Intercept Only Model Example (Random Effects ANOVA)

SPSS

```
MIXED mathach
  /METHOD = REML
  /PRINT = SOLUTION TESTCOV
  /FIXED =  | SSTYPE(3)
  /RANDOM = INTERCEPT  | SUBJECT(schoolid) COVTYPE(UN).
```

Mixed Model Analysis

Warnings

The covariance structure for random effect with only one level will be changed to Identity.

Model Dimension[a]

		Number of Levels	Covariance Structure	Number of Parameters	Subject Variables
Fixed Effects	Intercept	1		1	
Random Effects	Intercept	1	Identity	1	schoolid
Residual				1	
Total		2		3	

a. Dependent Variable: mathach.

Information Criteria[a]

-2 Restricted Log Likelihood	47116.793532
Akaike's Information Criterion (AIC)	47120.793532
Hurvich and Tsai's Criterion (AICC)	47120.795203
Bozdogan's Criterion (CAIC)	47136.552756
Schwarz's Bayesian Criterion (BIC)	47134.552756

The information criteria are displayed in smaller-is-better form.

a. Dependent Variable: mathach.

Fixed Effects

Type III Tests of Fixed Effects[a]

Source	Numerator df	Denominator df	F	Sig.
Intercept	1	160.829	2671.555	<.001

a. Dependent Variable: mathach.

Estimates of Fixed Effects[a]

Parameter	Estimate	Std. Error	df	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Intercept	12.637	.244	160.829	51.687	<.001	12.154	13.120

a. Dependent Variable: mathach.

Covariance Parameters

Estimates of Covariance Parameters[a]

Parameter		Estimate	Std. Error	Wald Z	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound
Residual		39.148	.661	59.267	<.001	37.875	40.464
Intercept [subject = schoolid]	Variance	8.622	1.065	8.092	<.001	6.767	10.985

a. Dependent Variable: mathach.

R

```
> #install nlme package upon first use on a computer
> #install.packages("nlme")
> library(nlme)
>
> #random effects ANOVA model
> model <- lme(mathach ~ 1, random = ~ 1|schoolid, data = mydata, method="REML")
> summary(model)
Linear mixed-effects model fit by REML
 Data: mydata
       AIC      BIC    logLik
  47122.79 47143.43 -23558.4

Random effects:
 Formula: ~1 | schoolid
        (Intercept) Residual
StdDev:    2.934966 6.256862

Fixed effects: mathach ~ 1
               Value Std.Error   DF  t-value p-value
(Intercept) 12.63697 0.2443936 7025 51.70747       0

Standardized Within-Group Residuals:
        Min          Q1         Med          Q3         Max
-3.06312473 -0.75387398  0.02670132  0.76062171  2.74262579

Number of Observations: 7185
Number of Groups: 160
> #nlme provides standard deviations of the random effects by default, use VarCorr to obtain
variances
> VarCorr(model)
schoolid = pdLogChol(1)
            Variance  StdDev
(Intercept)  8.614025 2.934966
Residual    39.148322 6.256862
> #obtain confidence intervals for fixed and random effects (in SD units), similar to SPSS values
> intervals(model)
Approximate 95% confidence intervals

 Fixed effects:
               lower     est.    upper
(Intercept) 12.15789 12.63697 13.11606
attr(,"label")
[1] "Fixed effects:"

 Random Effects:
  Level: schoolid
                   lower     est.    upper
sd((Intercept)) 2.595995 2.934966 3.318198

 Within-group standard error:
   lower     est.    upper
6.154239 6.256862 6.361196
```

Note: the lmer function in the lme4 package also can be used for the same results. We will be using the lme4 package later in the course. For both the SPSS analysis and the R analysis, a one-tailed test should be used for the test of significance for the random effect (intercept variance) and its confidence limit (Snijders & Bosker, 2012), which we will discuss in the "Significance Testing in Multilevel Regression" handout. A second model can be tested requesting a 90% confidence interval adding the subcommand `/CRITERIA=CIN(90).` in SPSS and on the intervals function in the R `nmle` package `intervals(model,.90)`. The confidence limits for the intercept variance (random effect) can then be used for this second model, but the traditional two-tailed 95% confidence limits should be used for the fixed effects. Results for both packages produce confidence limits of 7.01 and 10.58, and the significance level was unchanged ($p < .001$).

HLM

```
The data source for this run = C:\jason\HLM\mlrclass\hsb.mdm
The command file for this run = C:\jason\HLM\mlrclass\hsb.hlm
Output file name = C:\jason\HLM\mlrclass\hlm2.html
The maximum number of level-1 units = 7185
The maximum number of level-2 units = 160
The maximum number of iterations = 100

Method of estimation: restricted maximum likelihood

The outcome variable is MATHACH
```

Summary of the model specified
Level-1 Model

$MATHACH_{ij} = \beta_{0j} + r_{ij}$

Level-2 Model

$\beta_{0j} = \gamma_{00} + u_{0j}$

Mixed Model

$MATHACH_{ij} = \gamma_{00} + u_{0j} + r_{ij}$

Final Results - Iteration 4
Iterations stopped due to small change in likelihood function

$\sigma^2 = 39.14831$

τ

INTRCPT1, β_0	8.61431

Random level-1 coefficient	Reliability estimate
INTRCPT1, β_0	0.901

The value of the log-likelihood function at iteration 4 = -2.355840E+004

Final estimation of fixed effects:

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	12.636972	0.244412	51.704	159	<0.001

Final estimation of fixed effects
(with robust standard errors)

Fixed Effect	Coefficient	Standard error	*t*-ratio	Approx. *d.f.*	*p*-value
For INTRCPT1, β_0					
INTRCPT2, γ_{00}	12.636972	0.243628	51.870	159	<0.001

Final estimation of variance components

Random Effect	Standard Deviation	Variance Component	*d.f.*	χ^2	*p*-value
INTRCPT1, u_0	2.93501	8.61431	159	1660.23259	<0.001
level-1, r	6.25686	39.14831			

Statistics for current covariance components model

```
Deviance = 47116.793477
Number of estimated parameters = 2
```

Write-up Example

A multilevel model was tested to investigate whether math achievement varied significantly across schools. This intercept-only (or empty) model is equivalent to a random effects ANOVA. Results indicated that the average math achievement score was 12.63. The intercept variance for math achievement was significant, $\tau_0^2 =$ 8.61, p < .001, indicating that math achievement varied significantly across schools. The intraclass correlation coefficient (ρ = .18) indicated that approximately 18% of the variance in math achievement was between schools, with the remainder in math achievement variability occurring within schools.