Modern Physics

Fall/Winter 1900, Max Planck's paper *"Ueber das Gesetz der Energieverteilung im Normalspectrum"*, Annalen der Physik IV, 553 (1901) – peak in 1920s/30s

Two major parts: modern relativity, first 4 - 6 lectures

Quantum mechanics and its applications, rest of the course –also main content of Phys 312 to follow next quarter
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What is Physics all about?

concepts and their connection, i.e. mathematically formulated equations/laws,

concepts and laws are derived from interplay between theory and experiment, this makes sure only good theories survive and theories get better over time

some "fundamental" concepts such as space and time are much older than Physics and are sort of common sense knowledge (Kant's "a priori" concepts) and may even be inherited genetically but: Werner Heisenberg (in ***Physics and Philosophy***)

"Any concepts or words which we have formed in the past through the interplay between the world and ourselves are not really sharply defined with respect to their meaning; that is to say, we do not know exactly how far they will help us in finding our way in the world. ... This is true even of the simplest and most general concepts like "existence" and "space and time"…

The concepts may, however, be sharply defined with regard to their connections. **This is actually the fact when the concepts become a part of a system of axioms and definition which can be expressed consistently by a mathematical scheme.** Such a group of connected concepts may be applicable to a wide field of experience and will help us to find our way in this field. …

So modern physics will be in large parts contrary to our intuition because it deals with the very fast, i.e. speeds comparable to the speed of light and the very small, atoms, molecules, elemental particles.

Our (possibly inherited) lack of appreciation that the world of the very fast and the world of the very small may well be very different from the world we are used to makes modern physics difficult to comprehend, but Heisenberg showed the way, see above, we have to stick to the mathematical schemes that connect concepts and have to redefine well known concepts, such as space, time, causality, … to fit these schemes

Relativistic Mechanics - Special Relativity

Galileo, Newton: **Inertial reference frame:**
Newton's (first) law: ***a body continues to be at rest or continues moving with constant velocity if there is no net force acting on it***

v = 0 or constant if $\Sigma a = \Sigma F = 0$

$$because \Sigma F = ma \quad \text{and} \quad v = \frac{ds}{dt}$$

m = inertia, that is where the reference frame gets its name from

Galileo, Newton: *all laws of mechanics are the same in inertial reference frames, all reference frames are equally valid, there is no preferred inertial reference frame* **– *classical concept of relativity***

Classic relations between an **event** as observed in two different (v = 0, v' > 0) inertial reference frames are related by Galilean transformation

Galilean transformations

two different sets of coordinates: space coordinates (x,y,z), time coordinate t at rest; (x',y',z';t') in motion

$x = x' + v\,t$ $\quad$ $x' = x - v\,t$

$y = y'$ $\quad$ $y' = y$

$z = z'$ $\quad$ $z' = z$

$t = t'$ $\quad$ $t' = t$,

event has one set of coordinates in one system and another set of coordinates in another system

(back transformation are the same except for sign of v)

leads to vector addition law of velocities, if event moves in unprimed frame with velocity u, v and u add up

$$u_x = \frac{x_2 - x_1}{t_2 - t_1} = \frac{(x'_2 + vt'_2) - (x'_1 + vt'_1)}{t'_2 - t'_1} = \frac{(x'_2 - x'_1) + v(t'_2 - t'_1)}{t'_2 - t'_1} = \frac{\Delta x'}{\Delta t'} + v$$

$u_x = u_x' + v$

$u_y = u_y'$

$u_z = u_z'$

Nota Bene: space and time coordinates do not mix,

importance of these equation is that they ensure the physical laws that are invariant with respect to these equations are valid everywhere and at all times (if we use our common sense ideas of space and time)

<u>**Result of Galileian relativity:**</u> there is no mechanical experiment that can detect absolute motion, you can eat your dinner in an air plane (when it is not accelerating) which is moving rather fast with respect to the earth – just as well as on your dinner table at home – which is moving even faster with respect to the sun

In 1870s -1904 some new idea of how to measure absolute motion

$$c = \sqrt{\frac{1}{m_0 e_0}} = \sqrt{\frac{1}{4p \cdot 10^{-7} TmA^{-1} \bullet 8.854187817 \cdot 10^{-12} C^2 N^{-1} m^{-2}}}$$

prediction of **Maxwell's 1860 set of equations**, μ_0 permeability, e_0 permittivity of free space (i.e. vacuum)

c = 2.99792458 10^8 $\frac{m}{s}$ constant (and now exact per definition)

<u>***according to what/whom has c this value ???***</u>

Maxwell's own answer: ***luminiferous ether*** (something quite strange, present everywhere even in the nearly absolute vacuum of free space, but allows planets and other objects to move through it freely, …, and which is in absolute rest)

Not only c = constant in vacuum but the other laws by Maxwell's do not obey a Galilean transformation, so at last there seemed to be a way of detecting motion, if you do an electromagnetic experiment such as measuring the speed of light in an airplane or on earth, you should get the relative speed with respect to the ether which is supposed to be at absolute rest.

Recall sound: travels in air and any kind of body, speeds: 243 $\frac{m}{s}$ in air at 293 K, 249 $\frac{m}{s}$ in air at 303 K and normal pressure, 3800 $\frac{m}{s}$ in concrete at 293 K, needs actually a medium to propagate, if you have a potential source of sound in vacuum – you can't hear it as the wave can't propagate

So upwind sound travels faster – as it is carried along with the wind itself, downwind sound travel slower since the medium (air) travels in the opposite direction – Galilean transformations seem to apply

Conundrum:

Since ether seemed to be so special – it should define a very particular frame of reference, i.e. the only one in which Maxwell's equations are correct, in all other frames of reference, i.e. our earth, there should be deviations from Maxwell's laws, … on the other hand, these laws work quite well, how can this be?

Michelson-Morley Experiment, 1887-1904

Designed to detect the ether and earth's relative motion with respect to the ether by detecting small changes in the speed of light, i.e. deviations from Maxwell's "c = constant law" by interferometry

Light source A, semitransparent mirror = beam splitter B, two mirrors C and E all mounted on a rigid base

Mirrors C and E are placed at equal distances L from beam splitter, so that the two resulting beams have (apparently the) same path length (2L) to go in perpendicular directions, reach the mirrors C and D and get reflected back to the beam splitter where they are joined together again

If time taken for the light to go from B to E and back is the same as the time from B to C and back, emerging beams D and F will be in phase and reinforce each other

It these two times differ slightly, beams D and F will produce interference pattern.

If apparatus is at rest with respect to the ether, times should be exactly equal because the lengths the light must travel are exactly equal – if it is moving towards the right with a velocity u, there should be a difference in the times, resulting in an interference pattern.

Why should that be?

Time to go from B to E and back = t_1
return time E to B = t_2 (t_1 ? t_2 because of movement of apparatus to right)

it the apparatus moves, while light is on its way to from B to E, the mirror together with the whole apparatus moves away, this distance is u t_1, i.e. the light must travel with speed c the length L + ut_1 in order to reach the mirror

$ct_1 = L + ut_1$ $\qquad\qquad$ $t_1 = L / (c - u)$

which means that velocity of the light with respect to apparatus is c - u

for return travel velocity of light with respect to apparatus must therefore be c + u, because the beam splitter B and the light beam are moving in opposite directions

$t_2 = L / (c + u)$ and $ct_2 = L - ut_2$

total time for B to E and back is $t_1 + t_2 = 2Lc/(c^2-u^2) = \dfrac{2L}{c(1-u^2/c^2)}$

now the other path: B – C and back, again assumption is apparatus is moving to the right (because we want to measure this movement by an anticipated shift in the interference pattern)

during time t_3 mirror C will move to the right by a distance ut_3 light has therefore to travel along the hypotenuse of right triangle BC'½B'

$(ct_3)^2 = L^2 + (ut_3)^2$

$L^2 = c^2t_3^2 - u^2t_3^2 = (c^2-u^2)t_3^2$

So $t_3 = L/\sqrt{c^2-u^2}$, $= \dfrac{L}{c \cdot \sqrt{1-u^2/c^2}}$

triangle is symmetric, so time it takes for the light to return to B is 2 t_3

$2\ t_3 = \dfrac{2L}{c\sqrt{1-u^2/c^2}}$? $t_1 + t_2 = \dfrac{2L}{c(1-u^2/c^2)}$

difference is just factor $\frac{1}{\sqrt{1-u^2/c^2}}$ = ? (Lorentz factor) >1

denominators represent modifications in time caused by motion of the apparatus, they are not the same so we should see an interference pattern and from this we could calculate u velocity of earth with respect to ether– the whole point of the experiment

a minor technical point, we can't make the lengths L exactly equal, we can compensate for this in the interference pattern, then we can turn apparatus around by 90° degrees and should see a shift of interference pattern between two sets of settings 1) arbitrary orientation and 2) 90° rotated with respect to 1)

