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COMMUN. STATIST.-THEOR. METH., 15(4), 1377-1388 (1986)

ON TAIL-ORDERING AND COMPARISON OF FAILURE RATES

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Key Words and Phrases: two sample problem; distribution-free tests; asymptotic relative efficiency.

ABSTRACT

In this paper, we have studied some implications between tail-ordering (also known as dispersive ordering) and failure rate ordering (also called TP_2 ordering) of two probability distribution functions. Based on independent random samples from these distributions, a class of distribution-free tests has been proposed for testing the null hypothesis that the two life distributions are identical against the alternative that one failure rate is uniformly smaller than the other. The tests have good efficiencies as compared to their competitors.

1. INTRODUCTION AND SUMMARY

Let X be a random variable with distribution function $F(x) = P[X \leq x]$ and survival function $\overline{F}(x) = P[X > x] = 1 - F(x)$. We consider the following partial ordering for any arbitrary pair of distributions.

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Definition 1.1 : $\bar{G} \overset{TP_2}{<} \bar{F}$ if and only if

$$\begin{vmatrix} \bar{G}(x) & \bar{G}(y) \\ \bar{F}(x) & \bar{F}(y) \end{vmatrix} \geq 0, \text{ for all } x \leq y \qquad (1.1)$$

This ordering has been considered by Keilson and Sumita (1982), among others.

Let $\bar{F}_x$ be the conditional survival function for the remaining life conditional on survival upto time x, that is, $\bar{F}_x(y) = \bar{F}(x+y)/\bar{F}(x)$ for $y \geq 0$ and $0 \leq x < F^{-1}(1)$. If F is absolutely continuous with density function f, the failure (or hazard) rate r_F of F is defined by $r_F = f/\bar{F}$. The equivalence of the following statements can be easily established (also, see, Keilson and Sumita (1982)).

$$\left.\begin{array}{l} \text{(i) } \bar{G} \overset{TP_2}{<} \bar{F} \\ \text{(ii) } \bar{F}(x)/\bar{G}(x) \text{ is nondecreasing in } x \in [0, G^{-1}(1)). \\ \text{(iii) } \bar{F}_x(y) \geq \bar{G}_x(y), \text{ for all } x,\ y \geq 0. \\ \text{(iv) } r_F(x) \leq r_G(x), \text{ for } x \geq 0, \text{ if F and G are absolutely continuous.} \end{array}\right\} \qquad (1.2)$$

Suppose there are two independent systems, System I and System II, with life distributions F and G. Then $\bar{G} \overset{TP_2}{<} \bar{F}$ means that conditional on the event that both System I and System II have survived upto time x, for any $x \geq 0$, the remaining life of System I is stochastically larger than that of System II. Occasionally, we shall also call $\bar{G} \overset{TP_2}{<} \bar{F}$ as failure rate ordering. It can be easily seen that $\bar{G} \overset{TP_2}{<} \bar{F}$ implies that F is stochastically larger than G.

In the second section another partial ordering, known as 'tail-ordering' or 'dispersive ordering,' (Doksum (1969)), is considered and relation between

TP_2 ordering as defined by (1.1) or (1.2) and tail-ordering is investigated. In Section 3, we propose a class of distribution-free tests for testing the null hypothesis

$$H_0 : F(x) = G(x), \quad \text{for every } x \geq 0 \qquad (1.3)$$

against the alternative

$$H_A : \bar{G} \overset{TP_2}{<} \bar{F} \qquad (1.4)$$

or equivalently, the failure rate ordering alternative

$$H_A' : r_F(x) \leq r_G(x), \text{ for every } x \geq 0 \qquad (1.5)$$

if F and G are absolutely continuous.

This testing problem has earlier been considered by Chikkagouder and Shuster (1974) and Kochar (1979, 1981). The last section is devoted to efficiency comparisons. The newly proposed tests have been compared with the existing tests in the Pitman asymptotic relative efficiency (ARE) sense for some particular distributions belonging to H_A.

2. RELATIONS BETWEEN SOME PARTIAL ORDERINGS

Recently Lewis and Thompson (1981), Shaked (1982) and Lynch, Mimmack and Proschan (1983) have considered the following partial ordering on the space of probability distribution functions.

