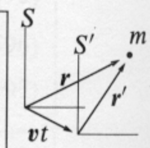
## 3.2 Frames of reference

### Galilean transformations

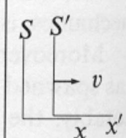
| | | | | |
|---|---|---|---|---|
| Time and position[a] | $\mathbf{r}=\mathbf{r}'+\mathbf{v}t$ | (3.1) | $\mathbf{r},\mathbf{r}'$ | position in frames $S$ and $S'$ |
| | $t=t'$ | (3.2) | $\mathbf{v}$ | velocity of $S'$ in $S$ |
| | | | $t,t'$ | time in $S$ and $S'$ |
| Velocity | $\mathbf{u}=\mathbf{u}'+\mathbf{v}$ | (3.3) | $\mathbf{u},\mathbf{u}'$ | velocity in frames $S$ and $S'$ |
| Momentum | $\mathbf{p}=\mathbf{p}'+m\mathbf{v}$ | (3.4) | $\mathbf{p},\mathbf{p}'$ | particle momentum in frames $S$ and $S'$ |
| | | | $m$ | particle mass |
| Angular momentum | $\mathbf{J}=\mathbf{J}'+m\mathbf{r}'\times\mathbf{v}+\mathbf{v}\times\mathbf{p}'t$ | (3.5) | $\mathbf{J},\mathbf{J}'$ | angular momentum in frames $S$ and $S'$ |
| Kinetic energy | $T=T'+m\mathbf{u}'\cdot\mathbf{v}+\frac{1}{2}mv^2$ | (3.6) | $T,T'$ | kinetic energy in frames $S$ and $S'$ |




[a]Frames coincide at $t=0$.

### Lorentz (spacetime) transformations[a]

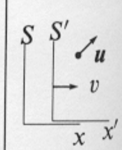
| | | | | |
|---|---|---|---|---|
| Lorentz factor | $\gamma=\left(1-\frac{v^2}{c^2}\right)^{-1/2}$ | (3.7) | $\gamma$ | Lorentz factor |
| | | | $v$ | velocity of $S'$ in $S$ |
| | | | $c$ | speed of light |
| Time and position | $x=\gamma(x'+vt');\quad x'=\gamma(x-vt)$ | (3.8) | $x,x'$ | $x$-position in frames $S$ and $S'$ (similarly for $y$ and $z$) |
| | $y=y';\quad y'=y$ | (3.9) | | |
| | $z=z';\quad z'=z$ | (3.10) | | |
| | $t=\gamma\left(t'+\frac{v}{c^2}x'\right);\quad t'=\gamma\left(t-\frac{v}{c^2}x\right)$ | (3.11) | $t,t'$ | time in frames $S$ and $S'$ |
| Differential four-vector[b] | $\mathrm{d}\mathbf{X}=(c\,\mathrm{d}t,-\mathrm{d}x,-\mathrm{d}y,-\mathrm{d}z)$ | (3.12) | $\mathbf{X}$ | spacetime four-vector |




[a]For frames $S$ and $S'$ coincident at $t=0$ in relative motion along $x$. See page 141 for the transformations of electromagnetic quantities.
[b]Covariant components, using the $(1,-1,-1,-1)$ signature.

### Velocity transformations[a]

| | | | | |
|---|---|---|---|---|
| Velocity | | | $\gamma$ | Lorentz factor $=[1-(v/c)^2]^{-1/2}$ |
| | $u_x=\dfrac{u'_x+v}{1+u'_xv/c^2};\quad u'_x=\dfrac{u_x-v}{1-u_xv/c^2}$ | (3.13) | $v$ | velocity of $S'$ in $S$ |
| | $u_y=\dfrac{u'_y}{\gamma(1+u'_xv/c^2)};\quad u'_y=\dfrac{u_y}{\gamma(1-u_xv/c^2)}$ | (3.14) | $c$ | speed of light |
| | $u_z=\dfrac{u'_z}{\gamma(1+u'_xv/c^2)};\quad u'_z=\dfrac{u_z}{\gamma(1-u_xv/c^2)}$ | (3.15) | $u_i,u'_i$ | particle velocity components in frames $S$ and $S'$ |




[a]For frames $S$ and $S'$ coincident at $t=0$ in relative motion along $x$.