Multilevel Regression Equations with a Level-2 Predictor

Until this point, we have only included a level-1 predictor, but we can add a level-2 predictor, called Z_j. The level-2 predictor does not the i subscript, because it only varies by group, not by individual. Only the subscript, j, is used to represent the index for groups. I am using Snijders and Bosker (2012) notation in this handout.

Level 1 (within-group, micro)
The first level (within-group) MLR regression equation is the following, which is the same as what we have used thus far.

$$Y_{ij} = \beta_{0j} + \beta_{1j} X_{ij} + R_{ij}$$

The second-level equation involves a prediction of the intercept and slope from the first equation. The second-level equation is structured in exactly the same way as any regression equation, although the symbols used are different. There are also two equations, one that predicts the intercepts from each group, and one that predicts the slope for each group.

Level 2 (between-group, macro)
It may be desirable to just add Z_j to the first equation below, with β_{0j} as the outcome. The level-2 variable does not need to be included in both equations, and including it to predict β_{1j} implies an important new effect in the model—the *cross-level interaction*—that becomes clearer with the single multilevel equation.

$$\beta_{0j} = \gamma_{00} + \gamma_{01} Z_j + U_{0j}$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11} Z_j + U_{1j}$$

where γ_{00} and γ_{10} are intercepts, γ_{01} and γ_{11} are slopes, and U_{0j} and U_{1j} are errors or residuals.

Single multilevel equation
The level-1 and level-2 equations can be combined into a single equation by plugging the level-2 equations into the level-1 equations. I illustrate with both a level-1 and level-2 predictor.

$$Y_{ij} = (\gamma_{00} + \gamma_{01} Z_j + U_{0j}) + (\gamma_{10} + \gamma_{11} Z_j + U_{1j}) X_{ij} + R_{ij}$$

Then rearranging the terms, we have:

$$Y_{ij} = \gamma_{00} + \gamma_{01} Z_j + \gamma_{10} X_{ij} + \gamma_{11} Z_j X_{ij} + U_{0j} + U_{1j} X_{ij} + R_{ij}$$

Notice that the last three terms of the equation, $U_{0j} + U_{1j} X_{ij} + R_{ij}$, if considered as one error term, transform this more complicated equation into a regular regression equation:

$$Y_{ij} = \gamma_{00} + \gamma_{01} Z_j + \gamma_{10} X_{ij} + \gamma_{11} Z_j X_{ij} + error$$

The only special term remaining is $\gamma_{11} Z_j X_{ij}$, which is an interaction between X and Z.

This gets pretty complex quickly as you can see.

References
Snijders, T.A.B., & Bosker, R.J. (2012). *Multilevel analysis: An introduction to basic and advanced multilevel modeling (2nd Edition)*. London: Sage.