

Phys 311, 1st third term paper

your name

36 points in total

your SS#

1. What does the Michelson-Morley experiment prove directly:

that there is time dilation parallel and perpendicular to the direction of motion ✓

that there must be mass dilation

that one can't transform mass into potential energy

that there is length contraction perpendicular to the direction of motion

that the Galilean transformations go over into the Lorentz transformations for small speeds

that one prediction of Maxwell's electrodynamics, i.e. $c = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$ is correct for all observers ✓

that the rest of Maxwell's electrodynamics must necessarily be incorrect

that all of Newton's mechanics must necessarily be correct as it is invariant with respect to a Galilean transformation

that there cannot be any ether

that there is no need for an ether ✓

inertial reference frames do not exist in nature

non of the above as the experiment proved:

3 points

be careful there is more than one correct answer, marking wrong answers in addition to correct answers leads to the subtraction of points, so ponder carefully, the key is here “**directly**”

2. If it was not intended as a proof of principle, what was the Michelson-Morley experiment actually set up to measure originally? 4 points

from the viewpoint that special relativity is correct, the experiment is proof that the relative motion of the earth with respect to the frame of reference in which the ether was assumed to be at rest 2 points

it was indented to prove the principle that one can prove with electrodynamics that the earth is moving relative to something else, 2 points

3. Do the Lorentz transformations imply that there is 4 dimensional space time? If so make a qualitative statement on the basis of one of these equations? 1 point

yes, simply by having space coordinates and “time coordinates” mixed in the transformations

4. Are the rest mass and the electric charge of any object invariant with respect to a Lorentz transformation, i.e. the same for all observers regardless of the state of the motion? 3 points

yes, rest energy is the same for all observers, rest mass is just rest energy divided by c^2 , since c is a constant in all frames of references, rest mass must be as well 1 point

electric charge is a fundamental concept, if it is moving with respect to us, we see magnetism if it is at rest with respect to us, we see electrostatics, both magnetism and electrostatics are two “different sides of the same coin” 2 points

5. Prove algebraically that simultaneous events in one frame of reference are not simultaneous in another frame of reference. 4 points

Hint. Lorentz transformation, $\Delta t = \Delta t'$?

$$t' = \frac{1}{\sqrt{1 - v^2/c^2}} \left(t - \frac{vx}{c^2} \right) \text{ to start with}$$

so it must be true that

$$t_1' = \frac{1}{\sqrt{1 - v^2/c^2}} \left(t_1 - \frac{vx_1}{c^2} \right) \text{ and}$$

$$t_2' = \frac{1}{\sqrt{1 - v^2/c^2}} \left(t_2 - \frac{vx_2}{c^2} \right) \text{ and also if I define a time interval in the frame at rest } \Delta t = t_2 - t_1$$

$$t_2' - t_1' = \frac{1}{\sqrt{1 - v^2/c^2}} \left((t_2 - t_1) - \frac{v(x_2 - x_1)}{c^2} \right)$$

now if my time interval $\Delta t = 0$ because two things happened simultaneous to me, i.e. at the same time, so that $t_1 = t_2$ if follows

$$t_2' - t_1' = \Delta t' = \frac{1}{\sqrt{1 - v^2/c^2}} ([0] - \frac{vx}{c^2}) \quad \text{is not zero ! so in the frame that is moving with respect to me the}$$

interval is not zero, so it must be true $t_1' \neq t_2'$, so the same two events as observed from the moving frame are not simultaneously

6a. Can a physical theory that requires the simultaneity of two events *at different locations* be valid?