But no shift in interference pattern was ever observed, we do know u ? 0

$$2\,t_3 = \frac{2L}{c\sqrt{1-u^2/c^2}} \quad ? \quad t_1 + t_2 = \frac{1}{\sqrt{1-u^2/c^2}}\,\frac{2L}{c\sqrt{1-u^2/c^2}}$$

so it seems as if length of the path is B to E and back is contracted by a factor ? (Lorentz and Fitzgerald)

2. Result, the speed of light (in air at earth) is in all directions equal regardless of any relative movement of the earth with

respect to the ether that should result in an "ether wind" analogous to the wind that affects the speed of sound

apparent 0 velocity of earth and constant velocity of light results can both be explained by **Lorentz Transformations** (1904) , moving frame ' at t = 0 both frames coincide

$$x = \frac{1}{\sqrt{1 - v^2/c^2}} (x' + vt') \qquad [m] = [m + \tfrac{ms}{s}]$$

y = y', z = z'

$$t = \frac{1}{\sqrt{1 - v^2/c^2}} (t' + \frac{vx'}{c^2}) \qquad [s] = [s + \tfrac{m^2 s^2}{s m^2}]$$

in which Maxwell's law are invariant, i.e. have the same form regardless of the movement of the observer !

reverse transformations

$$x' = \frac{1}{\sqrt{1 - v^2/c^2}} (x - vt) \qquad [m] = [m + \tfrac{ms}{s}]$$

y' = y, z' = z

$$t' = \frac{1}{\sqrt{1 - v^2/c^2}} (t - \frac{vx}{c^2}) \qquad [s] = [s + \tfrac{m^2 s^2}{s m^2}]$$

for v << c, Lorentz transformations go over in Galilean transformations

consequences (some of which not fully realized by Lorentz):

1. space and time coordinates mix, i.e. they are the same sort of thing,
? better description 4 dimensional space–time

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definitions:

proper length: Length L_0 of an object measured in the rest frame of the object is its proper length. Measurements of the length from an reference frame that is in relative motion parallel to that length are always less than the proper length

proper time: When two events occur at same location in an inertial reference frame, the time interval between them, measured in that frame, is called proper time interval or the proper time. Measurements of the same time interval from nay other inertial reference frame are always greater

2. effect on length (both distances parallel v and length of moving objects parallel v)

L_0 length of an object at rest, **proper length**

$$L_0 = x_2 - x_1 = \frac{1}{\sqrt{1-v^2/c^2}}(x'_2 + vt'_2 - x'_1 - vt_1')$$

since $t'_2 = t'_1$ (same time of measurement)

$$L_0 = \frac{1}{\sqrt{1-v^2/c^2}}(x'_2 - x'_1), \qquad \text{with } L = (x'_2 - x'_1)$$

$$L_0 = \frac{1}{\sqrt{1-v^2/c^2}} L \quad \text{or} \quad L = \sqrt{1-v^2/c^2}\, L_0, \qquad (\text{lets call } v^2/c^2 = a)$$

$\sqrt{1-v^2/c^2} = ? = ?^{-1}$ inverse Lorentz factor or contraction factor > 1

Result: $L < L_0$ length contraction for any velocity, a > 0

<u>say v = 0.1 c, L_0 = 1m</u>

L ~ 0.995 cm or 0.854 m or 0.666 m ?

<u>say v = 100 km h^{-1} , L_0 = 1m</u>

L ~ 0.99995 m ***or*** 0.999999995 m ***or*** 0.999999999999995 m

if a << 1 $\sqrt{1-v^2/c^2} = \sqrt{1-a} \sim 1 - \frac{1}{2} a$, so $\sqrt{1-8.5853\cdot 10^{-15}} \sim 1 - 5\ 10^{-15}$

it does not get noticed in everyday experience

3. effect on time (rate at which clocks and all natural processes run)

t_0 time coordinate of a object at rest, **proper time**

$$\Delta t' = t_2' - t_1' = \frac{1}{\sqrt{1-v^2/c^2}}\left(t_2 + \frac{vx_2}{c^2} - t'_1 - \frac{vx_1}{c^2}\right)$$

since $x_2 = x_1$ (same place of event)

$$\Delta t' = \frac{1}{\sqrt{1-v^2/c^2}}(t_2 - t_1), \qquad \text{with } \Delta t_0 = t_2 - t_1$$

$$\Delta t' = \frac{1}{\sqrt{1-v^2/c^2}}(\Delta t_0)$$ = ? ? t_0 = ?$^{-1}$? $t_{0,}$? = Lorentz factor > 1

(lets call $v^2/c^2 = a$)

Result: ? t' > ? t_0 time dilation for any velocity, a > 0

<u>**say v_0 = 0.1 c, ? t_0 = 1h = 3600 s**</u>

? t' ~ 1 h+18 sec ***or*** 1 h+1min+25 sec ***or*** 1 h+3 min+5 s ?

<u>**say v_0 = 100 km h^{-1} , ? t_0 = 1h = 3600 s**</u>

? t' ~ 1 h + 5 s ***or*** 1 h + 15 µs ***or*** 1h + 18 ps ?

if a << 1 $1/\sqrt{1-a}$ ~ 1 + ½ a, so $(\sqrt{1-8.5853\cdot 10^{-15}})^{-1}$ ~ 1 + 5 10^{-15}

does not get noticed in everyday experience, *but can be measured*

Lorentz noted dependence of time on motion in his equations but call all these times "apparent times" in order to distinguish them from the one "true time" he believed to be absolute in the Newtonian sense

4. relativistic addition of velocities

for convenience: ? = $^1\!\Big/\!\sqrt{1-{}^{v^2}\!/\!_{c^2}}$ = $^1\!/\!_h$ = $^1\!/\!\sqrt{1-a}$ Lorentz factor

$$u_x = \frac{\Delta x}{\Delta t} = \frac{g\,(\Delta x' + v\Delta t')}{g\,(\Delta t' + v\,{}^{\Delta x'}\!/\!_{c^2})} = \frac{({}^{\Delta x'}\!/\!_{\Delta t'}) + v}{1 + ({}^{v}\!/\!_{c^2})\cdot({}^{\Delta x'}\!/\!_{\Delta t'})} = \frac{u_x' + v}{1 + {}^{u_x'\cdot v}\!/\!_{c^2}} \qquad / :\Delta t'$$

$$u_y = \frac{\Delta y}{\Delta t} = \frac{\Delta y'}{g\cdot\{\Delta t' + {}^{vx'}\!/\!_{c^2}\}} = \frac{({}^{\Delta y'}\!/\!_{\Delta t'})}{g\cdot\{1 + ({}^{v}\!/\!_{c^2})\cdot({}^{\Delta x'}\!/\!_{\Delta t'})\}} = \frac{u_y'}{g\cdot\{1 + {}^{u_x'\cdot v}\!/\!_{c^2}\}}$$

$$u_z = \frac{\Delta z}{\Delta t} = \frac{\Delta z'}{g\cdot\{\Delta t' + {}^{vx'}\!/\!_{c^2}\}} = \frac{({}^{\Delta z'}\!/\!_{\Delta t'})}{g\cdot\{1 + ({}^{v}\!/\!_{c^2})\cdot({}^{\Delta x'}\!/\!_{\Delta t'})\}} = \frac{u_z'}{g\cdot\{1 + {}^{u_x'\cdot v}\!/\!_{c^2}\}}$$

reverse transformations

$$u_x' = \frac{\Delta x}{\Delta t} = \frac{g\,(\Delta x - v\Delta t)}{g\,(\Delta t - v\,{}^{\Delta x}\!/\!_{c^2})} = \frac{({}^{\Delta x}\!/\!_{\Delta t}) - v}{1 - ({}^{v}\!/\!_{c^2})\cdot({}^{\Delta x}\!/\!_{\Delta t})} = \frac{u_x - v}{1 - {}^{u_x\cdot v}\!/\!_{c^2}} \qquad / :\Delta t'$$

$$u_y' = \frac{\Delta y}{\Delta t} = \frac{\Delta y}{g \cdot \{\Delta t - vx/c^2\}} = \frac{(\Delta y/\Delta t)}{g \cdot \{1-(v/c^2)\cdot(\Delta x/\Delta t)\}} = \frac{u_y}{g \cdot \{1-u_x \cdot v/c^2\}}$$

$$u_z' = \frac{\Delta z}{\Delta t} = \frac{\Delta z}{g \cdot \{\Delta t - vx/c^2\}} = \frac{(\Delta z/\Delta t)}{g \cdot \{1-(v/c^2)\cdot(\Delta x/\Delta t)\}} = \frac{u_z}{g \cdot \{1-u_x \cdot v/c^2\}}$$

Result: velocities do not add up simply (Galilean)

<u>say v = 0.5 c *super super* fact rocket sending out light, u_x = 1 c</u>

how fast is the light going to be ? 1.5 c ***or*** 1c **or** 0.75 c when measured from earth, u_x ?

