## Ordinal Examples

The examples shown below come from a study conducted by Karen Seccombe that examined health coverage among low income families. I illustrate ordinal analyses with two variables-the extent to which the participant reported cutting meals (cutmeals) and a three-category education measure (ed).

## SPSS

A simple way to get a linear-by-linear association model is through the cross-tabs procedure which we used for nominal contingency chi-square tests. The linear-by-linear (i.e., ordinal loglinear) test is found in the default output. The linear-by-linear association reported is the same as the loglinear model test.

```
crosstabs /tables=ed by cutmeal
    /cells=count row column
    /statistics=chisq phi.
```

ed * cutmeal how often cut meal size Crosstabulation

|  |  |  | cutmeal how often cut meal size |  |  | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | 0 never or rarely | 1 some months but not every month | 2 almost every month |  |
| ed | 1.00 less than hs | Count | 85 | 16 | 13 | 114 |
|  |  | \% within ed | 74.6\% | 14.0\% | 11.4\% | 100.0\% |
|  |  | \% within cutmeal how often cut meal size | 19.8\% | 28.6\% | 19.7\% | 20.7\% |
|  | 2.00 hs | Count | 183 | 17 | 27 | 227 |
|  |  | \% within ed | 80.6\% | 7.5\% | 11.9\% | 100.0\% |
|  |  | \% within cutmeal how often cut meal size | 42.7\% | 30.4\% | 40.9\% | 41.2\% |
|  | 3.00 college | Count | 161 | 23 | 26 | 210 |
|  |  | \% within ed | 76.7\% | 11.0\% | 12.4\% | 100.0\% |
|  |  | \% within cutmeal how often cut meal size | 37.5\% | 41.1\% | 39.4\% | 38.1\% |
| Total |  | Count | 429 | 56 | 66 | 551 |
|  |  | \% within ed | 77.9\% | 10.2\% | 12.0\% | 100.0\% |
|  |  | \% within cutmeal how often cut meal size | 100.0\% | 100.0\% | 100.0\% | 100.0\% |


| Chi-Square Tests |  |  |  |
| :---: | :---: | :---: | :---: |
|  | Value | df | Asymptotic Significance (2-sided) |
| Pearson Chi-Square | $3.888{ }^{\text {a }}$ | 4 | . 421 |
| Likelihood Ratio | 3.867 | 4 | . 424 |
| Linear-by-Linear Association | . 000 | 1 | . 984 |
| $N$ of Valid Cases | 551 |  |  |
| a. 0 cells ( $0.0 \%$ ) minimum $\exp$ | ed coun <br> is 11.59 | than |  |

Or, the loglinear procedure genlog can be used if you compute an association variable and add that to the model. The genlog approach is easily expanded to include additional variables and effects. Adding keyword freq to the /print subcommand prints the frequencies (omitted below).

```
compute assoc = cutmeal*ed.
genlog ed cutmeal with assoc
/print=est freq
/plot=none
/design = ed cutmeal assoc.
```

Although the values are similar, the likelihood ratio and Pearson chi-square tests do not provide a test of the association term by itself. A comparison of the fit to a model without the association term, which I will not illustrate here, would be needed for that. (If this was a binary loglinear model, the fit would be perfect-a saturated model).

Goodness-of-Fit Tests a,b

|  | Value | df | Sig. |
| :--- | :---: | ---: | ---: |
| Likelihood Ratio | 3.866 | 3 | .276 |
| Pearson Chi-Square | 3.890 | 3 | .274 |

a. Model: Poisson
b. Design: Constant + ed + cutmeal + assoc

The test of the association term is found in the last row (highlighted) of the parameter estimates ( $b$ values) table, however. Notice that this sig. ( $p$-value) matches the linear-by-linear sig. value in the crosstabs output above.

Parameter Estimates b,c

| Parameter | Estimate | Std. Error | Z | Sig. | 95\% Confidence Interval |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  | Lower Bound | Upper Bound |
| Constant | 3.217 | . 411 | 7.836 | . 000 | 2.412 | 4.022 |
| [ $\mathrm{ed}=1.00$ ] | -. 610 | . 130 | -4.705 | . 000 | -. 864 | -. 356 |
| [ $\mathrm{ed}=2.00$ ] | . 078 | . 100 | . 785 | . 433 | -. 117 | . 274 |
| [ $\mathrm{ed}=3.00$ ] | $0^{\text {a }}$ |  |  |  |  |  |
| [cutmeal $=0$ ] | 1.879 | . 388 | 4.849 | . 000 | 1.120 | 2.639 |
| [cutmeal $=1$ ] | -. 161 | . 257 | -. 624 | . 533 | -. 665 | . 344 |
| [cutmeal $=2$ ] | $0^{\text {a }}$ | . |  | . |  |  |
| assoc | . 002 | . 084 | . 021 | . 984 | -. 162 | . 166 |

a. This parameter is set to zero because it is redundant.
b. Model: Poisson
c. Design: Constant + ed + cutmeal + assoc

Retest the ordinal model without the association variable to compare to the saturated model. The difference in likelihood ratio tests is a likelihood ratio chi-square that can be tested to significance to determine of there is a relationship between education and cutting back on meals.

```
genlog ed cutmeal with assoc
/model=poisson
/print=est freq
/plot=none
/design = ed cutmeal .
```

Likelihood ratio from saturated model $=3.866$. Likelihood ratio from model without assoc $=3.867$. Likelihood ratio difference $=3.867-3.866=.001$. This can be compared to the critical chi-square value with $\mathrm{df}=1$, which is 3.84 , so the difference is not significant, indicating no significant relationship between the two variables.