No, all physical theories have to be the same regardless of the frame of reference of the observer, as different observers don't agree on simultaneous events, no theory can be valid that required simultaneity at two different locations

see Beiser, p. 45 **3 points**

6b. Can a physical theory that requires the simultaneity of two events *at the same locations* be valid? Explain your reasoning clearly

yes, now the situation is different so $x_1 = x_2$ and $t_1 = t_2$ are required, so this makes $\Delta x = 0$ and $\Delta t = 0$

putting this in the Lorentz transformation

$$t_2' - t_1' = \Delta t' = \frac{1}{\sqrt{1 - v^2/c^2}} ([0] - \frac{v(x_2 - x_1)}{c^2}) = \frac{1}{\sqrt{1 - v^2/c^2}} ([0] - \frac{v(0)}{c^2}) = 0 - 0 = 0$$

so that a theory will be OK, see Beiser p. 45 **3 points**

7. Prove algebraically that for speeds small compared to the speed of light, the relativistic formula for kinetic energy reduces to (4 points)

$$KE \sim \frac{1}{2} m_0 v^2 = \frac{p^2}{2m_0} \quad \text{Hint: Binomial expansion}$$

Beiser p. 29

$$KE_{\text{rela}} = m c^2 - m_0 c^2 = \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) m_0 c^2$$

for small velocities we can use binomial expansion

$$\frac{1}{\sqrt{1-v^2/c^2}} \sim 1 + \frac{1}{2} \frac{v^2}{c^2}$$

so we can write $KE \sim (1 + \frac{1}{2} \frac{v^2}{c^2}) m_0 c^2 - m_0 c^2 \sim \frac{1}{2} m_0 v^2$

$$p = m_0 v \text{ so } p^2 = m_0 v^2$$

$$p^2 = m_0 v^2 \text{ divided by } 2m \quad \frac{p^2}{2m} = \frac{m_0 v^2}{2m} = \frac{m_0 v^2}{2} = KE_{\text{classic}}$$

8. What speed measurement will well trained husband and wife physicists agree upon (if they can't agree about much else)? 2 points

speed of light regardless of their relative motion with respect to each other

9. A rocket traveling at speed 0.8 c relative to the earth shoots forward a beam of particles with speed 0.9 c relative to the rocket. (9a) What is the particle's speed relative to the rocket? 3 points

(9b) What is the particles speed relative to the earth? 3 points

(9c) The same rocket shoots forward a signal, i.e. pulse of light, with speed c relative to the rocket. What is the signal's speed relative to the earth? 3 points

Taylor, Zafiratos, Dubson, text, p. 33-34

A rocket traveling at speed $0.8c$ relative to the earth shoots forward a beam of particles with speed $0.9c$ relative to the rocket. What is the particles' speed relative to the earth?

Let S be the rest frame of the earth and S' that of the rocket, with x and x' axes both aligned along the rocket's velocity. The relative speed of the two frames is $v = 0.8c$. We are given that the particles are traveling along the x' axis with speed $u' = 0.9c$ (relative to S') and we want to find their speed u relative to S . The classical answer is, of course, that $u = u' + v = 1.7c$, that is, because the two velocities are collinear, u' and v simply add in classical physics.

The correct relativistic answer is given by the inverse of (1.43) (from which we omit the subscripts x , since all velocities are along the x axis).

$$\begin{aligned} u &= \frac{u' + v}{1 + u'v/c^2} \\ &= \frac{0.9c + 0.8c}{1 + (0.9 \times 0.8)} = \frac{1.7}{1.72} c \approx 0.99c \end{aligned} \quad (1.45)$$

The striking feature of this answer is that when we “add” $u' = 0.9c$ to $v = 0.8c$ relativistically, we get an answer that is less than c . In fact, it is fairly easy to show that for any value of u' that is less than c , the speed u is also less than c (see Problem 1.47); that is, a particle whose speed is less than c in one frame has speed less than c in any other frame.

Example 1.8

The rocket of Example 1.7 shoots forward a signal (for example, a pulse of light) with speed c relative to the rocket. What is the signal's speed relative to the earth?

In this case $u' = c$. Thus according to (1.45)

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{c + v}{1 + v/c} = c \quad (1.46)$$

That is, anything that travels at the speed of light in one frame does the same as observed from any other frame. (We have proved this here only for the case that u is in the same direction as v . However, the result is true for any direction; for another example, see Problem 1.48.) We can paraphrase this to say that the speed of light is invariant as we pass from one inertial frame to another. This is, of course, just the second postulate of relativity, which led us to the Lorentz transformation in the first place.