Quadratic Growth Curve Example

In these growth curve examples, I do not allow the quadratic term to vary over time. The reason for this is that a curvilinear effect with three time points is likely to lead to convergence problems. The number of time points (or n_j in the group nested case) creates limitations for the number of random effects that can be included in the model, and the model may either be theoretically not identified (non-positive degrees of freedom) or there may be estimation problems. If one tries to include too many random effects, HLM will give an explicit warning message about degrees of freedom. In SPSS, the output for the significance tests for one or more random slope may be suppressed. I centered the time variable in this example which has the advantage of reducing collinearity between the linear and quadratic time variables.[1] Researchers may or may not want to center the time codes. The disadvantage is that the intercept is interpreted as the value of the dependent variable at the middle time point rather than at baseline.

SPSS

I use the GENLINMIXED to get robust standard errors.

```
recode time (0=-1) (1=0) (2=1) into ctime.
compute ctimesq=ctime*ctime.

*need to convert time values to numeric scale.
variable level time ctime ctimesq (scale).

GENLINMIXED
/DATA_STRUCTURE SUBJECTS=id
/FIELDS TARGET= depress
/TARGET_OPTIONS DISTRIBUTION=NORMAL LINK=IDENTITY
/BUILD_OPTIONS DF_METHOD=SATTERTHWAITE COVB=ROBUST
/FIXED  EFFECTS= ctime ctimesq USE_INTERCEPT=TRUE
/RANDOM EFFECTS=ctime  USE_INTERCEPT=TRUE SUBJECTS=id
COVARIANCE_TYPE=UNSTRUCTURED.


*here is the syntax for the standard mixed command
*MIXED depress WITH ctime ctimesq
 /METHOD = REML
 /PRINT = SOLUTION TESTCOV HISTORY
 /FIXED = ctime ctimesq | SSTYPE(3)
 /RANDOM = INTERCEPT ctime | SUBJECT(rid) COVTYPE(UN).
```

Model Summary

Target		depress Summed CESD score
Probability Distribution		Normal
Link Function		Identity
Information Criterion	Akaike Corrected	4908.356
	Bayesian	4926.497

Information criteria are based on the -2 log likelihood (4900.299) and are used to compare models. Models with smaller information criterion values fit better.

Coefficients of Determination

Pseudo-R Square Measures	Marginal	.030
	Conditional	.603

Fixed Coefficients [a]

					95% Confidence Interval	
Model Term	Coefficient	Std. Error	t	Sig.	Lower	Upper
Intercept	11.733	.6342	18.502	<.001	10.486	12.980
ctime	-1.911	.2883	-6.628	<.001	-2.479	-1.343
ctimesq	-.951	.4747	-2.004	.046	-1.888	-.015

Probability distribution: Normal
Link function: Identity
a. Target: Summed CESD score

[1] Centering does not reduce collinearity except in cases where one variable in the model is computed as a product of one or more variables also used in the model.

Random Effect

Random Effect Covariance	Estimate	Std. Error	Z	Sig.	95% Confidence Interval	
					Lower	Upper
UN (1,1)	49.798	5.807	8.575	<.001	39.623	62.587
UN (2,1)	-2.884	2.280	-1.265	.206	-7.352	1.585
UN (2,2)	1.883	2.440	.772	.440	.149	23.861

