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TESTING FOR SUPERADDITIVITY OF THE MEAN VALUE FUNCTION OF A NON-HOMOGENEOUS POISSON PROCESS

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ABSTRACT

Consider a non-homogeneous Poisson process, $N(t)$, with mean value function $\Lambda(t)$ and intensity function $\lambda(t)$. A conditional test of the hypothesis that the process is homogeneous, versus alternatives for which $\Lambda(t)$ is superadditive, was proposed by Hollander and Proschan (1974). A new test for superadditivity of $\Lambda(t)$, which is based on a linear combination of the occurrence times of the process $N(t)$ is suggested in this paper. Though this test has the same Pitman efficiency as the Hollander-Proschan test, it is shown by Monte-Carlo simulation that our test has more power for many important alternatives. Tables for the exact null distribution of the test statistic have been given.

1. INTRODUCTION

Non-homogeneous Poisson Processes (NHPP's) provide models for various physical phenomena. The monograph by Ascher and Feingold (1984) contains an excellent survey and discussion of the role of NHPP's in probabilistic modeling of repairable systems and reliability growth.

Let $\{N(t),\ t \geq 0\}$ be a non-homogeneous Poisson process with mean value function $\Lambda(t) = E[N(t)]$. Assume $\Lambda(t)$ is differentiable with intensity function $\lambda(t) = d\Lambda(t)/dt$. If $\lambda(t) \equiv \lambda$, for some constant $\lambda > 0$, the process $\{N(t), t \geq 0\}$ is said to be homogeneous. It is of interest to determine whether the associated intensity function $\lambda(t)$ is constant (corresponding to a homogeneous Poisson process) or not. A large number of tests are available in the literature for testing the homogeneity of the process $N(t)$ against the alternative that $\lambda(t)$ is increasing; that is, the mean value function $\Lambda(t)$ is convex. For a detailed survey, see Bain et. al. (1985).

Hollander and Proschan (1974) gave several physical models wherein, roughly speaking, the expected number of events in any initial interval (i.e., of the form $[0,t]$) is no greater than the expected number of events in any interval of the same length occurring later (i.e. of the form $[x,x+t]$). More precisely, the mean value function $\Lambda(t)$ has, in such situations, the property that

$$\Lambda(t_1) + \Lambda(t_2) \leq \Lambda(t_1+t_2), \text{ where } 0 \leq t_1, t_2 \qquad (1.1)$$

In other words, $\Lambda(t)$ is superadditive. Note that $\Lambda(t)$ is superadditive whenever it is convex, provided $\Lambda(0) = 0$, but the converse is not true. In many of the examples motivating superadditivity of $\Lambda(t)$ given by Hollander and Proschan (1974), $\Lambda(t)$ is superadditive but not convex.

Suppose that the NHPP $N(t)$ is observed over the time interval $[0,t^*]$, where t^* is a prefixed positive number. The random number of occurrences, N, of the events of the process during this time period is recorded. For an observed value $n(>0)$ of N, let the occurrence times be given by $0 < T_1 < \ldots < T_n < t^*$. We consider the problem of testing the null hypothesis

$$H_0\colon \lambda(t) \equiv \lambda, \ 0 \leq t \leq t^*, \ (\lambda \text{ unknown}) \qquad (1.2)$$

against the alternative

$$H_1\colon \Lambda(t_1) + \Lambda(t_2) \leq \Lambda(t_1+t_2), \ 0 \leq t_1 + t_2 \leq t^*, \qquad (1.3)$$

where, in (1.3), the inequality is assumed to be strict for at least one pair (t_1,t_2). The alternative H_1 specifies that, in the interval $[0,t^*]$, $\Lambda(t)$ is superadditive. It should be noted that, though we are interested in the whole process, we cannot check for superadditivity beyond the period of observation. Hence, in (1.3), the restrictions $0 \leq t_1 + t_2 \leq t^*$ are motivated by practical considerations. Furthermore, if we reverse the inequalities in (1.1) and in (1.3), we obtain the dual problem of testing homogeneity of the NHPP against subadditive alternatives. Because the test and results thereof are similar, we shall not consider testing for subadditivity of $\Lambda(t)$ in detail.