<u>say v = 100 km h^{-1} (between frames) , u_x' = 5 km h^{-1}</u>

(you walk in a super fast train in the direction of motion)

95 km h^{-1} ***or*** 105 - 5.25 10^{-14} km h^{-1} ***or*** 105 + 5.25 10^{-14} km h^{-1} ?

does not get noticed in everyday experience

just as for v << c, Lorentz transformations go over in Galilean transformations, relativistic velocity addition goes over into Galilean velocity addition for u_x, v and u_x' << c

but for c = u_x

$$u_x' = \frac{\Delta x'}{\Delta t'} = \frac{u_x - v}{1-u_x \cdot v/c^2} = \frac{c-v}{1-c\cdot v/c^2} = \frac{c(c/c - v/c)}{1-v/c} = \frac{c(1-v/c)}{1-v/c} = c$$

so then c = u_x = u_x'

as one may easily mix up u_x, u_x'and v two spaceships A and B are moving in opposite directions observer on earth measures speed of A as 0.7 c and speed of B as 0.85 c, find velocity of B with respect to A

so we have u_x^A and u_x^B as both are measured from rest frame at earth and must assign signs, lets A move to the right and call it + direction, B move to the left and call it - direction

we should find $u_x^{B'}$ the velocity of B in moving frame ‘ of A

A moves with u_x^A with respect to earth, i.e. that is v with respect to earth and it is positive, so

$u_x^{B'} = \dfrac{u_x^B - v}{1 - u_x^B \cdot v / c^2\}}$ when numbers are put in make sure to remember u_x^B is negative, result -0.9771 c, seems to be OK with intuition, B goes pretty fast towards A which is receding pretty fast as well, but speed must be smaller than c, so discount the - sign

5: relativistic Doppler effect, transverse Doppler effect

remember Doppler effect for sound waves?
when a car or truck is moving while its horn is blowing, frequency (pitch) of sound is higher as the vehicle approaches you and lower as it moves away from you
different formulae for observer at rest - source moving; source at rest – observer moving, both source and observer moving

<u>for light (electromagnetic) waves only relative velocity v is important</u>

$f_{obs} = \sqrt{\frac{1+v/c}{1-v/c}}\, f_{source}$ source approaching observer on same axis

$f_{obs} = \sqrt{\frac{1-v/c}{1+v/c}}\, f_{source}$ source receding from observer on same axis

f_{source} proper frequency

transverse Doppler effect - source and observer on perpendicular axes - is **consequence of relativity – exists only for electromagnetic waves**

$f_{obs} = \sqrt{1-v^2/c^2}\; f_{source}$ analog to $T_{obs} = 1\Big/\sqrt{1-v^2/c^2}\; T_{source}$

nothing else then time dilation formula - with T_{source} **proper period** (***proper time*** it takes to complete one oscillation)

discovered 1938 by Ivens and Stilwell, who did not believe in relativity prior to their discovery

Summary so far - 1904

1) Galileo/Newton's classical relativity/mechanics, all mechanical laws are invariant to Galilean transformation, work very well in the realm of our everyday human experience

2) according to Lorentz for **electrodynamics**, Maxwell's laws are invariant to the Lorentz transformations, Lorentz transformations contain Galilean transformation as limiting cases for small speeds – if Maxwell's equations and Lorentz transformation are both true, and Michelson-Morley experiment suggest they are, then there "all kinds of strange" effects to be expected at high velocities

Lorentz's place in history, besides his 1902 Nobel prize for theory of electrons: setting scene for

Poincaré (1904): "According to the principle of relativity, the laws of physical phenomena must be the same for a fixed observer as for an observer who has a uniform motion of translation relative to him, so that we have not, nor can we possibly have, any means of discerning whether or not we are carried along in such a motion."

so all electrodynamics experiments (e.g. along the lines of Michelson-Morley) are doomed to get a "zero" velocity/no effect result just as no mechanical experiment could detect motion either

Einstein's special theory of relativity, 1905

(deals only with inertial frames – therefore special) Einstein general theory of relativity deals with accelerated reference frames and gravity, 1915)

(when Einstein proposed both theories, people would hardly believe him, even M. Planck, Nobel - laureate himself, though by 1921 that this can't all be right - it is simply too weird
- when Einstein got his Nobel prize 1921 it was for the photoelectric effect - not for relativity

Einstein's Postulates:

The laws of all physics are the same in all inertial reference frames. That is, basic laws such as $\sum \vec{F} = \frac{d\vec{p}}{dt}$
have the same mathematical form, for all observers moving at constant velocity with respect to each other, this velocity may be either of the order of magnitude of our human experience or close to the speed of light.

Light propagates through empty space with a definitive speed c independent of the speed of the source or observer. That is, all observers will measure the same speed for c regardless of their frame of reference, there is a definitive speed limit for all objects and this is c, anything that has mass will be slower, anything without mass will rush around at this speed all the time

There is, hence, only one kind of relativity in nature, as electrodynamics are not consistent with the Galilean transformation but agree well with experiments, the Galilean transformation must be wrong,

As Newton's mechanics are consistent with the Galilean transformation, it can't be correct although it seems to agree with experiment well at the typically encountered speeds on the human experience scale, so a new kind of mechanics must be developed and was subsequently developed by Einstein!

The apparent times of the Lorentz transformation are the real times, there are in fact many different real times depending on the velocity of the observer, so time is not absolute

There is no ether to be discovered experimentally, there is not a preferred inertial system attached to the ether, so space is not absolute either

<u>Condition:</u> the new mechanics must contain the Newtonian mechanics as limiting cases for small speeds just as the Lorentz transformations contain the Galilean transformations as limiting case.

From this kind of reasoning, Einstein got same conclusions as from Lorentz transformation, (he derived and interpreted Lorentz's equations independently)

1: four dimensional spacetime

2: length contraction

3: time dilation

4: relativistic addition of velocities

5: relativistic Doppler effect, transverse Doppler effect

6. full blown modern relativity – not only length contraction and time dilation as Lorentz but, **relativistic dynamics**

7. rest mass, "relativistic" mass / "mass dilation"

8. relativistic momentum, force and acceleration

9. relativistic kinetic energy

10. rest energy, total energy and mass-energy relation relativistic energy and momentum, massless particles

the one nice thing about **<u>special relativity:</u>**

the mathematical scheme is just high school algebra and calculus, so if you are lost by the blunt disagreement between your everyday experience and modern relativity, you have to do the maths, they will guide you to the correct conclusions

before we pick up the story again with point 7, let's see how cleverly Einstein derived the Lorentz equations

directly translated from A. Einstein, “Uber die spezielle und allgemeine Relativitätstheorie” 1916

extra lines are added in the algebraic derivations in order to make it easier for you guys to comprehend what he is doing

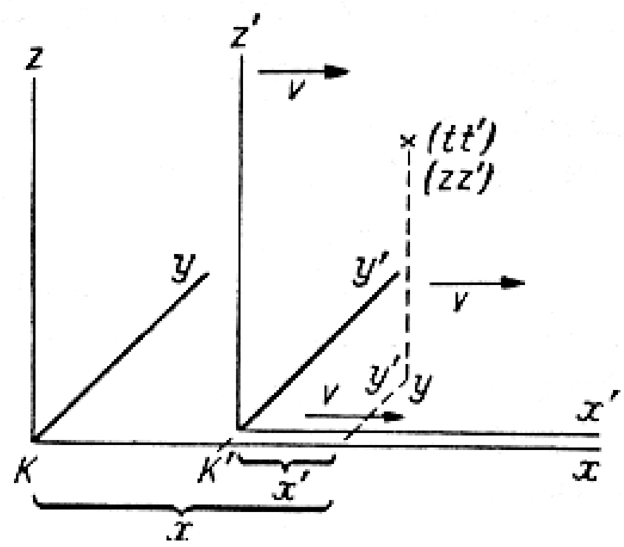


Abb. 2

… in figure 2 (Abb. 2) the x axes of both systems are coinciding all the time. We can, thus, dived the problem and first look only at events that are located on the x-axis …