Covariance Structure: Unstructured
Subject Specification: id

R

If a random effect for the quadratic variable (`ctime2`) is attempted, R gives a warning message that the model is "nearly unidentified." I did not include the random effect for the quadratic effect in this example.[2]

```
> #quadratic growth curve model
> #create centered time variable and squared-centered time variable
> mydata$ctime <- mydata$time - 1
> mydata$ctime2 <- mydata$ctime*mydata$ctime
>
> model <- lmer(depress ~ ctime + ctime2 + (ctime|rid), data = mydata,REML=TRUE)
> summary(model)
Linear mixed model fit by REML. t-tests use Satterthwaite's method ['lmerModLmerTest']
Formula: depress ~ ctime + ctime2 + (ctime | rid)
   Data: mydata

REML criterion at convergence: 4900.3

Scaled residuals:
    Min      1Q  Median      3Q     Max 
-2.9569 -0.4749 -0.1224  0.3117  4.9577 

Random effects:
 Groups   Name        Variance Std.Dev. Corr 
 rid      (Intercept) 49.798   7.057         
          ctime        1.883   1.372    -0.30
 Residual             35.310   5.942         
Number of obs: 702, groups:  rid, 234

Fixed effects:
            Estimate Std. Error       df t value             Pr(>|t|)    
(Intercept)  11.7332     0.6031 388.4446  19.455 < 0.0000000000000002 ***
ctime        -1.9112     0.2890 232.9958  -6.614       0.000000000253 ***
ctime2       -0.9515     0.4758 232.9937  -2.000               0.0467 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Correlation of Fixed Effects:
       (Intr) ctime 
ctime  -0.071       
ctime2 -0.526  0.000

> VarCorr(model)
 Groups   Name        Std.Dev. Corr  
 rid      (Intercept) 7.0568         
          ctime       1.3722   -0.298
 Residual             5.9422         
>
> library(MLMusingR)
> robust_mixed(model1)

Standard error type = CR2 
Degrees of freedom = Satterthwaite 

            Estimate  mb.se robust.se t.stat  df               Pr(>t)    
(Intercept)   14.827  0.625     0.678 21.870 195 <0.0000000000000002 ***
time          -2.042  0.285     0.285 -7.162 232 <0.0000000000000002 ***
health        -2.638  0.367     0.410 -6.426 148 <0.0000000000000002 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
>
> library(lmerTest)
> rand(model)
ANOVA-like table for random-effects: Single term deletions

Model:
depress ~ ctime + ctime2 + (ctime | rid)
                       npar  logLik    AIC    LRT Df Pr(>Chisq)
<none>                    7 -2450.2 4914.3
```

[2] If convergence problems occur, you may get a warning message about confidence intervals for the covariance (e.g.,.sig02 are from -1 to 1) or for the variance of the time slope (e.g., .sig03 with the lower CI equal to 0) that suggests untrustworthy results. Running the function `warnings()` provides additional details, which refer to problems with the "model matrix" (zeta; Bates, D.M., 2010, lme4: Mixed-effects modeling with R) and estimating the confidence intervals in the iterative process.

```
ctime in (ctime | rid)    5 -2451.3 4912.5 2.2141  2     0.3305
> confint(model, oldNames=TRUE)
Computing profile confidence intervals ...
                2.5 %      97.5 %
.sig01       6.279326  7.89507530
.sig02      -1.000000  1.00000000
.sig03       0.000000  2.60464451
.sigma       5.430323  6.45309743
(Intercept) 10.550779 12.91559542
ctime       -2.478688 -1.34376542
ctime2      -1.885803 -0.01718869
```

HLM

Summary of the model specified

Level-1 Model

$DEPRESS_{ij} = \beta_{0j} + \beta_{1j}*(CTIME_{ij}) + \beta_{2j}*(CTIMESQ_{ij}) + r_{ij}$

Level-2 Model

$\beta_{0j} = \gamma_{00} + u_{0j}$
$\beta_{1j} = \gamma_{10} + u_{1j}$
$\beta_{2j} = \gamma_{20}$

Mixed Model

$DEPRESS_{ij} = \gamma_{00}$
$+ \gamma_{10}*CTIME_{ij}$
$+ \gamma_{20}*CTIMESQ_{ij} + u_{0j} + u_{1j}*CTIME_{ij} + r_{ij}$

Final Results - Iteration 4
Iterations stopped due to small change in likelihood function

$\sigma^2 = 35.23966$

τ

INTRCPT1,β_0	49.82527	-2.88216
CTIME,β_1	-2.88216	1.95763

τ (as correlations)

INTRCPT1,β_0	1.000	-0.292
CTIME,β_1	-0.292	1.000

Random level-1 coefficient	Reliability estimate
INTRCPT1,β_0	0.809
CTIME,β_1	0.100

