}^2)(2.2\times10^{5}\text{ J/m}^3)]^{1/2}=2.35\text{ T}$$

14-45 (a) For the first layer: $I_1=I_0e^{-(\mu_{\text{Al}}d)}$, for the second layer: $I_2=I_1e^{-(\mu_{\text{Cu}}d)}$, and for the third layer: $\frac{I_0}{3}=I_2e^{-(\mu_{\text{Pb}})d}$ so that $\frac{I_0}{3}=I_0e^{-d(\mu_{\text{Al}}+\mu_{\text{Cu}}+\mu_{\text{Pb}})}$. Using Table 14.2,

$$d=\frac{\ln3}{\mu_{\text{Al}}+\mu_{\text{Cu}}+\mu_{\text{Pb}}}=\frac{\ln3}{(5.4+170+610)(\text{cm}^{-1})}=1.4\times10^{-3}\text{ cm.}$$

(b) If the copper and aluminum are removed, then $I=I_0e^{-(610+1.40\times10^{-3})}=0.426I_0$. About 43% of the x-rays get through whereas 33% got through before.

14-47 $$x=\frac{\ln2}{\mu}=\frac{\ln2}{0.18}=3.85\text{ cm}$$
This means that x-rays can probe the human body to a depth of at least 3.85 cm without severe attenuation and probably farther with reasonable attenuation.

14-49 (a) Assume he works 5 days per week, 50 weeks per year and takes 8 x-rays per day. # x-rays $= 2\,000$ x-rays per year and $\frac{5}{2\,000} = 0.002\,5$ rem per x-ray.

(b) 5 rem/yr is 38 times the background radiation of 0.13 rem/yr.

14-51 The second worker received twice as much radiation energy but he received it in twice as much tissue. Radiation dose is an intensive, not extensive quantity—measured in joules per kilogram. If you double this energy and the exposed mass, the number of rads is the same in the two cases.

14-53 One rad $\rightarrow$ Deposits $10^{-2}$ J/kg, therefore 25 rad $\rightarrow 25\times10^{-2}$ J/kg
If $M = 75$ kg, $E = (75 \text{ kg})(25\times10^{-2} \text{ J/kg}) = 18.8$ J

14-55 One electron strikes the first dynode with 100 eV of energy: 10 electrons are freed from the first dynode. These are accelerated to the second dynode. By conservation of energy the number freed here, $N$ is: $(10)(\Delta V) = (N)(10)$ or $10(200-100) = N(10)$ so $N = 100$. By the seventh dynode, $N = 10^6$ electrons. Up to the seventh dynode, we assume all energy is conserved (no losses). Hence we have $10^6$ electrons impinging on the seventh dynode from the sixth. These are accelerated through (700 – 600) V. Hence $E = (10^6)(100) = 10^8$ eV. In addition some energy is needed to cause the $10^6$ electrons at the seventh dynode to move to the counter.

14-57 To conserve momentum, the two fragments must move in opposite directions with speeds $v_1$ and $v_2$ such that

$$m_1 v_1 = m_2 v_2 \qquad \text{or} \qquad v_2 = \left(\frac{m_1}{m_2}\right) v_1.$$

The kinetic energies after the break-up are then $K_1 = \frac{1}{2} m_1 v_1^2$ and

$$K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_2 \left(\frac{m_1}{m_2}\right)^2 v_1^2 = \left(\frac{m_1}{m_2}\right) K_1.$$

The fraction of the total kinetic energy carried off by $m_1$ is

$$\frac{K_1}{K_1 + K_2} = \frac{K_1}{K_1 + (m_1/m_2)K_1} = \frac{m_2}{m_1 + m_2}$$

and the fraction carried off by $m_2$ is $1 - \frac{m_2}{m_1 + m_2} = \frac{m_1}{m_1 + m_2}$.