The conditional test of H_0 vs H_1 due to Hollander and Proschan (1974) is based on a U-statistic, involving the occurrence times $T_1, \ldots, T_n$. Here, we propose a new test for this problem. Our test statistic, a linear combination of the T_i's, is discussed in Section 2. Whereas Hollander and Proschan (1974) provide only a table of Monte-Carlo estimates of the cut-off values of their statistic for selected values of n and significance levels, α, the exact null distribution of our statistic has been given in a closed form. Adapting a general approach to critical values of linear combinations of orders statistics (L-statistics) from a uniform distribution given by Ramallingam (1988), critical values for $n = 5(1)20$ are tabulated in Section 2. The asymptotic distribution of the new test statistic and its asymptotic relative efficiencies (ARE) are discussed in Section 3. It is noted that, in the Pitman ARE sense, both tests are equally efficient. The results of a Monte-Carlo study of the powers of the two tests for small samples have also been given in this section. For Weibull and linearly increasing intensity functions $\lambda(t)$, the new test has much better performance than the Hollander-Proschan test. However, the Hollander-Proschan test does better than the new test in the case of a triangular intensity.

2. THE PROPOSED TEST

Gupta and Kirmani (1988) have observed connections between occurrences times of non-homogeneous Poisson processes, record values and minimal repair times. They have established a number of relations among occurrence times or interoccurrence times of a NHPP when the mean value function $\Lambda(t)$ is convex or superadditive. In particular, they prove the following result:

Theorem 2.1: If $\Lambda(t)$ is superadditive, then for each $k,l \geq 1$,

$$T_{k+l} - T_k \overset{st}{\leq} T_l \qquad (2.1)$$

Inequality is reversed in (2.1) if Λ is subadditive.

We shall make use of this result for developing our procedure for testing H_0 against H_1. Let

$$V_n = \frac{1}{n^2 t^*} \sum\sum_{k+l \le n} [T_{k+l} - T_k - T_l]$$
$$= \frac{1}{n^2} \sum_{i=1}^{n} (3i-2n-1)(\frac{T_i}{t^*}) \qquad (2.2)$$

We base our test on the statistic V_n, small values of V_n being significant for testing H_0 against H_1. For testing H_0 against the dual alternative of sub-additive mean function $\Lambda(t)$, large values of V_n will be significant.

Conditional on $N = n$, it is well-known that the random variables $T_1, T_2, \ldots, T_n$ are distributed as the order statistics of a random sample of size n from a distribution with cumulative distribution function

$$H(t) = \begin{cases} 0 & , \ t \le 0 \\ \dfrac{\Lambda(t)}{\Lambda(t^*)} & , \ 0 \le t \le t^* \\ 1 & , \ t > t^* \end{cases} \qquad (2.3)$$

See, for example, Ross (1983, p. 53).

Under H_0, $H(t)$ is the distribution function of a uniform random variable over $[0,t^*]$ and, consequently, the null distribution of V_n is independent of t^*. In particular, under H_0,

$$E[V_n] = \frac{1}{n^2} \sum_{i=1}^{n} (3i-2n-1) \frac{i}{(n+1)} = 0$$
$$\mathrm{Var}[V_n] = \frac{(2n+1)(n^2+n-2)}{60n^3(n+2)} \qquad (2.4a)$$

In view of Theorem 2.1, under H_1, $T_{k+l} - T_k \overset{st}{\le} T_l$ and, consequently, we obtain that

$$E(T_{k+l} - T_k) < E(T_l), \ k + l \le n.$$

This clearly implies that, under H_1,

$$E(V_n) = (n^2 t^*)^{-1} \sum_{\substack{k=1 \\ k+l \le n}}^{n} \sum_{l=1}^{n} [E(T_{k+l} - T_k) - E(T_l)] < 0 \qquad (2.4b)$$