The value of the log-likelihood function at iteration 4 = -2.449231E+003

Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	11.733188	0.634164	18.502	233	<0.001
For CTIME slope, β_1					
INTRCPT2, γ_{10}	-1.911226	0.288339	-6.628	233	<0.001
For CTIMESQ slope, β_2					
INTRCPT2, γ_{20}	-0.951496	0.474741	-2.004	233	0.046

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	*d.f.*	χ^2	*p*-value
INTRCPT1, u_0	7.05870	49.82527	233	1221.24362	<0.001
CTIME slope, u_1	1.39915	1.95763	233	258.36673	0.122
level-1, r	5.93630	35.23966			

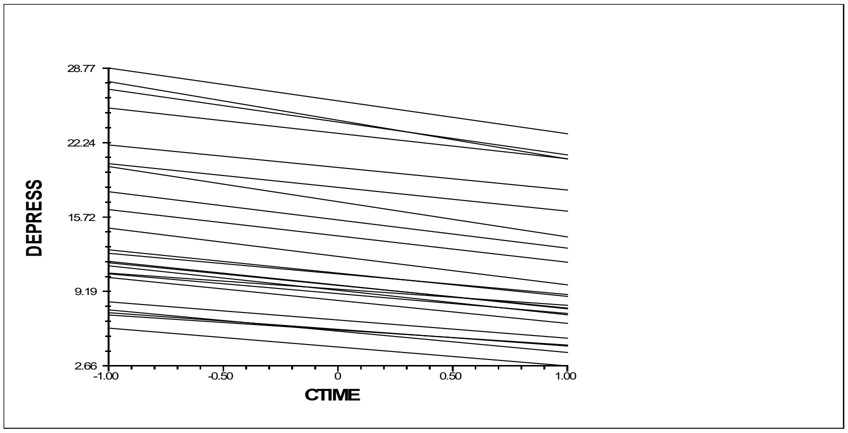
Statistics for current covariance components model

Deviance = 4898.461714
Number of estimated parameters = 4

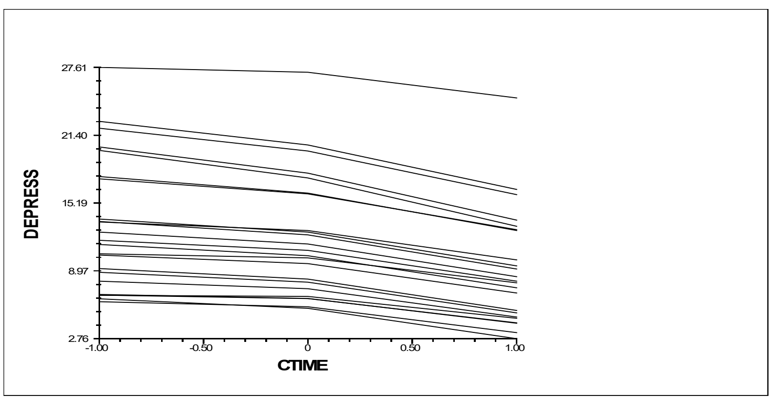
A growth curve model was tested to investigate whether there was a nonlinear change in depression over time. Both linear and quadratic components were included in the model. The time variable was centered at the mid-point of the study to reduce collinearity between the linear and quadratic components. Because of the limited number of parameters that could be estimated with three time points, the linear slope was allowed to vary across individuals, but the quadratic slope was not allowed to vary. Robust standard errors were used to account for non-normality (Liang & Zeger, 1986). At the midpoint of the study, the mean depression score was 11.73. There was a significant linear decline in depression over the three waves, γ_{10} = -1.91, SE = .29, $p < .001$, suggesting a decrease in depression scores of nearly two points every six months. The quadratic effect was also significant, γ_{20} = -.95, SE = .47, p < 05, and, based on the examination of Figure 1, suggested an accelerated decline in depression over time. The linear slope did not vary significant across individuals, τ^2_1 = 1.96, χ^2 = 258.37, p = .122, suggesting a similar rate of linear decline among participants.

Note: report the Wald z-values for the variance (with halved p-values) and covariance effects in SPSS and R or use the profile likelihood confidence intervals in R.

Individual linear slopes



Individual quadratic curves



Average quadratic curve (Figure 1).