Definition 2.1 : G is said to be dispersed with respect to F ($G \overset{disp}{<} F$) if and only if

$$G^{-1}(\beta) - G^{-1}(\alpha) \leq F^{-1}(\beta) - F^{-1}(\alpha) \qquad (2.1)$$

whenever $0 < \alpha < \beta < 1$.

In the above papers, many interesting and important properties of this partial ordering have been discussed. Deshpandé and Kochar (1983) have pointed out that this concept of dispersive ordering is the same as that of tail-ordering as defined by Doksum (1969). That is (2.1) holds if and only if

$$F^{-1}G(x) - x \text{ is nondecreasing in } x \tag{2.2}$$

If (2.2) holds, we say that G is tail-ordered with respect to F ($G \overset{t}{<} F$). Doksum (1969), Deshpandé and Kochar (1982) and Deshpandé and Mehta (1982) have used this concept of tail-ordering in some inferential problems to obtain bounds on efficiencies of tests, probabilities of correct selections etc.

Below we study some relations between tail-ordering and failure rate ordering as defined by H_A.

Definition 2.2 : We say that a life distribution F is increasing failure rate (IFR) distribution if $\overline{F}_x(y) = \overline{F}(x+y)/\overline{F}(x)$ is nonincreasing in x for $0 \leq y < F^{-1}(1)$. If density exists, it is equivalent to saying that $r_F(x)$ is nondecreasing in x for $x \geq 0$.

Definition 2.3 : A life distribution F is said to be a decreasing failure rate (DFR) distribution if $\overline{F}_x(y) = \overline{F}(x+y)/\overline{F}(x)$ is nondecreasing in x for $0 \leq y < F^{-1}(1)$. If probability density function exists, this is equivalent to saying that $r_F(x)$ is nonincreasing in x for $x \geq 0$.

The following Lemma can be easily proved.

Lemma 2.1 : Let F be an increasing failure rate distribution, then

(a) for every nondecreasing function $h \geq 0$, $\overline{F}(x+h(x))/\overline{F}(x)$ is nonincreasing in x, for $x \geq 0$.

(b) for every nondecreasing function k such that $0 \leq k(y) \leq y$, $\overline{F}(y)/\overline{F}(y-k(y))$ is nondecreasing in y, for $y \geq 0$.

With the help of this lemma, we prove the following theorem.

Theorem 2.1 :

(a) Let F or G be IFR. If $G \overset{t}{<} F$, then $\overline{G} \overset{TP_2}{<} \overline{F}$.

(b) Let F or G be DFR. If $\overline{G} \overset{TP_2}{<} \overline{F}$, then $G \overset{t}{<} F$.

Proof : Let

$$h(x) = F^{-1}G(x) - x = \overline{F}^{-1}\overline{G}(x) - x, \tag{2.3}$$

so that we can write $\overline{G}(x)$ as

$$\overline{G}(x) = \overline{F}[h(x) + x] \tag{2.4}$$

(a) Since $G \overset{t}{<} F$, h(x) is nondecreasing in x, for $x \geq 0$.

Let F be IFR. Then it follows from the above Lemma (a) that $\overline{F}(x)/\overline{G}(x) = \overline{F}(x)/\overline{F}[h(x) + x]$ is nondecreasing in x, that is, $\overline{G} \overset{TP_2}{<} \overline{F}$.

Similarly, we can prove the required result assuming that G is IFR and by using the (b) part of the above Lemma.

(b) Let h(x) be as defined by (2.3). Since $\overline{G} \overset{TP_2}{<} \overline{F}$, $\overline{F}(x)/\overline{G}(x) = \overline{F}(x)/\overline{F}[h(x) + x]$ is nondecreasing in x for $x \geq 0$. It follows that $\overline{F}(x) \geq \overline{G}(x)$ and hence $h(x) \geq 0$ for $x \geq 0$. For $x_1 \leq x_2$.

$$\frac{\overline{F}(x_1)}{\overline{F}[h(x_1)+x_1]} \leq \frac{\overline{F}(x_2)}{\overline{F}[h(x_2)+x_2]} \tag{2.5}$$

If F is DFR,

$$\frac{\overline{F}[x_1+h(x_1)]}{\overline{F}(x_1)} \leq \frac{\overline{F}[x_2+h(x_1)]}{\overline{F}(x_2)} \tag{2.6}$$

From (2.5) and (2.6), we find that $h(x_1) \leq h(x_2)$, that is, $h(x) = F^{-1}G(x) - x$ is nondecreasing in x for $x \geq 0$.