a ray of light along the x-axis of K obeys

$$x = ct$$

$$x - ct = 0 \qquad (1)$$

the same ray of light along the x-axis of K’ obeys

$$x' - ct' = 0 \qquad (2)$$

as all space-time points have to obey (1) and (2) it must be true that

$$(x' - ct') = ? \, (x - ct) \qquad (3)$$

where ? is a constant

analogously we must have

$$x' + ct' = \mu \, (x + ct) \qquad (4)$$

where μ is also a constant

adding or subtracting (3) and (4), whereby we replace for simplicity the constants ? and μ by

$$a = \frac{l + m}{2}$$

$$b = \frac{l - m}{2}$$

we obtain the system

$$x' = a\,x - b\,c\,t \qquad (5a)$$
$$c\,t' = a\,c\,t - b\,x \qquad (5b)$$

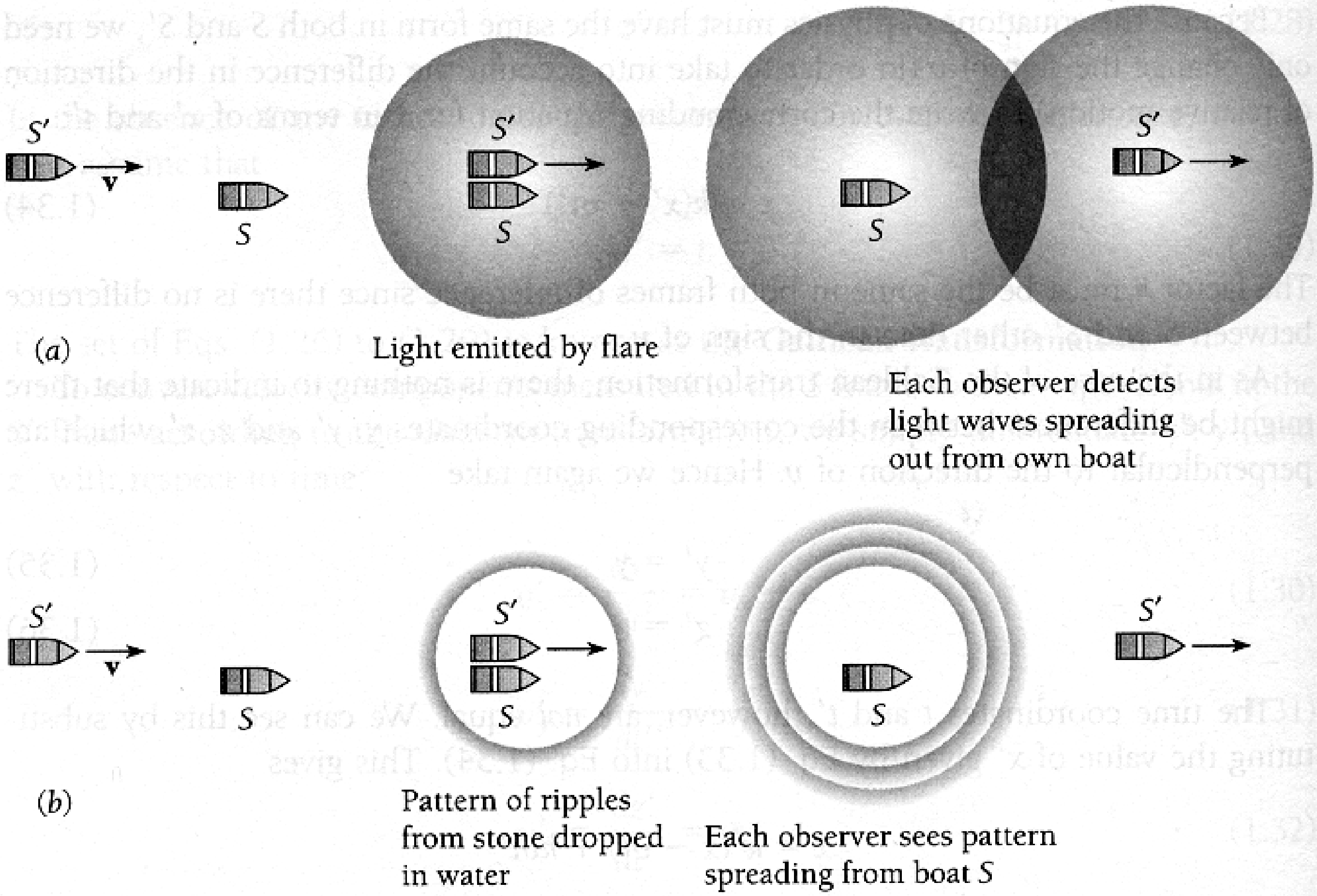


Figure 1.23 (a) Inertial frame S' is a boat moving at speed v in the $+x$ direction relative to another boat, which is the inertial frame S. When $t = t_0 = 0$, S' is next to S, and $x = x_0 = 0$. At this moment a flare is fired from one of the boats. An observer on boat S detects light waves spreading out at speed c from his boat. An observer on boat S' also detects light waves spreading out at speed c from her boat, even though S' is moving to the right relative to S. (b) If instead a stone were dropped in the water at $t = t_0 = 0$, the observers would find a pattern of ripples spreading out around S at different speeds relative to their boats. The difference between (a) and (b) is that water, in which the ripples move, is itself a frame of reference whereas space, in which light moves, is not.

7a: relativistic momentum

remember classic momentum, p_{class} = m v, conservation of momentum in collisions ?

Since impulse is function of mass has to be treated relativistic if v not much smaller than c

same (Lorentz) factor ? = $1 \Big/ \sqrt{1 - v^2 / c^2}$ applies

$p_{rela} = 1 \Big/ \sqrt{1 - v^2 / c^2} \; m_o \, v$

<u>say m_0 1 kg , v = 100 km h^{-1}</u>

p_{class} = 27.777778 [kg m s^{-1}] [N s^{-1}]

p_{rela} = 27.777778 + 1.5 10^{-14} [N s^{-1}]

<u>say m_0 1 kg , v = 0.1 c</u>

p_{class} = 2.99792458 10^7 [N s^{-1}]

p_{rela} ~ 3.0130275 10^7 [N s^{-1}], tested countless times in particle accelerators

7b: rest mass, "relativistic" mass / mass "dilation"

same (Lorentz) factor **as for time dilation ? =** $1 \Big/ \sqrt{1 - v^2 / c^2}$

mass "dilation": m = $1 \Big/ \sqrt{1 - v^2 / c^2}$ m_o $m > m_0$ for any $v > 0$

where m_0 is called rest mass **proper mass**, (as measured in a inertial reference frame at rest)

m > m_0 for any speed, a > 0

<u>say m_0 = 1 kg, v = 0.5 c</u> for the sake of it

m ˜ 30 kg ***or*** 5 kg ***or*** 1.15 kg ?

say <u>m_0 = 1 kg, v = 100 km h^{-1}</u> for the sake of getting an idea of the magnitude

m ˜ 1.1 kg ***or*** 1.005 kg ***or*** 1 kg + 5 pg (pico 10^{-12}) ?

does not get noticed in everyday experience

<u>"relativistic mass"</u> is not a **real effect** as relativistic time, older texts and formulae collections use it often

one can't replace "relativistic mass" in formulae in the same way as one could relativistic time, - in different formulae - there will be different factors to account for relativistic effects

check

http://www.physics.pdx.edu/~pmoeck/lectures/311/one-map.pdf

for the modern "one map-two clock approach"

actually this one formula above and its consequences within the scheme of high school algebra/calculus are all there is to modern relativity, this modification by Einstein makes Newtonian mechanics fully compatible with the Lorentz transformation,

e.g. if E = mc^2 is correct, what will be the formula for relativistic mass?