14-59 (a) $\Delta V = 4\pi r^2 \Delta r = 4\pi (14.0\times10^3 \text{ m})^2 (0.05 \text{ m}) = 1.23\times10^8 \text{ m}^3 \sim 10^8 \text{ m}^3$

(b) The force on the next layer is determined by atmospheric pressure.

$$W = P\Delta V = (1.013\times10^5 \text{ N/m}^2)(1.23\times10^8 \text{ m}^3) = 1.25\times10^{13} \text{ J} \sim 10^{13} \text{ J}$$

(c) $1.25\times10^{13}\ \text{J}=\frac{1}{10}(\text{yield})$, so yield $=1.25\times10^{14}\ \text{J}\sim10^{14}\ \text{J}$

(d) $\frac{1.25\times10^{14}\ \text{J}}{4.2\times10^{9}\ \text{J/ton TNT}}=2.97\times10^{4}\ \text{ton TNT}\sim10^{4}\ \text{ton TNT}$ or $\sim10$ kiloton

# 15

# Particle Physics

15-1 The time for a particle traveling with the speed of light to travel a distance of $3\times10^{-15}$ m is $\Delta t=\dfrac{d}{v}=\dfrac{3\times10^{-15}\ \text{m}}{3\times10^{8}\ \text{m/s}}=10^{-23}\ \text{s}$.

15-3 The minimum energy is released, and hence the minimum frequency photons are produced. The proton and antiproton are at rest when they annihilate. That is, $E=E_0$ and $K=0$. To conserve momentum, each photon must carry away one-half the energy. Thus, $E_{\text{min}}=hf_{\text{min}}=\dfrac{2E_0}{2}=E_0=938.3\ \text{MeV}$. Thus,

$$f_{\text{min}}=\frac{(938.3\ \text{MeV})(1.6\times10^{-19}\ \text{J/MeV})}{6.63\times10^{-34}\ \text{J s}}=2.26\times10^{23}\ \text{Hz}$$

and

$$\lambda_{\text{max}}=\frac{c}{f_{\text{min}}}=\frac{3\times10^{8}\ \text{m/s}}{2.26\times10^{23}\ \text{Hz}}=1.32\times10^{-15}\ \text{m}$$

15-5 The rest energy of the $Z^0$ boson is $E_0=96\ \text{GeV}$. The maximum time a virtual $Z^0$ boson can exist is found from $\Delta E\Delta t=\hbar$.

$$\Delta t=\frac{\hbar}{\Delta E}=\frac{1.055\times10^{-34}\ \text{J s}}{(96\ \text{GeV})(1.6\times10^{-10}\ \text{J/GeV})}=6.87\times10^{-27}\ \text{s}.$$

The maximum distance it can travel in this time is

$$d=c(\Delta t)=(3\times10^{8}\ \text{m/s})(6.87\times10^{-27}\ \text{s})=2.06\times10^{-18}\ \text{m}.$$

The distance $d$ is an approximate value for the range of the weak interaction.

15-7 Use Table 15.2 to find properties that can be conserved in the given reactions

| | | Reaction 1 | Reaction 2 |
|---|---|---|---|
| (a) | Charge: | $\pi^- + p \to K^- + \Sigma^+$<br>$(-)+(+) \to (-)+(+)$<br>$0 \to 0$ √ | $\pi^- + p \to \pi^- + \Sigma^+$<br>$(-)+(+) \to (-)+(+)$<br>$0 \to 0$ √ |
| (b) | Baryon number: | $(0)+(+1) \to (0)+(+1)$<br>$+1 \to +1$ √ | $(0)+(+1) \to (0)+(+1)$<br>$+1 \to +1$ √ |
| (c) | Strangeness: | $(0)+(0) \to (+1)+(-1)$<br>$0 \to 0$ √ | $(0)+(0) \to (0)+(+1)$<br>$0 \to (-1)$ X |

Thus, the second reaction is not allowed since it does not conserve strangeness.

