

Examples Illustrating Kenward-Roger Degrees of Freedom in SPSS and R

I use the HSB example with SES as a predictor with a random slope to illustrate obtaining the Kenward-Roger degrees of freedom approach (see the previous handout “Significance Testing in Multilevel Regression” for more details and references) for the fixed effects test in SPSS and R.¹ This model is the same as the model tested in handout “Random Slopes Example: SPSS, R, and HLM.” The Kenward-Roger degrees of freedom testing approach for fixed effects appears to perform better for small sample sizes (e.g., when $N < 50$ and especially when $N < 25$; McNeish, 2017). So, I recommend use of these settings for data with a smaller number of groups and I recommend using software that uses a t-test (or use of a t-table) when the number of groups is less than about 120. For this example, we should not expect to see any differences in statistical conclusions from our prior analyses because there were 160 schools, but it will be useful to see the code and output for Kenward-Roger approach illustrated anyway.

SPSS

```
MIXED mathach WITH ses
  /METHOD = REML
  /PRINT = SOLUTION TESTCOV
  /FIXED = ses | SSTYPE(3)
  /RANDOM = INTERCEPT ses | SUBJECT(schoolid) COVTYPE(UN)
  /CRITERIA=DFMETHOD(KENWARDROGER) .
```

Estimates of Fixed Effects^a

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Intercept	12.665	.190	145.568	66.661	<.001	12.290	13.041
ses	2.394	.118	157.544	20.223	<.001	2.160	2.628

a. Dependent Variable: mathach.

Notice that the df for the `ses` variable is 157.544. That is the same value obtained in the prior analysis. That is because SPSS is using Satterthwaite degrees of freedom by default. The Kenward-Roger degrees of freedom will be the same but will have a different standard error. In the previous analysis, the standard error was .118 and that is the same here. The standard error adjustment made by the Kenward-Roger method is quite small in this case, as we would expect given the N . And because of rounding the difference can be only seen in the t-value, which is slightly lower here, $t = 20.223$, compared the prior value of 20.226 in the earlier analysis. Very little practical difference in this case!

R

The Kenward-Roger degrees of freedom can be obtained with the `nlme` package in conjunction with the `lmerTest` package, so I load that first. I omit some of the output.

```
> library(lmerTest)
> #get kenward-roger df test for fixed effects
> model <- lmer (mathach ~ ses +(ses|schoolid), data=mydata)
> summary(model, ddf=c("kenward-roger"))
```

```
Fixed effects:
      Estimate Std. Error      df t value      Pr(>|t|)
(Intercept) 12.6650    0.1900 158.4895   66.66 <0.0000000000000002 ***
ses          2.3938    0.1184 147.6675   20.22 <0.0000000000000002 ***
```

In the previous analysis, the default fixed effects tests with `nlme` used $df = 7024$ with a t-test and had a slightly higher t-value, 20.26576, than those obtained here with the Kenward-Roger. Again, we should expect very little difference in the test results with 160 schools. Notice that the KR method with `nlme` seems to arrive at a different value from that obtained with SPSS (147.67 vs. 157.54).

¹ There is no Satterthwaite or Kenward-Roger adjustments in the HLM package, although the t-tests used in HLM should perform better than the default tests of fixed effects in SPSS and R `nlme` with default degrees of freedom when there is a small number of groups. Note that the `lme4` package uses Satterthwaite degrees of freedom by default.