Start with body at rest, apply a force to the body, →→→ starts moving and gives it kinetic energy (since energy is increased mass is increased as well)

as long as force continues, energy and mass both increase

rate of change of energy $\frac{dE}{dt} = \vec{F} \bullet \vec{v}$ **(1)**

because $W = \int \vec{F} \bullet \vec{s}$ **, change in kinetic energy (dE) is equivalent to work done, and** $\vec{v} = \frac{d\vec{s}}{dt}$

now $\vec{F} = \frac{d(m\vec{v})}{dt}$ **(impulse = change of momentum** $\vec{F} \cdot dt = d\vec{p}$ **)**

inserting in (1) $\frac{d(mc^2)}{dt} = \vec{v} \bullet \frac{d(m\vec{v})}{dt}$ **(2)**

trick to resolve for m multiply both sides by 2m

$c^2(2m)\frac{dm}{dt} = 2mv\frac{d(mv)}{dt}$ **now** $2m\frac{dm}{dt} = \frac{d(m^2)}{dt}$ **and** $2mv\frac{d(mv)}{dt} = \frac{d(mv)^2}{dt}$

replacing

$$c^2 \frac{d(m^2)}{dt} = \frac{d(m^2v^2)}{dt}, \qquad \textbf{(3)}$$

if derivates of two quantities are equal, the quantities themselves differ by a constant C

$m^2c^2 = m^2v^2 + C$ / to define C we consider special case v = 0 and say m is mass at rest m_0, $m_0^2c^2 = 0 + C$

$m^2c^2 = m^2v^2 + m_0^2c^2$ / dived by c^2 and rearranging

$m^2(1-v^2/c^2) = m_0^2$

finally $$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

the one formula that is needed to derive relativistic mechanics which confirms to the Lorentz equations

this formula being consistent with $E = mc^2$ does not mean it is a real effect

check

http://www.physics.pdx.edu/~pmoeck/lectures/311/one-map.pdf

for the modern "one map-two clock approach" so how about

8. relativistic force and acceleration

Newton's second law?

$$\overline{F}_{class} = \frac{d\overline{p}}{dt} = \frac{m \cdot d\overline{v}}{dt} = m\,\overline{a}$$

$$\overline{a} = \frac{\overline{F}}{m}$$

$\overline{I} = \overline{F}\,dt = m\,d\overline{v}$ - impulse equals change of momentum

Since impulse, force and acceleration are functions of mass have to be treated relativistic if v not much smaller than c

If force, acceleration, velocity parallel x-axis

$$\overline{F} = \frac{1}{\left(\sqrt{1 - v^2/c^2}\right)^3}\, m\,\overline{a}$$

$$\mathbf{\overline{a} =} \sqrt{(1 - v^2/c^2)^3}\, \frac{\overline{F}}{m}$$

NOTE THAT THE FACTORS ARE NEITHER ? NOR ?

<u>Consequences:</u>

constant force no longer causes constant acceleration – Newton's 2nd law is to be treated relativistically in case v not <<c

as velocity increases, acceleration caused by given force constantly decreases,

if v close to c, a goes to zero, it’s impossible to accelerate an object with mass to c *(however hard one may try)*

9. relativistic kinetic energy

remember kinetic energy $KE_{class} = ½\ m\ v^2$

can’t be right if m = m(v)

$KE\ ?\ \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\ m_0\ ½\ v^2$ nature is not that simple, I am afraid

$$KE_{rela} = m\ c^2 - m_0\ c^2 = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1 \right) m_0\ c^2$$

factor is **neither ? nor ?**

LET’S compare KE_{class} and KE_{rela}

Say m_0 = 1 kg, v = 100 km h^{-1}

KE_{class} = 385.80247 [kg m^2 s^{-2}] [Nm], [Ws], [J]

KE_{rela} = 385.80247 + 2 10^{-12} [J]

<u>**Say m_0 = 1 kg, v = 0.1 c**</u>

KE_{class} = 4.4937757 10^2 [**TJ**]

KE_{rela} = 4.5277623 10^2 [**TJ**] [Terra Joule] **WHOW!!!** World "consumption" of electrical energy is only ˜ 1.5 10^4 TJ per hour

10: rest energy, total energy and mass-energy relation, massless particles

rearranging $KE_{rela} = m\,c^2 - m_0\,c^2$

$mc^2 = KE_{rela} + m_0c^2$ is total energy,
there is no potential energy around

with $m_0\,c^2$ rest energy, energy of body at rest

$mc^2 = E = ?\ m_0\,c^2$
most famous formulae of physics *at last*

E = total energy = kinetic energy + rest energy

so mass is in effect a form of energy, can be converted into other forms of energy

(is actually done in nuclear fission, fusion, any chemical reaction, e.g. in your stomach when you digest your food)

energy released in Hiroshima bomb equivalent to about 1 gram, ***that gram was mass loss/energy gain***

law of conservation of energy has to be expanded:
in all processes the "combination of energy and mass" are conserved

Conserving energy, energy/mass, and momentum for processes such as collisions, radioactive decay, chemical reactions
A+B ? C   A+B ? C+D   A ? B+C

sum of momenta of the initial objects is equal to sum of momenta of the final objects

sum of energies of initial objects is equal to sum of energies of final objects

there is no independent conservation law for mass, sum of masses of reactants is different from sum of masses of products,

there is no independent conservation law for kinetic energy, but total energy = kinetic energy + rest energy (m_0c^2) is conserved

products are lighter than reactants if reaction is releasing energy, exothermic, 1 g in Hiroshima bomb
products are heavier than reactants if reaction consumed energy, endothermic

----

hypothetical fission event, exothermic of course, so we will lose some mass, one nucleus (M_0) at rest splits into two identical fragments (m_0) that head of with equal momentum in different directions

$\mathbf{p_1}$ = ? $m_0\mathbf{v}$ $\mathbf{p_2}$ = -? $m_0\mathbf{v}$

M_0c^2 = 2? m_0c^2 rearranged ? = $M_0/2m_0$ > 1

and which taken together are lighter than the original nucleus

in a sense: energy and mass can be "created" from each other

how about $m_0 = 0$, a massless particle? Does not violate the $E = mc^2$ law, it is just all "pure kinetic" energy $KE_{rela} = m\,c^2 - m_0\,c^2$

all other objects in the university are a mixture of "impure" energy, i.e. mass, and "pure" energy

everything in the universe is in a sense energy

close to Greek philosopher Heraclitus, 535 – 475 BC: "pa?ta s??" – all things are in flux, everything is constantly moving and changing, only change is permanent, you can't step into the same river twice

Particles/objects with mass at rest have certain rest energy, if you divide rest energy by c^2 you get rest mass

Electron (at rest)	0.511 MeV
Proton (at rest)	938.29 MeV
Alpha (at rest)	3727.41 MeV
u = atomic mass unit = $^1/_{12}$ of mass ^{12}C = 1.6605402(10) 10^{-27} kg	931.49432(28) MeV ~ 931.5 MeV
Human being, 80 kg (at rest)	4.488 10^{31} MeV

if two particles of exactly equal velocity (with opposite signs), i.e. equal kinetic energy, and mass collide and stick together, they form a new particle
that particle will have zero kinetic energy, i.e. will be at rest, but it will be heavier [by summand 2m (? -1)] than the sum of the two original particles because the kinetic energy is converted into mass

combining rearranged relativistic total energy *and* relativistic momentum formulae

$$\left(\frac{E}{m_0c^2}\right)^2 = \frac{1}{1-{v^2}/{c^2}} \qquad \textbf{\textit{and}} \qquad \left(\frac{p}{m_0c}\right)^2 = \frac{{v^2}/{c^2}}{1-{v^2}/{c^2}}$$

(after subtracting the second from first and some rearranging)

$$E^2 = (m_0c^2)^2 + (pc)^2 \qquad \text{(alternatively } \frac{f}{c} = \frac{pc}{E}\text{)}$$

if there is a particle at rest, p = 0: $E = m_0 c^2$

but an particle may have energy and momentum even if it has no m_0, in other words no rest mass

such a particle, e.g. a photon, has E = pc = h**f** = hn where

h: Planck's constant: 6.626075 10^{-34} Js [Ws^2]
f: frequency = c/l and ? wavelength

and is doomed to rush around with speed of light

Nota Bene*: p cannot be calculated by p = m v since $m_0 = 0$*

Experimental verifications

Length contraction: Michelson-Morley and similar experiments, muon decay

muon is formed in atmosphere at a few km height

at rest in laboratory lifetime 2.2 µs, v = 0.99 c

in 2.2 µs lifetime (as measured in frame at rest with respect to the muon) it can only travel 0.99 c times 2.2 µs = 653 m

but it is observed on earth, what's happens is the 653 m are a contracted length, corresponding to a length of 653 m times ? ˜ 7.1, i.e. 4636 m for the observer at earth

Time dilation: muon decay - alternative view

as the muon is moving so fast, its life time, i.e. 2.2 µs, is dilated by a factor 7.1, i.e. is really 15.6 µs with respect to an observer on earth, in this time it will cover 4636 m in the observers frame

other experimental evidence: flying atomic clocks around the earth, application in global positioning system