2.1 THE EXACT NULL DISTRIBUTION OF V_n

As noted in (2.2), under H_0,

$$n^2 V_n \overset{d}{=} \sum_{i=1}^{n} a_i U_{(i)} \qquad (2.5)$$

where $a_i = 3i - 2n - 1$ and $U_{(i)}$ is the i-th order statistic in a random sample of size n from the uniform distribution over the interval (0,1). Matsunawa (1985) gives the exact probability density function (p.d.f.) of the general L-statistics of the type (2.5). However, there are some errors in his formula (see his equations 3.7 and 3.8). The corrected version of his p.d.f is provided by Ramallingam (1988), who also derives the cumulative distribution fucntion (c.d.f) of statistics of the form (2.5). In order to state the p.d.f and c.d.f of the statistic $n^2 V_n$, we let $b_j = \sum_{l=j}^{n} a_l$, $j = 1,2, \ldots, n$ and let k^* be the number of distinct non-zero b_j's. Denote such b_j's by $b_1^*, b_2^*, \ldots, b_{k^*}^*$ and their corresponding multiplicities by $\nu_1, \nu_2, \ldots, \nu_{k^*}$. Finally, for a complex variable s, define functions $G(s)$ and $G_l(s)$ as

$$G(s) = \prod_{j=1}^{k^*} (s + (1/b_j^*))^{-\nu_j} \qquad (2.6)$$

$$G_l(s) = (s + (1/b_l^*))^{\nu_l} G(s) = \prod_{j=1, j \ne l}^{k^*} [s + (1/b_j^*)]^{-\nu_j} \qquad (2.7)$$

Following Ramallingam (1988), we obtain the exact p.d.f of $n^2 V_n$ as

$$f_{n^2 V_n}(t) = \sum_{l=1}^{k^*} \sum_{m=1}^{\nu_l} [\mathrm{sgn}(b_l^*) C_{l,m}^{\#} \chi(t/b_l^*) \chi(1 - t/b_l^*)$$
$$t^{m-1} (1 - t/b_l^*)^{n-m} / B(m, n-m+1)], \qquad (2.8)$$

where

$$\mathrm{sgn}(x) = \begin{cases} 1 & \text{if } x \ge 0 \\ -1 & \text{if } x < 0, \end{cases}$$

$$\chi(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \le 0, \end{cases} \qquad (2.9)$$

$$C_{l,m} = (G_l^{(\nu_l - m)}(-1/b_l^*))/(\nu_l - m)!, \qquad (2.10)$$

and

TABLE I.

Critical values of $\sqrt{30n}\ V_n$

α	Lower Tail		Upper Tail	
n	0.01	0.05	0.95	0.99
5	-2.6186	-1.7506	1.3314	1.6623
6	-2.6602	-1.7574	1.3448	1.6977
7	-2.7322	-1.7982	1.3937	1.7655
8	-2.6779	-1.7571	1.3830	1.7610
9	-2.6737	-1.7623	1.3962	1.7717
10	-2.6714	-1.7648	1.4028	1.8069
11	-2.6680	-1.7655	1.4167	1.8217
12	-2.6655	-1.7555	1.4236	1.8362
13	-2.6592	-1.7623	1.4338	1.8507
14	-2.6573	-1.7605	1.4441	1.8661
15	-2.6467	-1.7572	1.4585	1.8827
16	-2.6368	-1.7463	1.4623	1.9005
17	-2.6409	-1.7486	1.4800	1.9082
18	-2.6360	-1.7465	1.4648	1.9171
19	-2.6341	-1.7511	1.4711	1.9358
20	-2.6303	-1.7537	1.4816	1.9437

$$C^{\#}_{l,m} = \left(\prod_{j=1}^{k^*} (b_j^*)^{-\nu_j}\right) C_{l,m}, \qquad (2.11)$$

$G_l^p(s)$ being the p^{th} derivative of $G_l(s)$ in (2.7).

