Generalized Multilevel Regression Example for a Binary Outcome

HLM

To estimate a multilevel logistic model, go to *Basic Settings* and choose *Bernoulli*. I used adaptive quadrature here (under Other Settings → Estimation Settings) but the sixth-order Laplace estimates (labeled Laplace-2 in the output) would also be acceptable. You must indicate the maximum number of iterations (100 was acceptable here) and the number of quadrature points. I used 7 for this example.[1] The first section of the HLM output gives the PQL estimates, which I omit here given their known bias.[2] To compare with the R results I did not include any random slopes.

Summary of the model specified

Step 2 model

Level-1 Model

$\text{Prob}(GUN_{ij}=1|\beta_j) = \phi_{ij}$

$\log[\phi_{ij}/(1 - \phi_{ij})] = \eta_{ij}$

$\eta_{ij} = \beta_{0j} + \beta_{1j}*(RACE_{ij}) + \beta_{2j}*(EROSION_{ij})$

Level-2 Model

$\beta_{0j} = \gamma_{00} + \gamma_{01}*(DROPOUT_j) + u_{0j}$

$\beta_{1j} = \gamma_{10}$

$\beta_{2j} = \gamma_{20}$

RACE EROSION have been centered around the grand mean.

DROPOUT has been centered around the grand mean.

Level-1 variance = $1/[\phi_{ij}(1-\phi_{ij})]$

Mixed Model

$\eta_{ij} = \gamma_{00} + \gamma_{01}*DROPOUT_j$
$+ \gamma_{10}*RACE_{ij}$
$+ \gamma_{20}*EROSION_{ij}$
$+ u_{0j}$

Results For Unit-Specific Model, Adaptive Gaussian Quadrature

iteration 3

τ

INTRCPT1,β_0 0.32323

Standard error of τ

INTRCPT1,β_0 0.21389

Final estimation of fixed effects (Unit-specific model)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	-3.177305	0.169494	-18.746	26	<0.001
DROPOUT, γ_{01}	0.021184	0.056359	0.376	26	0.710
For RACE slope, β_1					
INTRCPT2, γ_{10}	0.328915	0.254004	1.295	2154	0.195
For EROSION slope, β_2					
INTRCPT2, γ_{20}	0.810334	0.146447	5.533	2154	<0.001

Fixed Effect	Coefficient	Odds Ratio	Confidence Interval
For INTRCPT1, β_0			
INTRCPT2, γ_{00}	-3.177305	0.041698	(0.029,0.059)
DROPOUT, γ_{01}	0.021184	1.021410	(0.910,1.147)
For RACE slope, β_1			
INTRCPT2, γ_{10}	0.328915	1.389459	(0.844,2.287)
For EROSION slope, β_2			
INTRCPT2, γ_{20}	0.810334	2.248659	(1.687,2.997)

Statistics for the current model

[1] Raudenbush, Yang, & Yousef (2000) results suggest 10 to 20. Capanu, Gönen, and Begg (2013) found no substantial differences between using 3, 5, or 7 quadrature points and that Laplace estimates did well (the PQL estimates of fixed effects performed the poorest).

[2] HLM Version 8.02 did not print the adaptive quadrature results when a random slope was added. However, if Laplace estimates are also requested when estimating a random slope, then the adaptive quadrature estimates are printed.

Deviance = 4803.853963
Number of estimated parameters = 5

Results for Population-Average Model
The value of the log-likelihood function at iteration 3 = -2.974940E+03

Final estimation of fixed effects: (Population-average model)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	-3.044720	0.161741	-18.825	26	<0.001
DROPOUT, γ_{01}	0.021800	0.055686	0.391	26	0.699
For RACE slope, β_1					
INTRCPT2, γ_{10}	0.314908	0.237768	1.324	2154	0.185
For EROSION slope, β_2					
INTRCPT2, γ_{20}	0.795631	0.138454	5.747	2154	<0.001

Fixed Effect	Coefficient	Odds Ratio	Confidence Interval
For INTRCPT1, β_0			
INTRCPT2, γ_{00}	-3.044720	0.047610	(0.034,0.066)
DROPOUT, γ_{01}	0.021800	1.022040	(0.911,1.146)
For RACE slope, β_1			
INTRCPT2, γ_{10}	0.314908	1.370133	(0.859,2.185)
For EROSION slope, β_2			
INTRCPT2, γ_{20}	0.795631	2.215838	(1.689,2.907)