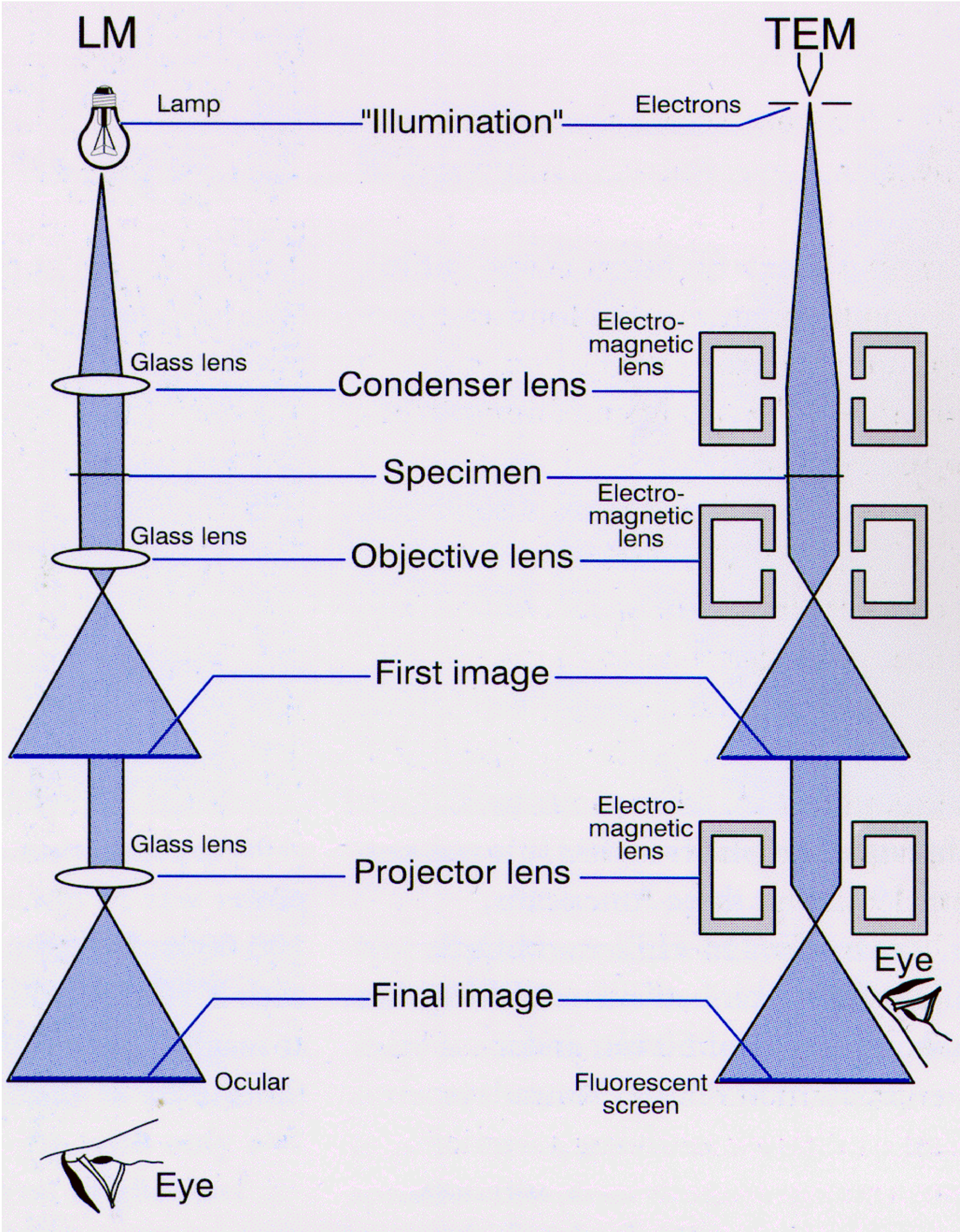
just as the muon "lives longer" due to its rapid movement, if a person moves around close to the speed of light, she/he will "live longer" than always

staying at rest, **but it won't extend that person's life span,** if he is destined to reach age 85, he will reach it anyhow,
her/his twin sibling which we assume to be destined to reach age 85 and one day as well will die earlier **relative to him as her**
(twin paradox, see your book)

Relativistic Doppler Effect: used by police to catch speeding cars

Relativistic mechanics: every single day at a particle accelerator, **every single day in an electron microscope**

so now be careful, electrons are small, i.e. light, can be accelerated to high velocities, if the velocities are high we have to make relativistic calculations



acceleration voltage 200 kV, what is the wavelength of the electron wave?

$p = m\,v = {}^{h}/_{?}$

both ***classis*** $m = m_0$ and “***relativistic***” $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$

----

200 keV = e U = potential energy (PE) = KE

$KE_{class} = \frac{1}{2} m_0 v^2$

$p_{class} = (2\, m_0\, KE_{class})^{0.5} = 3.4165\ 10^{-22}$ kgms^{-1}

?$_{class}$ = $^h/_{p\text{-}class}$ = 6.625 10^{-34} Nms / 3.4165 10^{-22} Ns ~1.94 pm

but $v_{class} = (2\, KE_{class}/m_0)^{0.5} = 2.6525\ 10^8\ ^m/_s$

so classical mechanics definitely not apply, all our results will be off

----

200 keV = e U = PE= $KE_{rela} = (\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1) \cdot m_0 c^2$ (1)

trick to simplify: $\beta = {}^v/_c$

so $p_{rela} = \frac{m_0 c}{\sqrt{1-b^2}} b$

$e U = (\frac{1}{\sqrt{1-b^2}} - 1) m_0 c^2$ resolving for ß yields

$\beta = \frac{\sqrt{(eU)^2 + 2 m_0 c^2 eU}}{eU + m_0 c^2}$ (2)

(2) in (1) $p_{rela} = \sqrt{2 m_0 eU \cdot (1 + \frac{eU}{2 m_0 c^2})} = 2.6417\ 10^{-22}$ kgms^{-1}

?$_{rela}$ = $^h/_{p\text{-}rela}$ = 6.625 10^{-34} Nms / 2.6417 10^{-22} Ns ~ 2.508 pm and $v_{rela} = 2.08452\ 10^{8}\ {}^{m}/_{s}$, i.e. 0.6953 c

that is what is observed in experiments at the microscope

so lets summarize deviations

$\Delta p = (p_{class} – p_{rela})\ 100\ \%\ /\ p_{rela} = 29.4\ \%$

$\Delta ? = (?_{class} – ?_{rela})\ 100\ \%\ /\ ?_{rela} = -\ 22.6\ \%$

$\Delta v = (v_{class} – v_{rela})\ 100\ \%\ /\ v_{rela} = 27.2\ \%$

New concept of time; present is not a moment, it is a time interval, it includes all the events from the past that we can in principle know about from different places and all the events from the future that we can in principle influence, e.g. if sun goes out, we won’t know for 8 minutes, if a far away star 100 000 light years away has died some 99 000 years ago we won’t know about this for a year

New concept about space: space itself is not a frame of reference

Questions a bright student may ask

Is Earth an inertial reference frame? **No only approximately**

Is Newton's 1st law valid on Earth? **Yes,** for small v

Is Newton's 2nd law valid on Earth? **Yes,** for small v

Is the rest of physics, electrodynamics,
quantum mechanics valid on Earth? **Yes**

Why is that? Einstein's theory of general relativity,

acceleration is equivalent to **gravitational field**,

Einstein himself (after some meditation on his desk at the patent office in Zurich 1907)

"… there occurred to me the happiest thought of my life, in the following form. The gravitational field has only a relative existence, because for an observer falling freely from the roof of a house there exist – at least in his immediate surroundings – no gravitational field. Indeed, if the observer drops some bodies then these remain relative to him in a state of rest or of uniform motion, independent of their particular chemical or physical nature. ***The matter independence of the acceleration of fall is a powerful argument for the fact that the relativity postulate has to be extended to coordinate systems which, relative to each other, are in non-uniform motion."***

<u>in other words:</u> relative motion does not matter in physics at all, be it linear with constant velocity or circularly with constant speed, or accelerated in whatever other manner

– so it does not matter that Earth is not an inertial reference frame, as a matter or fact, inertial reference frames do not exist in our universe - but they are quite helpful to explain basic physics to undergraduates

just as a physicist in a closed laboratory can not tell if he is in motion or at rest by all kind of experiments he is doing – if there is a ground for him to walk on, he can not tell by any of his experiments if he is in a gravitational field or being accelerated in the opposite direction

(good for us, so we are on Earth in what we consider a gravitational field and we do physics and our physics is applicable anywhere else in the universe)

general relativity is a generalization of special relativity in at least 3 senses:

it’s a new theory of gravity that replaces Newton’s theory of gravity, that explains the Perihelion of the orbit of Mercury, all planets have it mercury has it most as it is closest to the sun where space-time is more severely curved – observed effect can not be explained by Newton’s theory

light is bend by gravity, although it does not have mass, i.e. could not be subjected to a force of gravity in Newton’s sense, actually the light follows a straight line in a curved space a convex lens shaped galaxy will act like a focusing lens, we speak about "gravitational lens", there can be black holes, objects so massive that even light cannot escape from them, these holes suck everything around them in

time depends also on the strength of gravitational fields, the higher the gravitational potential, the slower the clock – two identical ultrahigh precision clocks (radioactive ?-ray emitters) 24 meters apart in a building at Harvard University, the clock in the basement is slower by about a second in a million years, **but the effect is significantly large that it must be corrected for in the global positioning system**

it's an important tool in cosmology, big bang, …. the frontier future "theory of everything ?"

besides the new effects due to relativity: any physical law (to be discovered in the future) or any sensible hypothesis must by invariant to the Lorentz transformations (special relativity) and invariant to another transformation of the so called Gaussian coordinates (general relativity) that describe bend space-time

so space-time is curved (non Euclidian) by masses, then there is no force of gravity needed at all, it is all geometry, everything moves in a straight line in curved space-time

a lower dimension analogue: consider two expeditions heading to the North pole, one starting in Siberia, one in the USA, both of them follow exactly a meridian and don't know about each other and don't feel any force exerted on them by the other team, but they get closer and closer together by geometry – that looks like the effect of a force but it is pure geometry

Space travel for human beings ???