Similarly, we can prove the result if G is DFR.

The following Corollarly follows immediately from this theorem.

Corollary 2.1 : Let F or G be an exponential distribution. Then $\overline{G} \overset{TP_2}{<} \overline{F}$ if and only if $G \overset{t}{<} F$.

3. THE PROPOSED TESTS

Let $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_m$ be independent random samples from the life distributions F and G, respectively. Let $N = n+m$. In this section we propose a class of distribution-free tests for testing H_0 against H_A. Tests for this problem have been earlier proposed by Chikkagouder and Shuster (1974) and Kochar (1979 and 1981).

H_A holds if and only if

$$\frac{\overline{F}(x)}{\overline{G}(x)} \geq \frac{\overline{F}(y)}{\overline{G}(y)} \quad \text{for } x \geq y$$

or equivalently,

$$\delta(x,y) = \left[\frac{\overline{F}(x)}{\overline{F}(y)}\right]^k - \left[\frac{\overline{G}(x)}{\overline{G}(y)}\right]^k \geq 0 \qquad (3.1)$$

for $x \geq y$ and for every $k > 0$. For fixed $k > 0$, and $k \neq 1$ consider the following measure of deviation between F and G

$$\Delta_k(F,G) = \iint_{x \geq y} \delta(x,y)\, dF(x)\, dG(y)$$

which simplifies to

$$\frac{1}{k+1} - \frac{k}{k-1} \int_0^{\infty} F(x) \left[\frac{2}{k+1} - \overline{G}^{k-1}(x)\right] dG(x) \qquad (3.2)$$

Under H_0, $\Delta_k(F,G) = 0$, but under H_A, $\Delta_k(F,G) > 0$.

Let F_n and G_m be the empirical distribution functions based on the random samples $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_m$ respectively. We propose a class of distribution-free tests based on the statistics

$$V_{k,N} = \int_0^{\infty} F_n(x)\left[\frac{2}{k+1} - \left\{1 - \frac{m}{m+1} G_m(x)\right\}^{k-1}\right] dG_m(x) \qquad (3.3)$$

Let $Y_{(1)}, \ldots, Y_{(m)}$ be the order statistics corresponding to the Y-sample and let $R_{(j)}$ denote the rank of $Y_{(j)}$ in the combined increasing arrangement of X's and Y's. We can also express $V_{k,N}$ in the form

$$V_{k,N} = \frac{1}{nm} \sum_{j=1}^{m} a_j \left(R_{(j)} - j\right) \qquad (3.4)$$

where $a_j = \left[\frac{2}{k+1} - \left(1 - \frac{j}{m+1}\right)^{k-1}\right]$, $j = 1, 2, \ldots m$

Statistics of this type have been considered by Sen (1964), Govindarajulu (1966, 1976) and Deshpandé (1972).

It follows from Deshpandé (1972) that under H_0

$$E[V_{k,N}] = \frac{1}{m(m+1)} \sum_{j=1}^{m} j a_j$$

$$\mathrm{Var}[V_{k,N}] = \frac{N+1}{n(m+2)m^2(m+1)^2}\, \underline{a}' \Sigma \underline{a}$$

where the matrix $\Sigma = (\sigma_{ij})$ $(i, j = 1, 2, \ldots, m)$ has $\sigma_{ij} = i(m+1-j) = \sigma_{ji}$ for $i \leq j$ and $\underline{a}' = (a_1, \ldots, a_m)$.

Large values of $V_{k,N}$ are significant for testing H_0 against H_A.

Govindarajulu (1966, 1976) has obtained the following asymptotic normality theorem.

Theorem 3.1 : Let $T_N = \int_{-\infty}^{\infty} F_n(x)\, J\left\{\frac{m}{m+1} G_m(x)\right\}\, dG_m(x)$

where J is a real-valued function on (0.1). If J is absolutely continuous on (0,1) with $|J'(u)| \leq$

$c\{u(1-u)\}^{-3/2+\delta}$ for some positive constants c and δ, then $N^{1/2}(T_N-\mu)/\sigma_N \xrightarrow{D} N(0,1)$ as $n \to \infty$ provided $0 < \underline{\lim}\, \frac{n}{N} \le \overline{\lim}\, \frac{n}{N} < 1$ as $N \to \infty$, where

$$\mu = \mu(J) = \int_{-\infty}^{\infty} F(x)\, J\{G(x)\}\, dG(x) \tag{3.5}$$

$$m\sigma_N^2 = m\sigma_N^2(J)$$

$$= \frac{2m}{n} \iint_{x<y} F(x)\overline{F}(y) J\{G(x)\} J\{G(y)\}\, dG(x)\, dG(y)$$

$$+ 2 \iint_{x<y} G(x)\overline{G}(y)\, J\{G(x)\}\, J\{G(y)\}\, dF(x)\, dF(y)$$

provided $\sigma_N^2 \neq 0$.