Final estimation of fixed effects
(Population-average model with robust standard errors)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	-3.044720	0.144009	-21.143	26	<0.001
DROPOUT, γ_{01}	0.021800	0.075314	0.289	26	0.775
For RACE slope, β_1					
INTRCPT2, γ_{10}	0.314908	0.181401	1.736	2154	0.083
For EROSION slope, β_2					
INTRCPT2, γ_{20}	0.795631	0.130639	6.090	2154	<0.001

Fixed Effect	Coefficient	Odds Ratio	Confidence Interval
For INTRCPT1, β_0			
INTRCPT2, γ_{00}	-3.044720	0.047610	(0.035,0.064)
DROPOUT, γ_{01}	0.021800	1.022040	(0.875,1.193)
For RACE slope, β_1			
INTRCPT2, γ_{10}	0.314908	1.370133	(0.960,1.956)
For EROSION slope, β_2			
INTRCPT2, γ_{20}	0.795631	2.215838	(1.715,2.863)

The predicted probability can be computed from the results. I use the population average results to more accurately estimate the proportion of students who have carried a gun in the population. The following formula can be used assuming mean centering of the predictors (or if testing the intercept only model) and the proportion is desired for the case when all predictors equal their means:

$$\varphi = \frac{1}{\left(1 + e^{-\eta_{ij}}\right)}$$

Where η_{ij} is the predicted log odds given the regression, $\eta_{ij} = \beta_0 + \beta_1\left(DROPOUT\right) + \beta_2\left(RACE\right) + \beta_3\left(EROSION\right)$. η_{ij} is easy to calculate if all predictors are 0 (i.e., equal to their means when centered), because η_{ij} is simply equal to β_0:

$$\varphi = \frac{1}{\left(1 + e^{-(-3.045)}\right)} = .045$$

Thus, approximately 4.5% of students in the population are expected to report carrying a gun in the previous year.

R

Both adaptive quadrature and Laplace approximation (not higher-order Laplace) are available from R `lme4`, but random slopes are not allowed with adaptive quadrature. To save space, I only included the adaptive quadrature results. For Laplace estimates, just remove the `nAGC=` statement. Profile likelihood confidence intervals for the random effects (through the `lmerTest` `confint` function) are available whether Laplace or adaptive quadrature estimates are requested. For Laplace estimates, just omit nAGQ=7

```
> #make sure predictors are numeric--did not need here so commented
> #d$gun <- as.numeric(d$gun)
> #d$gun = d$gun - 1.
> #d$race <- as.numeric(d$race)
>
> #listwise deletion to make sure centering is correct
> d = d[complete.cases(d[,c('race','erosion','dropout','gun')]),]
> #check your results
> #library(psych)
> #describe(d)
>
> #grand mean center variables
> d$crace <- d$race - mean(d$race)
> d$cerosion <- d$erosion  - mean(d$erosion)
> d$cdropout <- d$dropout - mean(d$dropout)
> #check your results
> #library(psych)
> #describe(d)
>
> #library(lme4)
> #rm(model1)
> #Laplace approximation
> #model1 <- glmer(gun ~ crace + cerosion + cdropout + (1|schnum), family = binomial,data=d)
> #summary(model1)
> #library(lmerTest)
> #confint(model1)
>
> rm(model2)
> #adaptive quadrature, 7 integration points (random slopes not possible with adapt quad in R)
> model2 <- glmer(gun ~ race + erosion + dropout + (1|schnum), family = binomial, nAGQ=7,data=d)
> summary(model2)
Generalized linear mixed model fit by maximum likelihood (Adaptive Gauss-Hermite Quadrature, nAGQ = 7) ['g
lmerMod']
 Family: binomial  ( logit )
Formula: gun ~ race + erosion + dropout + (1 | schnum)
   Data: d

     AIC      BIC   logLik deviance df.resid
   854.1    882.8   -422.1    844.1     2269

Scaled residuals:
    Min      1Q  Median      3Q     Max
-0.7677 -0.2359 -0.1872 -0.1584  6.9827

Random effects:
 Groups Name        Variance Std.Dev.
 schnum (Intercept) 0.291    0.5395
Number of obs: 2274, groups:  schnum, 28

Fixed effects:
            Estimate Std. Error z value             Pr(>|z|)
(Intercept) -3.76095    0.33717 -11.154 < 0.0000000000000002 ***
race         0.38143    0.24082   1.584                0.113
erosion      0.81171    0.14077   5.766        0.00000000811 ***
dropout      0.02875    0.05388   0.534                0.594
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Correlation of Fixed Effects:
        (Intr) race   erosin
race    -0.102
erosion -0.370 -0.041
dropout -0.803 -0.054  0.025
> library(lmerTest)
> confint(model2)
```

```
Computing profile confidence intervals ...
                  2.5 %     97.5 %
.sig01       0.17025395  0.9813158
(Intercept) -4.45819088 -3.0819219
race        -0.10705233  0.8406398
erosion      0.53301190  1.0861359
dropout     -0.08676033  0.1371978
```