<u>Say m_0 = 100 kg, v = 0.1 c</u>

KE_{rela} = 4.5277623 ^{16} J

WHOW!!! World "consumption" of electrical energy is only ˜ 1.5 10^{16} J per hour

since that is (at least) the energy needed to get 100 kg mass to 0.1 c speed,

space travel is hardly feasible as a pastime for the many,

but, very next solar system, a-centaury, is 4.6 light years away

i.e. to get here takes (at v = 0.1c) ~ 46 years earth time

within these ~ 46 years almost all of the guys who designed the hardware/software of the spaceship are retired – think about the state of the art of computers 46 years ago!!!

the astronauts must by then be pretty bored and have retirement on their mind as well as he has aged (46 – 0.5%) years

so what lessons are to be learned from special relativity?

Einstein's suggesting and numerous experimentalists confirming that Newton's laws and with them all of classical mechanics are only valid for small speeds should make us humble that all physicals laws may be wrong one way or the other,

even if we do not like strange ideas such as time dilation and length contraction, we have to deal with them because they describe very well what is seen in **experiments**

It took mankind so long after Newton to figure our his laws do not describe nature accurately because all of mankind's experience before say 1900 was for speeds that are not at all comparable to the speed of light, so mankind's experience was sort of incomplete

in quantum mechanics, there will be even weirder concepts because we are dealing with things that are much much smaller than ourselves and everything in our perceivable surrounding, so we can't perceive/imagine correctly what is "down there", we have to rely on mathematical theories that are in agreement with experiments – this is going to be the new truth there is

as intuition misleads us in regions where we do not have direct experience, e.g. the very fast, the very small, one has to stick to the mathematical scheme – see Heisenberg's opening statement

special relativity also explains why electricity and magnetism are one and the same thing

remember: moving electrical charge gives rise to magnetic force

i.e. an electric motor is based on a relativistic effect although speed of electrons is small compared to c i.e. is about 1 $^{mm}/_{s}$, but electric force between electron and proton in hydrogen atom is 10^{39} more powerful that gravitational force and there may be 10^{19} $^{electrons}/_{mm}$

example force between parallel current currying wires,
electric charge is (like c) relativistically invariant (a chare of magnitude q is the same in all reference frames)

facts: two parallel wires, electrically neutral for both current = zero, current ? zero

distance between moving charges undergoes length contraction by inverse Lorentz factor $\sqrt{1-v^2/c^2}$

currents parallel, wires attract each other (remember definition of Ampere) $F_{mag} = \dfrac{m_0 m_{rel} \vec{I}_1 \bullet \vec{I}_2 \cdot l}{2p \cdot r}$ but why???

In laboratory frame of reference, both wires are neutral, to us there appears to be an magnetic interaction between the currents, to each of the charges, there is only an electrical interaction adding up to

$F_{elec} = \dfrac{Q_1 \cdot Q_2}{4p \cdot e_0 r^2}$ **so special relativity is a great unifier !!!**

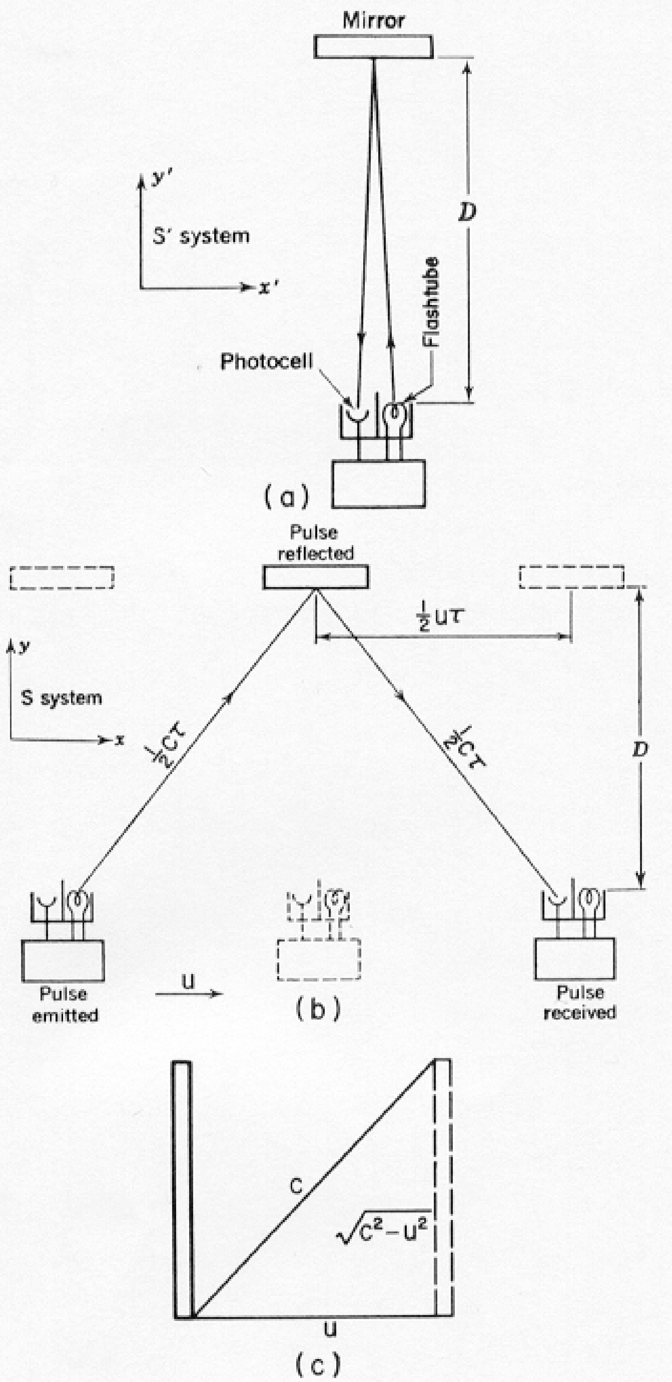


Fig. 15–3. (a) A "light clock" at rest in the S' system. (b) The same clock, moving through the S system. (c) Illustration of the diagonal path taken by the light beam in a moving "light clock."