15-9 (a) $p \to \pi^- + \pi^0$ (Baryon number is violated: $1 \to 0+0$)

(b) $p+p \to p+p+\pi^0$ (This reaction can occur)

(c) $p+p \to p+\pi^+$ (Baryon number is violated: $1+1 \to 1+0$)

(d) $\pi^+ \to \mu^+ + v_\mu$ (This reaction can occur)

(e) $n \to p + e^- + \overline{v}_e$ (This reaction can occur)

(f) $\pi^+ \to \mu^+ + n$ (Violates baryon number: $0 \to 0+1$, and violates muon-lepton number: $0 \to -1+0$.

15-11 (a) $\mu^- \to e + \gamma$ $L_e: 0 \to 1+0$ and $L_\mu: 1 \to 0+0$

(b) $n \to p + e^- + v_e$ $L_e: 0 \to 0+1+1$

(c) $\Lambda^0 \to p + \pi^0$ Strangeness $-1 \to 0+0$ and charge $0 \to +1+0$

(d) $p \to e^+ + \pi^0$ Baryon number $+1 \to 0+0$ and lepton number $0 \to 1+0$

(e) $\Xi^0 \to n + \pi^0$ Strangeness $-2 \to 0+0$

15-13 (a) In Equation 15.16, $K_{th} = \frac{(m_3+m_4+m_5+m_6)^2c^2 - (m_1+m_2)^2c^2}{2m_2}$ where $m_1$ is the mass of the incident particle, $m_2$ is the mass of the stationary target particle, and $m_3$, $m_4$, $m_5$, and $m_6$ are the product particle masses. For $\overline{p}$ production,

$$K_{th} = \frac{(4m_p)^2c^2 - (2m_p)^2c^2}{2m_p} = 6m_pc^2 = (6)(938.3\text{ MeV}) = 5\,630\text{ MeV or } 5.63\text{ GeV}.$$

(b) Using Equation 15.16 for the reaction $\mathrm{p+p+n+\overline{n}}$,

$$K_{th} = \frac{(2m_p + 2m_n)^2 c^2 - (2m_p)^2 c^2}{2m_p}$$

$$= \frac{(4)\left[(938.8+939.6)^2 \mathrm{MeV}^2 c^2 - \left(4(938.3)^2 \mathrm{MeV}^2 c^2\right)\right]}{(2)(938.3 \text{ MeV})} = 5.64 \text{ GeV}$$

15-15 Let $E$ = efficiency in %

For Example 15.5, $E = \left(\dfrac{m_{\pi^0} c^2}{K_{th}}\right) \times 100 = \left[\dfrac{135 \text{ MeV}}{280 \text{ MeV}}\right] \times 100 = 48\%$

For Exercise 3, $E = \left(\dfrac{m_{\pi^+} c^2}{K_{th}}\right) \times 100 = \left[\dfrac{139.6 \text{ MeV}}{292 \text{ MeV}}\right] \times 100 = 48\%$

$E = 2\left(\dfrac{m_{\pi^+} c^2}{K_{th}}\right) \times 100 = 2\left[\dfrac{139.6 \text{ MeV}}{600 \text{ MeV}}\right] \times 100 = 46\%$

For Problem 13, $E = 2\left(\dfrac{m_p c^2}{K_{th}}\right) \times 100 = 2\left[\dfrac{938.3 \text{ MeV}}{5.63 \text{ GeV}}\right] \times 100 = 33\%$

15-17 $\Sigma^0 + \mathrm{p} \rightarrow \Sigma^+ + \gamma + \mathrm{X}$

$\mathrm{dds + uud \rightarrow uds + 0 + ?}$

The left side has a net 3d, 2u, and 1s. The right hand side has 1d, 1u, and 1s leaving 2d and 1u missing. The unknown particle is a neutron, udd. Baryon and strangeness numbers are conserved.