Note: Special code is needed (`0+slopevar`) to suppress multiple intercepts that are generated by default when there is more than one random effect (e.g., intercept and slope). This seems atypical to me so I modify the model to have only one intercept. For example,

```
model1 <- glmer(gun ~ race + erosion + dropout + (race|schnum) + (0+erosion|schnum),
family = binomial,data=mydata)
```

SPSS

Because SPSS only provides PQL estimates, which I do not recommend using when other methods are available, I do not present a full example here. Below is syntax, however, for estimating a binary model. Be sure to make the id variable a nominal variable

```
*change id to nominal level.
VARIABLE LEVEL schnum (NOMINAL).

GENLINMIXED
/DATA_STRUCTURE SUBJECTS=schnum
/FIELDS TARGET=gun
/TARGET_OPTIONS DISTRIBUTION=BINOMIAL LINK=LOGIT
/FIXED EFFECTS=race cerosion cdropout USE_INTERCEPT=TRUE
/RANDOM USE_INTERCEPT=TRUE SUBJECTS=schnum
COVARIANCE_TYPE=VARIANCE_COMPONENTS
/BUILD_OPTIONS TARGET_CATEGORY_ORDER=DESCENDING
INPUTS_CATEGORY_ORDER=DESCENDING
MAX_ITERATIONS=1500 CONFIDENCE_LEVEL=95 DF_METHOD=SATTERTHWAITE.
```

The "TARGET" is the outcome and the "INPUTS" are the predictors. The SUBJECTS variable is the group designation. ORDER=DESCENDING is used to specify that the 0 group is used as the comparison (typically what is desired) for the dependent or the independent variable. If omitted, the 1 group is used as the default.

Write Up

(These results were taken from the R output—details about estimation method and software would be given earlier in the results section)

A multilevel logistic model, using adaptive quadrature, was tested to investigate the relationship of student race/ethnicity, student perceptions of neighborhood erosion, and school dropout rates to reports of carrying a gun. The erosion and dropout variables were centered and no random slopes were included in this model. The intercept for gun use was -3.76, which corresponds to a gun use rate of approximately 2.3% for white students with average ratings of erosion from a school with an average dropout rate.[3] This value varied significant across schools as indicated by the profile likelihood confidence intervals (Bates & DebRoy, 2004), $\tau_0^2 = .29$, 95%CI(.17,.98). Higher perceptions of erosion were associated with greater likelihood or carrying a gun, $\gamma_{20} = .81$, OR = 2.25, $z = 5.77$, $p < .001$, indicating more than twice the rate of reporting carrying a gun for each one-point increase on the erosion scale. Neither the race/ethnicity of the student, = .38, OR = 1.46, $z = 1.58$, $p = .11$, nor school dropout rate, $\gamma_{01} = .03$, OR = 1.03, $z = .53$, $p = .59$, were significantly related to gun use.

Computations: $OR_{erosion} = e^{.81} = 2.25$. $OR_{race} = e^{.38} = 1.46$. $OR_{dropout} = e^{.03} = 1.03$.

Bates, D. M., & DebRoy, S. (2004). Linear mixed models and penalized least squares. *Journal of Multivariate Analysis*, *91*(1), 1-17.

[3] The rate was computing using the same equation as illustrated above, but differs from that computation because the HLM and R intercept estimates differed.