The c.d.f of $n^2 V_n$ is

$$F_{n^2V_n}(x) = \sum_{l=1}^{k^*} \sum_{m=1}^{\nu_1} C^{\#}_{l,m} (b_l^*)^m [g_1(x,l,m)\chi(b_l^*) + g_2(x,l,m)\chi(-b_l^*)] \qquad (2.12)$$

where,

$$g_1(x,l,m) = \begin{cases} 0 & \text{if } x < 0 \\ IB(x/b_l^*, m, n-m+1) & \text{if } 0 \le x < b_l^* \\ 1 & \text{if } x \ge b_l^*, \end{cases}$$

$$g_2(x,l,m) = \begin{cases} 0 & \text{if } x < b_l^* \\ IB(1-x/b_l^*, n-m+1, m) & \text{if } b_l^* \le x < 0 \\ 1 & \text{if } x \ge b_l^*. \end{cases}$$

$$IB(y,\alpha,\beta) = \int_0^y t^{\alpha-1}(1-t)^{\beta-1} dt / B(\alpha,\beta), \ 0 < y < 1, \ 0 < \alpha,\beta.$$

It has been demonstrated by Ramallingam (1988) that the coefficients $C_{l,m}$ in (2.10) can be easily computed by symbolic manipulation of the expressions in (2.6) and (2.7). We used the symbolic computation software REDUCE (Seward (1985)) at Northern Illinois University to find the critical values of V_n that could be used for carrying out our new test for small samples. In Table I, we provide critical values of $\sqrt{30n}\ V_n$ for $n = 5(1)20$.

2.2 THE ASYMPTOTIC DISTRIBUTION OF V_n

Let us rewrite V_n as

$$V_n = \frac{1}{n} \sum_{i=1}^{n} \left(3\frac{i}{n} - 2 - \frac{1}{n}\right) \frac{T_i}{t^*}.$$

Being a linear combination of order statistics from the distribution function $H(t)$ in (2.3), we can study the asymptotic distribution of V_n via the behaviour of the following L-statistic:

$$V_n' = \frac{1}{n} \sum_{i=1}^{n} J(i/n)\, T_i / t^*,$$

where $J(u) = 3u - 2$. Since $\sqrt{n}(V_n - V_n')$ converges to zero in probability, we state the large-sample distribution of $V_n(V_n')$ using the results of Shorack (1969) as follows:

Theorem 2.2: Let

$$\mu(J,H) = \int_0^{t^*} uJ(H(u))dH(u)$$

$$\sigma^2(J,H) = 2 \int_{s=0}^{t^*} \int_{t=s}^{t^*} H(s)[1-H(t)]J(H(s))J[H(t)]dtds.$$

Then, provided $\sigma^2(J,H) > 0$, $\sqrt{n}\,(V_n - \mu(J,H))$ is asymptotically normal with mean 0 and variance $\sigma^2(J,H)$.

Under H_0, $\mu = 0$ and $\sigma^2 = 1/30$. Thus the normal approximation for large n to the superadditive test rejects H_0 when $\sqrt{30n}\; V_n \leq -z_\alpha$, where z_α is the $[100(1-\alpha)]$-percentile of the standard normal distribution.

3. ASYMPTOTIC RELATIVE EFFICIENCIES AND ESTIMATES OF POWERS

Hollander and Proschan (1974) proposed the following test statistic for testing H_0 against H_1:

$$Q_n = 2K_n/[n(n-1)(n-2)], \tag{3.1}$$

where,

$$K_n = \Sigma^*[\phi(T_{\alpha_3}+T_{\alpha_2},t^*) - \{\phi(T_{\alpha_3},T_{\alpha_1}+T_{\alpha_2})\phi(T_{\alpha_1}+T_{\alpha_2},t^*)\}] \tag{3.2}$$