This theorem is applicable to $V_{k,N}$ provided $1 \neq k > 1/2$. It can be shown that, under H_0,

$$\mu = \frac{k-1}{k(k+1)} \tag{3.6}$$

$$m\sigma_N^2 = \frac{2N(k-1)^2}{3n\,(k+1)^2(k+2)(2k+1)} \tag{3.7}$$

For large samples the distribution of the standardized version of $V_{k,N}$ may be approximated by the standard normal distribution.

4. ASYMPTOTIC EFFICIENCIES

We compare the $V_{k,N}$ tests with the Wilcoxon (1945) test, the Savage (1956) test and the tests W and S proposed earlier by Kochar (1979 and 1981) in the Pitman's asymptotic relative efficiency sense for the following alternatives belonging to H_A.

H_1 : $\overline{F}_\theta(x) = \{\overline{F}(x)\}^{1+\theta}$, $\theta > 0$ (Proportional hazards)

H_2 : $\overline{F}_\theta(x) = \exp[-\{x+\theta(x+e^{-x}-1)\}]$, $\theta > 0$ (Makeham)

TABLE I

Pitman asymptotic relative efficiencies with respect to the locally most powerful rank tests

Alternative hypothesis	Wilcoxon	Savage	W	S	$V_{.6,N}$	$V_{.9,N}$	$V_{2,N}$
H_1	0.75	1	0.8203	0.8438	0.8379	0.8435	0.8333
H_2	0.25	0.75	0.5353	0.7812	0.8743	0.8027	0.6250
H_3	0.0938	0.5	0.2307	0.4219	0.5302	0.4442	0.2893
H_4	0.5	0.8438	0.896	1	0.9813	0.9991	0.9570

H_3 : $\bar{F}_\theta(x) = \bar{F}(x) \exp[-\frac{1}{2}\theta\{\log \bar{F}(x)\}^2]$, $\theta \geq 0$

(Linearly increasing failure rate)

H_4 : $\bar{F}_\theta(x) = (1-\theta)\bar{F}(x)+\theta\bar{F}^2(x)\{1-\log \bar{F}(x)\}$, $0 \leq \theta \leq 1$

The formulae for Pitman asymptotic relative efficiencies can be obtained in Puri and Sen (1971) and Kochar (1981).

Locally most powerful rank tests for testing H_0 against the above alternatives can be easily obtained. (Hajek and Sidak (1967), Chikkagoudar and Shuster, (1974)).

Table I gives the Pitman asymptotic relative efficiencies of the above mentioned tests with respect to the corresponding locally most powerful rank tests for the above alternatives.

It is seen from this table that the newly proposed tests are quite efficient atleast for the above mentioned alternatives. Looking at this table we recommend $V_{.6,N}$ and it is expected that it will have good overall performance.

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USING THE CHINESE REMAINDER THEOREM IN CONSTRUCTING CONFOUNDED DESIGNS FOR MIXED FACTORIAL EXPERIMENTS

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Key Words and Phrases: Chinese Remainder Theorem; confounded designs; factorial experiments; mixed factorial experiments; asymmetrical factorial experiments.

ABSTRACT

A procedure for constructing confounded designs for mixed factorial experiments derived from the Chinese Remainder Theorem is presented. The present procedure as well as others, all through use of modular arithmetic, are compared.

1. INTRODUCTION

The design of a factorial experiment has been one of the techniques frequently employed by researchers from various areas. The advantages, under suitable circumstances, of using a confounded design for such an experiment are well-known. Up to the present, however, all the procedures for constructing confounded designs for mixed (asymmetrical) factorial experiments are troublesome. A general confounding procedure that works for all types of factorial experiments is not yet found. Thus it would seem important to develop new procedures for confounding in mixed factorials.

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