15-19 Quark composition of proton = uud, and of neutron = udd. Thus, if we neglect binding energies, we may write:

$$m_p = 2m_u + m_d \quad (1)$$

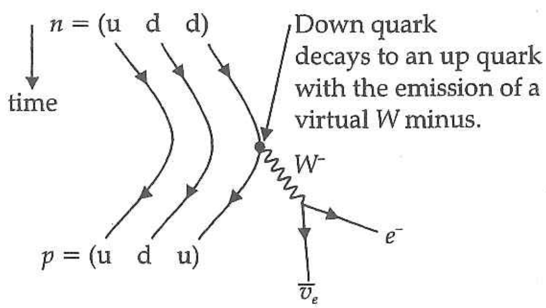
$$\text{and } m_n = m_u + 2m_d \quad (2)$$

Solving simultaneously, we find:

$$m_u = \frac{1}{3}(2m_p - m_n) = \frac{1}{3}\left[2(938.3 \text{ MeV}/c^2) - 939.6 \text{ MeV}/c^2\right] = 312.3 \text{ MeV}/c^2,$$

and from either Equations (1) or (2), $m_d = 313.6 \text{ MeV}/c^2$. These should be compared to the experimental masses $m_u \cong 5 \text{ MeV}/c^2$ and $m_d \simeq 10 \text{ MeV}/c^2$.

15-21



15-23 A photon travels the distance from the Large Magellanic Cloud to us in 170 000 years. The hypothetical massive neutrino travels the same distance in 170 000 years plus 10 seconds:

$$c(170\,000\text{ yr}) = v(170\,000\text{ yr} + 10\text{ s})$$

$$\frac{v}{c} = \frac{170\,000\text{ yr}}{170\,000\text{ yr} + 10\text{ s}} = \frac{1}{1 + \{10\text{ s}/[(1.7\times10^5\text{ yr})(3.156\times10^7\text{ s/yr})]\}} = \frac{1}{1 + 1.86\times10^{-12}}$$

For the neutrino we want to evaluate $mc^2$ in $E = \gamma mc^2$:

$$mc^2 = \frac{E}{\gamma} = E\sqrt{1 - \frac{v^2}{c^2}} = 10\text{ MeV}\sqrt{1 - \frac{1}{(1+1.86\times10^{-12})^2}} = 10\text{ MeV}\sqrt{\frac{(1+1.86\times10^{-12})^2 - 1}{(1+1.86\times10^{-12})^2}}$$

$$mc^2 \approx 10\text{ MeV}\sqrt{\frac{2(1.86\times10^{-12})}{1}} = 10\text{ MeV}(1.93\times10^{-6}) = 19\text{ eV}$$

Then the upper limit on the mass is

$$m = \boxed{\frac{19\text{ eV}}{c^2}}$$

$$m = \frac{19\text{ eV}}{c^2}\left(\frac{\text{u}}{931.5\times10^6\text{ eV}/c^2}\right) = 2.1\times10^{-8}\text{ u.}$$

15-25 $m_\Lambda c^2 = 1\,115.6\text{ MeV}$ $\Lambda^0 \to p + \pi^-$

$m_p c^2 = 938.3\text{ MeV}$ (See Table 15.2 for masses)

$m_\pi c^2 = 139.6\text{ MeV}$

The difference between starting mass-energy and final mass-energy is the kinetic energy of the products.

$$K_p + K_\pi = 37.7\text{ MeV and } p_p = -p_\pi$$

Applying conservation of relativistic energy,

$$[(938.3\text{ MeV})^2 + p^2c^2]^{1/2} - 938.3\text{ MeV} + [(139.6\text{ MeV})^2 + p^2c^2]^{1/2} - 139.6\text{ MeV} = 37.7\text{ MeV}.$$