ϕ being the function

$$\phi(a,b) = \begin{cases} 1 & \text{if } a \leq b \\ 0 & \text{if } a > b \end{cases}$$

and Σ^* denoting the summation over all choices of subscripts such that $1 \leq \alpha_1 < \alpha_2 < \alpha_3 \leq n$. Small values of Q_n are significant for testing H_0 against H_1. It should be remarked that Q_n is the U-statistic corresponding to the following overall measure of deviation between H_0 and H_1:

$$\beta(\Lambda) = \iint_{0 \leq t_1+t_2 \leq t^*} [H(t_1)+H(t_2)-H(t_1+t_2)]\,dH(t_1)\,dH(t_2) \tag{3.3}$$

where $H(t)$ is the distribution function in (2.3) that is induced by the particular $\Lambda(t)$. The test based on Q_n will henceforth be referred to as the *HP* test.

Invoking (2.4b) and Theorem 2.2, we can show that our test based on V_n in (2.2) is consistent for testing H_0 against H_1. In fact, this test is consistent against the broader class of alternatives satisfying (2.4b). Furthermore, it can be shown that the Pitman asymptotic relative efficiency of V_n with respect to Q_n is always unity when the usual regularity conditions are satisfied (see, Puri and Sen (1971), pp. 113-115).

It is of interest to compare the performance of the two tests for small sample sizes and for various alternatives. A Monte-Carlo study was conducted to compare the small sample powers of these two tests for the following types of intensity functions:

(i) Weibull type: $\lambda(t) = \sigma t^{\sigma-1}$, $\sigma \geq 1$ ($\sigma = 1$ corresponds to H_0).

(ii) Linearly increasing type: $\lambda(t) = 1 + \sigma t$ ($\sigma = 0$ corresponds to H_0).

(iii) Triangular type: $\lambda(t) = \begin{cases} 4t, & 0 \leq t \leq \frac{1}{2} \\ 4-4t, & \frac{1}{2} \leq t \leq 1 \end{cases}$

Note that in (iii) the rate of occurrence is increasing on $[0,\frac{1}{2}]$ and decreasing on $[\frac{1}{2},1]$. It is also easy to see that the mean value function $\Lambda(t)$ that corresponds to $\lambda(t)$ in (iii) is superadditive, though not convex.

In the Monte-Carlo study, the order statistics, $U_{(1)} < U_{(2)} < \ldots < U_{(n)}$ from a random sample of size n from the uniform distribution on $[0,1]$ are generated and, in view of (2.3), T_k is obtained by solving

$$\Lambda(T_k)/\Lambda(t^*) = U_{(k)} \tag{3.4}$$

Since the choice of the test that is to be used may possibly depend upon the number of failures observed, we study the powers of the two tests conditional on $N = n$ and, in particular, we consider $n = 5, 10, 25$ and 40. In the case of Weibull process, intensities with $\sigma = 1.2, 1.4, 1.6, 1.8, 2.0, 2.5, 4.0$, and 20.0 were considered. Also, as both tests are functions of T_k/t^* and $T_k/t^* = [U_{(k)}]^{1/\sigma}$, we assume without loss of generality that $t^* = 1$. In the case of linear intensity, (3.4) can easily be solved to yield

$$T_k/t^* = [-1+(1+2c\sigma U_{(k)})^{1/2}]/2\sigma t^*,$$

where $c = t^* + \frac{1}{2}\sigma t^{*2}$. Consequently, in this case, even though both tests are expressed in terms of T_k/t^*, we must vary both σ and t^* in order to compare the power functions of the two tests. We have considered eight cases corresponding to $\sigma = 0.2, 0.4, 0.8, 1.0, 1.5, 2.0, 2.5, 20.0$ for each of two values t^*, namely 5 and 20. Finally, we shall