Feynman's
light clock

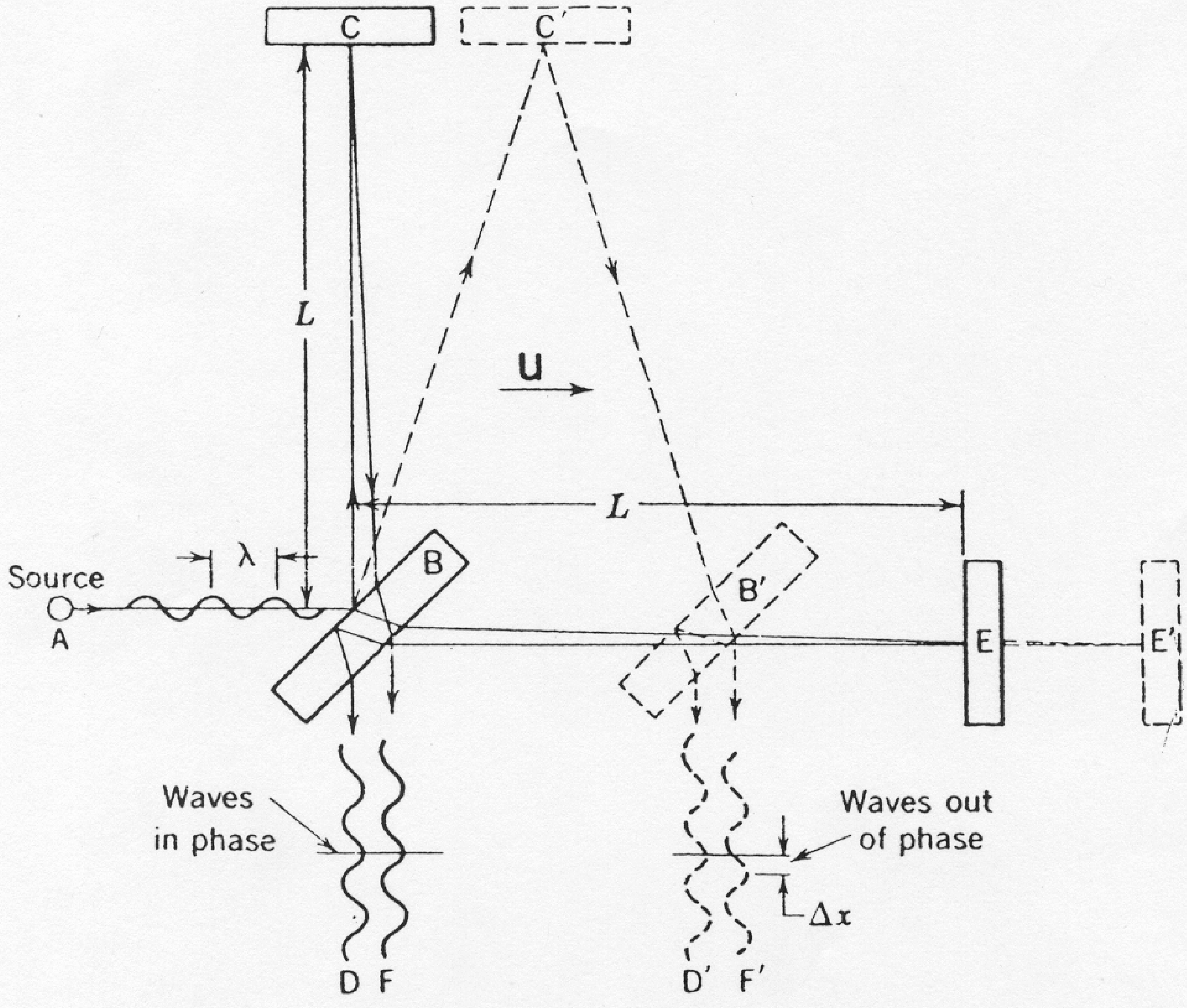
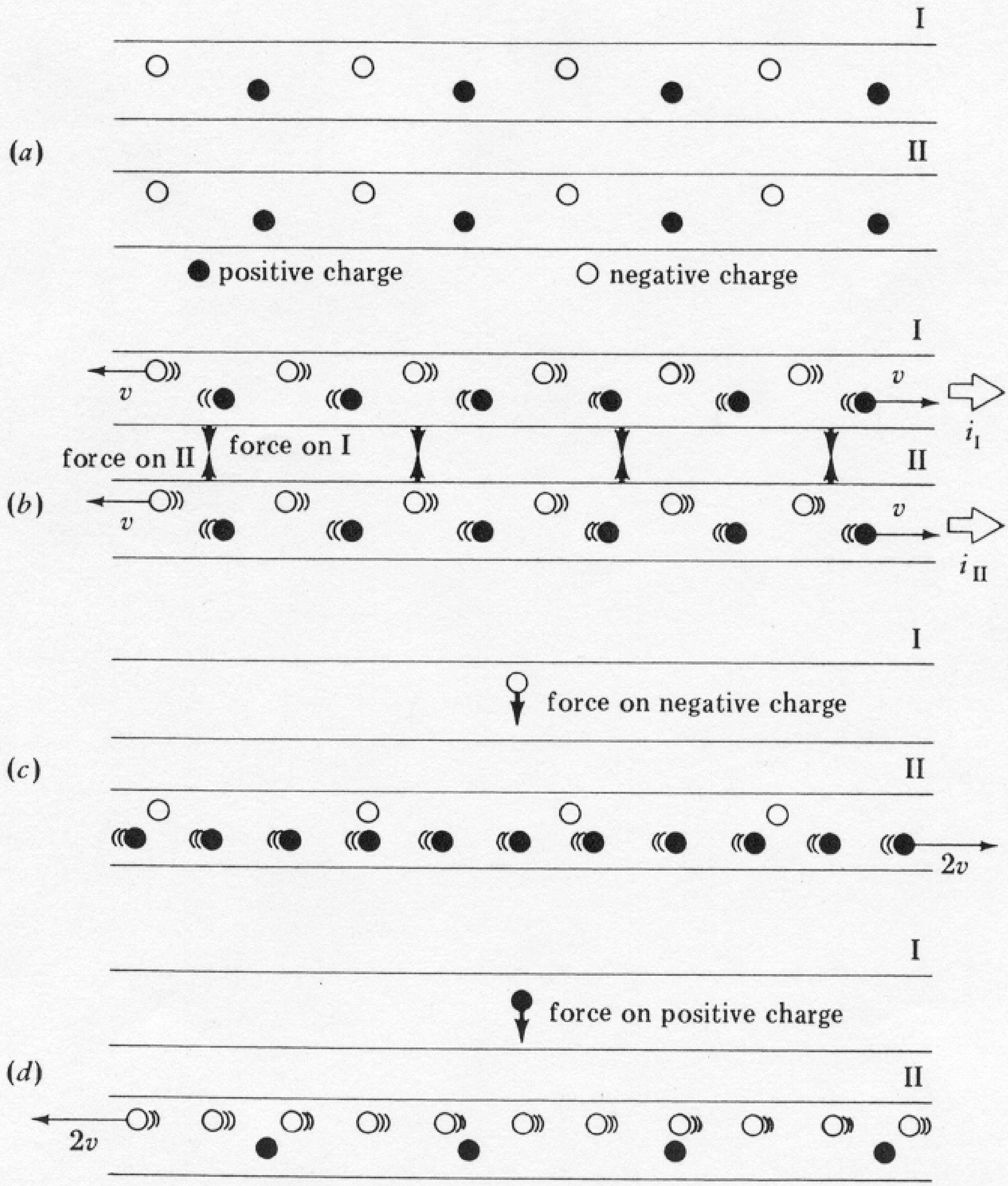


Fig. 15–2. Schematic diagram of the Michelson-Morley experiment.



How the magnetic attraction between parallel currents arises. (*a*) Idealized parallel conductors that contain equal numbers of positive and negative charges. (*b*) When the conductors carry currents, the spacing of their moving charges undergoes a relativistic contraction as seen from the laboratory. The conductors attract each other when i_I and i_{II} are in the same direction. (*c*) As seen by a negative charge in I, the negative charges in II are at rest whereas the positive charges are in motion. The contracted spacing of the latter leads to a net positive charge in II that attracts the negative charge in I. (*d*) As seen by a positive charge in I, the positive charges in II are at rest whereas the negative charges are in motion. The contracted spacing of the latter leads to a net negative charge on II that attracts the positive charge in I. The contracted spacings in *b*, *c*, and *d* are greatly exaggerated.

$\frac{u_{earth}}{c} = 10^{-4}$ in Michelson-Morley experiment

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Binominal expansion

$(1 \pm x)^n = 1 \pm nx \pm \frac{n(n-1)}{2!} \pm$ much smaller terms ….

If x << 1, $(1 \pm x)^n \sim 1 \pm nx$

$\sqrt{1+-x} \sim 1 \pm ½ x$

$1/\sqrt{1+-x} \sim 1 -+ ½ x$, so factor $\frac{1}{\sqrt{1-u^2/c^2}} \sim 1+ ½ ß$

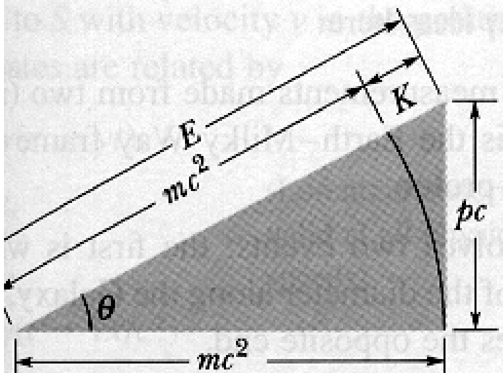

Fig. 38-15 A useful mnemonic device for remembering the relativistic relations among the total energy E, the rest energy or mass energy mc^2, the kinetic energy K, and the momentum p.

The right triangle of Fig. 38-15 can help you keep these useful relations in mind. You can also show that, in that triangle,

$$\sin\theta = \beta \quad \text{and} \quad \cos\theta = 1/\gamma. \tag{38-53}$$

With Eq. 38-52 we can see that the product pc must have the same unit as energy E; thus, we can express the unit of momentum p as an energy unit divided by c. In fact, momentum in fundamental particle physics is often reported in the units MeV/c or GeV/c.

✔CHECKPOINT 5: Are (a) the kinetic energy and (b) the total energy of a 1 GeV electron more than, less than, or equal to those of a 1 GeV proton?