Solving the algebra yields $p_p c = -p_\pi c = 100.4\text{ MeV}$. Then

$$K_p = \left[(m_p c^2)^2 + (100.4\text{ MeV})^2\right]^{1/2} - m_p c^2 = 5.4\text{ MeV}$$

$$K_\pi = [(139.6\text{ MeV})^2 + (100.4\text{ MeV})^2]^{1/2} - 139.6\text{ MeV} = 32.3\text{ MeV}$$

15-27 Time-dilated lifetime.

$$T = \gamma T_0 = \frac{0.9\times10^{-10}\text{ s}}{(1 - v^2/c^2)^{1/2}} = \frac{0.9\times10^{-10}\text{ s}}{(1 - (0.96)^2)^{1/2}} = 3.214\times10^{-10}\text{ s}$$

$$\text{distance} = (0.96)(3\times10^8\text{ m/s})(3.214\times10^{-10}\text{ s}) = 9.3\text{ cm}$$

15-29 $p+p \to p+\pi^+ + X$

$Q = M_p + M_p - M_p - M_{\pi^+} - M_X$

(From conservation of momentum, particle X has zero momentum and thus zero kinetic energy.)

$$Q = (2)(70.4\text{ MeV}) = 938.3\text{ MeV} + 938.3\text{ MeV} - 938.3\text{ MeV} - 139.5\text{ MeV} - M_X$$
$$M_X = 939.6\text{ MeV}$$

X must be a neutral baryon of rest mass 939.6 $\text{MeV}/c^2$. Thus X is a neutron.

15-31 (a) The mediator of this weak interaction is a $Z^0$ boson.

(b) The mediator of a strong (quark-quark) interaction is a gluon.

15-33 (a) $\Delta E = (m_n - m_p - m_e)c^2$. From Appendix B,

$\Delta E = (1.008\,665\text{ u} - 1.078\,25\text{ u})931.5\text{ MeV/u} = 0.782\text{ MeV}$

(b) Assuming the neutron at rest, momentum is conserved, $p_p = p_e$ relativistic energy is conserved, $\left[(m_p c^2)^2 + (p^2c^2)\right]^{1/2} + \left[(m_e c^2)^2 + (p_e^2 c^2)\right]^{1/2} = m_n c^2$. Since $p_p = p_e$.

$$\left[(938.3\text{ MeV})^2 + (pc)^2\right]^{1/2} + \left[(0.511\text{ MeV})^2 + (pc)^2\right]^{1/2} = 939.36\text{ MeV}$$

Solving the algebra $pc = 1.19\text{ MeV}$. If $p_e c = \gamma m_e v_e c = 1.19\text{ MeV}$, then,

$$\frac{\gamma v_e}{c} = \frac{1.19\text{ MeV}}{0.511\text{ MeV}} = \frac{x}{(1-x^2)^{1/2}} = 2.329 \text{ where } x = \frac{v_e}{c}$$

$$x^2 = (1-x^2)5.423$$
$$x = \frac{v_e}{c} = 0.919$$
$$v_e = 0.919c = 276 \times 10^6\text{ m/s}$$

$$\text{Then } m_p v_p = \gamma m_e v_e = \frac{(1.19\text{ MeV})(1.6\times10^{-13}\text{ J/MeV})}{3\times10^8\text{ m/s}}$$

$$v_p = \frac{\gamma m_e v_e}{m_p c} = \frac{(1.19\text{ MeV})(1.6\times10^{-13}\text{ J/MeV})}{(1.67\times10^{-27}\text{ kg})(3\times10^8\text{ m/s})} = 3.80\times10^5\text{ m/s}$$
$$v_p = 380\text{ km/s} = 0.001\,266c$$

(c) The electron is relativistic, the proton is not.