TABLE II

Estimated Powers for Testing H_0 versus H_1: Weibull Intensity

σ \ n	5	10	25	40
1.2	0.0920 0.0741	0.1193 0.0837	0.2026 0.1390	0.2655 0.2040
1.4	0.1385 0.0822	0.2140 0.1087	0.4398 0.2334	0.6093 0.4087
1.6	0.1967 0.0841	0.3332 0.1141	0.6885 0.2847	0.8629 0.5450
1.8	0.2621 0.0848	0.4650 0.1061	0.8558 0.2824	0.9676 0.5570
2.0	0.3257 0.0775	0.5856 0.0940	0.9455 0.2426	0.9937 0.4822
2.5	0.4879 0.0538	0.8173 0.0485	0.9983 0.0912	0.9999 0.1734
4.0	0.8436 0.0084	0.9934 0.0025	1.0000 0.0001	1.0000 0.0001
20	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000	1.0000 0.0000

tabulate the powers of the two tests at the single triangular intensity alternative indicated above for $n = 5, 10, 25, 40$.

Tables II - V contain Monte Carlo estimates of the powers for the two tests (at 5% level of significance) and for the three alternatives described above. These estimates were based on 10,000 replications that were generated using IMSL (version 10.0) subroutines. In each table, and for each choice of n and/or σ, the entries in the upperline are estimated powers for the new test based on V_n and the lower ones are for the Hollander-Proschan test.

It is clear from Table II that the V_n-test is uniformly (in n and σ) better than the *HP* test in the case of Weibull intensities. Curiously, whereas the power of the new test increases with increasing σ values, the power of the *HP* test decreases as σ increases and eventually becomes zero. This behavior of the *HP* test is to be expected because the distribution $H(t;\sigma)$ in (2.3) that is germane to the Weibull alternatives is stochastically increasing in $\sigma \geq 1$ and, as $\sigma \rightarrow \infty$, $H(t,\sigma)$ is degenerate in the limit at 1. Moreover, one can easily show that, for Weibull alternatives, $\beta(\Lambda)$ in (3.3) tends to zero, provided we let $\sigma \rightarrow \infty$. It is appropriate to recall from Hollander and Proschan (1974) that their test is consistent for alternatives $\Lambda(t)$ such that $\beta(\Lambda)$ in (3.3) is strictly negative. Thus the lack of power of the HP test for large values of σ, as suggested by Table II, is not a surprising phenomenon.

TABLE III

Estimated Powers for Testing H_0 against H_1 Linear Intensity with $t^* = 5$

σ \ n	5	10	25	40
0.2	0.1040 0.0675	0.1339 0.0743	0.2139 0.1075	0.2926 0.1685
0.4	0.1410 0.0705	0.1990 0.0817	0.3812 0.1433	0.5236 0.2561
0.8	0.1895 0.0731	0.3083 0.0931	0.5920 0.1956	0.7711 0.3649
1.0	0.2157 0.0790	0.3359 0.0914	0.6419 0.1863	0.8245 0.3761
1.5	0.2433 0.0748	0.3944 0.0923	0.7433 0.2105	0.9022 0.4107
2.0	0.2561 0.0767	0.4275 0.0913	0.7968 0.2071	0.9349 0.4353
2.5	0.2716 0.0806	0.4575 0.0957	0.8234 0.2244	0.9539 0.4487
20.0	0.3347 0.0748	0.5641 0.0969	0.9328 0.2305	0.9926 0.4743

From Tables III and IV, we see that, if we hold all but one of the quantities n, σ and t^*, the two tests almost always have increasing power with regard to the third quantity that is free to vary, although the rate at which the power of the HP test increases is significantly less than

TABLE IV
Estimated Powers for Testing H_0 against H_1
Linear Intensity with $t^* = 20$