The Kendall's tau-b, Gamma, Somer's d provide other ways to test association among ordinal variables but are not equivalent to the loglinear test.

| crosstabs /tables=ed by cutmeal |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| /cells=count row column |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
| Directional Measures |  |  |  |  |  |  |
|  |  |  | Value | Asymptotic Standard Error ${ }^{\text {a }}$ | $\begin{gathered} \text { Approximate } \\ \mathrm{T}^{\mathrm{b}} \\ \hline \end{gathered}$ | Approximate Significance |
| Ordinal by Ordinal | Somers' d | Symmetric | -. 001 | . 039 | -. 013 | . 989 |
|  |  | ed Dependent | -. 001 | . 053 | -. 013 | . 989 |
|  |  | cutmeal how often cut meal size Dependent | . 000 | . 030 | -. 013 | . 989 |

a. Not assuming the null hypothesis.
b. Using the asymptotic standard error assuming the null hypothesis.

Symmetric Measures

|  |  |  | Asymptotic <br> Standard <br> Error | Approximate <br> $\mathrm{T}^{\mathrm{b}}$ | Approximate <br> Significance |
| :--- | :--- | ---: | ---: | ---: | ---: |
| Ordinal by Ordinal | Kendall's tau-b | -.001 | .040 | -.013 | .989 |
|  | Gamma | -.001 | .081 | -.013 | .989 |
| N of Valid Cases |  | 551 |  |  |  |

a. Not assuming the null hypothesis.
b. Using the asymptotic standard error assuming the null hypothesis.

## R

Note: The glm function in R does not quite match SPSS or SAS, so may not be correct.

```
> rm(mymode1)
> rm(mytable)
>
#Loglinear models
    #two-way loglinear
> library(MASS)
> #make summary frequency table for two variables
> mytable = table(d$ed, d$cutmea1)
> tb1 = table(d$ed, d$cutmea1)
>
> #View(tbl)
#create data frame from the matrix tbl (need both below)
> mytable = as.data.frame(tbl)
>
> #conversion leads to new column names Var1 and Var2, so rename them
> colnames(mytable) [colnames(mytable)=="Var1"] <- "ed"
> colnames(mytab1e)[colnames(mytable)=="Var2"] <- "cutmeal"
>
> #linear-by-linear association model needs numeric variables for the association term
> mytable$ed = as.numeric(mytable$ed)
> mytable$cutmeal = as.numeric(mytable$cutmea1)
> mytable$assoc = mytable$ed*mytable$cutmea1
>
> #convert variables back to factors so that comparisons are used by g7m
> mytable$ed = as.factor(mytable$ed)
> mytab1e$cutmea1 = as.factor(mytab1e$cutmea1)
>
> #options statement needed to rectify SS type 3
> #results sort of close to SPSS and SAS but do not match
> options(contrasts=c("contr.sum","contr.poly"))
> mymodel = glm(Freq ~ ed + cutmeal + assoc, family=poisson(link="log"), data=mytable)
> coef(summary(mymode1))
    Estimate Std. Error z value Pr P P | | )
(Intercept) 3.609436604 0.35055919 10.29622591 0.000000000000000000000000007328013
ed1 -0.430917102 0.13236231 -3.25558760 0.0011315798454714475883980950854
ed2 0.255531808 0.05883593 4.34312534 0.0000140469935098658603705307213
```

```
cutmea11 1.306370971 0.19462469 6.71225722 0.0000000000191636491676279617710
cutmea12
assoc
    -0.733468878 0.09939322 -7.37946602 0.0000000000001589256381347463542
    0.001717556 0.08373613 0.02051153 0.9836353158062801638195082887250
> #DescTools can be used for association statistics that gives confidence intervals
> library("DescTools")
> Assocs(tb1,conf.leve1 = :95)
estimate 7wr.ci upr.ci
Phi Coeff. 0.0840 - -
Contingency Coeff. 
Goodman Kruska1 Gamma -0.0011 -0.1604 0.1583
Kenda11 Tau-b -0.0005 -0.0789 0.0779
Stuart Tau-c -0.0004 -0.0577 0.0569
Somers D C|R -0.0004 -0.0598 0.0590
Somers D R|C -0.0007 -0.1041 0.1027
Pearson Correlation }0.0009 -0.0827 0.0844
Spearman Correlation 
Lambda C|R 0.0000 0.0000 0.0000
Lambda R|C 0.0185 0.0000 0.0564
Lambda sym 0.0135 0.0000 0.0410
Uncertainty coeff. C|R 0.0051 -0.0051 0.0154
Uncertainty Coeff. R|C 0.0033-0.0033 0.0099
Uncertainty coeff. sym 0.0040 -0.0040 0.0120
Mutua1 Information 0.0051
#Assocs(tb1,conf.1eve1 = .95)
#note: Pearson does not match phi
> #vcd assocstats gives chi-square and likelihood ratio significance tests
> library(vcd)
> assocstats(tbl)
Likelihood Ratio 3.8668 df P(> X^2)
Pearson 3.8880 4 0.42137
Phi-Coefficient : NA
Contingency Coeff.: 0.084
Cramer's V : 0.059
```