15-35 (a)
$$p_{\Sigma^+} = eBr_{\Sigma^+} = \frac{(1.602\,177\times10^{-19}\text{ C})(1.15\text{ T})(1.99\text{ m})}{5.344\,288\times10^{-22}(\text{kg}\cdot\text{m/s})/(\text{MeV}/c)} = \frac{686\text{ MeV}}{c}$$
$$p_{\pi^+} = eBr_{\pi^+} = \frac{(1.602\,177\times10^{-19}\text{ C})(1.15\text{ T})(0.580\text{ m})}{5.344\,288\times10^{-22}(\text{kg}\cdot\text{m/s})/(\text{MeV}/c)} = \frac{200\text{ MeV}}{c}$$

(b) Let $\varphi$ be the angle made by the neutron's path with the path of the $\Sigma^+$ at the moment of decay. By conservation of momentum:

$$p_n \cos\varphi + (199.961\,581 \text{ MeV}/c)\cos 64.5° = 686.075\,081 \text{ MeV}/c$$

$$\therefore p_n \cos\varphi = 599.989\,401 \text{ MeV}/c \quad (1)$$

$$p_n \sin\varphi = (199.961\,581 \text{ MeV}/c)\sin 64.5° = 180.482\,380 \text{ MeV}/c \quad (2)$$

From (1) and (2):

$$p_n = \sqrt{(599.989\,401 \text{ MeV}/c)^2 + (180.482\,380 \text{ MeV}/c)^2} = 627 \text{ MeV}/c.$$

(c) $E_{\pi^+} = \sqrt{(p_{\pi^+}c)^2 + (m_{\pi^+}c^2)^2} = \sqrt{(199.961\,581 \text{ MeV})^2 + (139.6 \text{ MeV})^2} = 244 \text{ MeV}$

$E_n = \sqrt{(p_n c)^2 + (m_n c^2)^2} = \sqrt{(626.547\,022 \text{ MeV})^2 + (939.6 \text{ MeV})^2} = 1\,130 \text{ MeV}$

$E_{\Sigma^+} = E_{\pi^+} + E_n = 243.870\,445 \text{ MeV} + 1\,129.340\,219 \text{ MeV} = 1\,370 \text{ MeV}$

(d) $m_{\Sigma^+}c^2 = \sqrt{E_{\Sigma^+}^2 - (p_{\Sigma^+}c)^2} = \sqrt{(1\,373.210\,664 \text{ MeV})^2 - (686.075\,081 \text{ MeV})^2} - 1\,190 \text{ MeV}$

$\therefore m_{\Sigma^+} = 1\,190 \text{ MeV}/c^2$

$E_{\Sigma^+} = \gamma m_{\Sigma^+}c^2$, where $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \frac{1\,373.210\,664 \text{ MeV}}{1\,189.541\,303 \text{ MeV}} = 1.154\,4$. Solving for $v$,

$v = 0.500c$.

15-37 (a) If $2N$ particles are annihilated, the energy released is $2Nmc^2$. The resulting photon momentum is $p = \frac{E}{c} = \frac{2Nmc^2}{c} = 2Nmc$. Since the momentum of the system is conserved, the rocket will have momentum $2Nmc$ directed opposite the photon momentum. $p = 2Nmc$.

(b) Consider a particle that is annihilated and gives up its rest energy $mc^2$ to another particle that also has initial rest energy $mc^2$ (but no momentum initially).

$$E^2 = p^2c^2 + (mc^2)^2$$

Thus $(2mc^2)^2 = p^2c^2 + (mc^2)^2$. Where $p$ is the momentum the second particle acquires as a result of the annihilation of the first particle. Thus $4(mc^2)^2 = p^2c^2 + (mc^2)^2$, $p^2 = 3(mc^2)^2$. So $p = \sqrt{3}mc$. This process is repeated $N$ times (annihilate $\frac{N}{2}$ protons and $\frac{N}{2}$ antiprotons). Thus the total momentum acquired by the ejected particles is $\sqrt{3}Nmc$, and this momentum is imparted to the rocket.

$$p = \sqrt{3}Nmc$$

(c) Method (a) produces greater speed since $2Nmc > \sqrt{3}Nmc$.