σ \ n	5	10	25	40
0.2	0.1928	0.3002	0.5936	0.7722
	0.0773	0.0874	0.1877	0.3601
0.4	0.2376	0.3971	0.7635	0.9146
	0.0748	0.0867	0.2153	0.4241
0.8	0.2798	0.4843	0.8563	0.9653
	0.0754	0.0868	0.2344	0.4582
1.0	0.2962	0.4966	0.8725	0.9715
	0.0766	0.0977	0.2261	0.4631
1.5	0.3042	0.5265	0.9026	0.9826
	0.0760	0.0946	0.2308	0.4583
2.0	0.3083	0.5323	0.9129	0.9874
	0.0780	0.0951	0.2213	0.4655
2.5	0.3132	0.5516	0.9155	0.9881
	0.0770	0.0961	0.2359	0.4696
20.0	0.3393	0.5760	0.9429	0.9948
	0.0761	0.0974	0.2321	0.4777

TABLE V
Estimates of Powers for testing H_0 against H_1:
Triangular Intensity

n

5	10	25	40
.0811	0.2103	0.6185	0.8458
.3039	0.5304	0.8278	0.9368

that of the V_n-test. This is clear from the fact that, in our Monte-Carlo experiment for the linear intensity case, the estimated standard errors of the estimated powers do not exceed 0.005. Such low standard errors also explain why the occasional lack of monotonicity of power in these tables are statistically insignificant. Again, the new test based on V_n has more power than the *HP* test for all the choices of n, σ and t^* that were considered in our simulation study.

Finally, Table V demonstrates the superiority of the *HP* test over the V_n-test when we test H_0 against traingular intensity. Because the conclusions reached here do not depend greatly on n, similar results should hold for unconditional powers of the two tests.

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BIBLIOGRAPHY

Ascher, H. E. and Feingold (1984). *Repairable Systems Reliability.* Marcel Dekker, New York.

Bain, L. J., Engelhardt, M. and Wright, F. T. (1985). Tests for an increasing trend in the intensity of a Poisson process: a power study. *J. Amer. Statist. Assoc.* 80, 419-422.

Gupta, R. C. and Kirmani, S.N.U.A. (1988). Closure and montonicity properties of non-homogeneous Poisson processes and record values. To appear in *Probability in Engineering & Informational Sciences.*

Hollander, M. and Proschan, F. (1974). A test for superaddivity of the mean value function of a non-homogeneous Poisson process. *Stochastic Processes and Their Applications*, 2, 195-209.

Matsunawa, T. (1985). The exact and approximate distributions of linear combinations of selected order statistics from a uniform distribution. *Ann. Inst. Statist. Math.*, 37, 1-16.

Puri, M. L. and Sen, P. K. (1971). *Nonparametric Methods in Multivariate Analysis.* New York, John Wiley.

Ramallingam, T. (1988). Finding the exact distribution of linear combinations of selected order statistics from a uniform distribution by symbolic computing. *Submitted.*

Ross, S. M. (1983). *Stochastic Processes.* John Wiley & Sons, Inc.; New York.

Seward, L. R. (1985). REDUCE user's guide for IBM 360 and derivative computers version 3.2. The Rand Corporation, Santa Monica, CA 90406.

Shorack, G. (1969). Asymptotic normality of linear combinations of functions of order statistics. *Ann. Math. Statist.* 40, 2041-2050.

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AN APPROXIMATION TO THE DISTRIBUTION OF THE RATIO OF TWO GENERAL QUADRATIC FORMS WITH APPLICATION TO TIME SERIES VALUED DESIGNS

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ABSTRACT

Based on mixed cumulants up to order six, this paper provides a four moment approximation to the distribution of a ratio of two general quadratic forms in normal variables. The approximation is applied to calculate the percentile points of modified F-test statistics for testing treatment effects when standard F-ratio test is misleading because of dependence among observations. For the special case, when data is generated by an AR(1) process, the approximation is evaluated by a simulation study. For the general SARMA $(p,q)(P,Q)_s$ process, a modified F-test statistic is given, and its distribution for the $(0,1)(0,1)_{12}$ process, is approximated by the moment approximation technique.

1. INTRODUCTION

Highly automated data acquisition systems make possible the rapid gathering of large number of observations in industrial processes. These observations may be densely packed over time