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ASPECTS OF POSITIVE AGEING

JAYANT V. DESHPANDE,* *University of Poona*
SUBHASH C. KOCHAR,** *Panjab University*
HARSHINDER SINGH,** *Panjab University*

Abstract

The concept of positive ageing describes the adverse effects of age on the lifetime of units. Various aspects of this concept are described in terms of conditional probability distributions of residual lifetimes, failure rates, equilibrium distributions, etc. In this paper we further analyse this concept and relate it to stochastic dominance of first and higher orders. In the process we gain many insights and are able to define several new kinds of ageing criteria which supplement those existing in the literature.

RELIABILITY THEORY; SURVIVAL ANALYSIS; FAILURE RATE; MEAN RESIDUAL LIFE; EQUILIBRIUM DISTRIBUTION; IFR; IFRA; NBU; DMRL; NBUE; HNBUE

1. Introduction

Let X be a continuous positive random variable representing the lifetime of a unit. This unit could be a living organism or an organ thereof, a mechanical component or a system of components, etc. The lifetime is defined as the time for which the unit carries out its appointed functions satisfactorily and thereafter passes into the 'failed' state. No intermediate states are envisaged. The age of a working unit is the time for which it is already working satisfactorily, without failure.

Let F be the c.d.f. of X. Then $\bar{F}(x) = 1 - F(x) = P[X > x]$ is called its survival (survivorship) function and $\bar{F}_t(x) = P[X > x + t \mid X > t] = \bar{F}(x + t)/\bar{F}(t)$ is the survival function of a unit of age t, i.e., the conditional probability that a unit of age t (i.e., a unit having already survived up to time t) will survive for an additional x units of time. We see that $\bar{F}_0(x) = \bar{F}(x)$ is the survival function of a new (unused) unit. Obviously, any study of the phenomenon of ageing/no ageing has to be based on $\bar{F}_t(x)$ and functions related to it. The (instantaneous) hazard

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*Postal address: Department of Statistics, University of Poona, Ganeshkhind, Pune 411 007, India.
**Postal address: Department of Statistics, Panjab University, Chandigarh 160014, India.

rate of X is defined as $r_F(t) = f(t)/\bar{F}(t)$ provided f the density corresponding to F exists and $\bar{F}(t) > 0$. The mean residual lifetime of a unit of age t is defined as $e_F(t) = E(X - t \mid X > t) = \int_0^\infty \bar{F}_t(x)dx$. The equilibrium distribution corresponding to the c.d.f. F is defined as $H_F(t) = (1/e(0)) \int_0^t \bar{F}(x)dx$. All the above three transformations can be inverted to yield the original survival function $\bar{F}(x)$. Hence each of these is seen to be of relevance in the study of ageing.

'No ageing' is equivalent to the phenomenon that age has no effect on the residual survival function of a unit, i.e.

$$\bar{F}_t(x) = \bar{F}_0(x) = \bar{F}(x) \qquad \forall t, x > 0$$

or

$$\bar{F}(x + t) = \bar{F}(x)\bar{F}(t) \qquad \forall t, x > 0.$$

This last equation is the well-known Cauchy functional equation satisfied by only the exponential survival function, $\bar{F}(x) = \exp(-\lambda x)$, $x > 0$, $\lambda > 0$ among continuous survival functions. The lack-of-memory property of the exponential distribution is thus viewed as the no-ageing property in the context of reliability or survival analysis. Using the alternative functions introduced above it is again easily seen that $r_F(t) = \text{Const.}$, $e_F(t) = \text{Const.}$, and $H_F(t) = 1 - \exp(-\lambda t)$, $t > 0$, $\lambda > 0$ also characterize the exponential distribution. No ageing has thus been equivalently described as (i) constant failure rate, (ii) constant mean residual life function and (iii) exponential equilibrium distribution.

Positive ageing, meaning the adverse effect of age on the random residual lifetime of the unit, has been described in many ways in the literature. The dual concept of negative ageing, meaning a beneficial effect of age, has also been described. In this paper we concentrate on positive ageing, it being understood that a parallel development of negative ageing can also be carried out. It is considered to be axiomatic that any effect of age on the unit which contributes to the reduction of its residual lifetime (in some probabilistic sense) is to be taken as an adverse effect and the phenomenon is to be called positive ageing. Residual lifetimes of the units of different ages are random variables and hence stochastic comparisons must be carried out among them. These comparisons are made in terms of stochastic dominances of various orders between the residual survival functions of units of different ages, or in terms of other related functions.

2. Stochastic dominance

Let X_1 and X_2 be two random variables with c.d.f.'s F_1 and F_2 respectively. We say that X_1 is stochastically larger than X_2, ($X_1 \geqq^{st} X_2$) if $F_1(x) \leqq F_2(x)$ $\forall x$.

In economics theory this is known as first-order stochastic dominance and is denoted by $F_1 \text{ FSD } F_2$. Besides the well-known implication that $F_1 \text{ FSD } F_2 \Rightarrow E_{F_1}(X) \geqq E_{F_2}(X)$, we know that

(i) F_1 FSD $F_2 \Leftrightarrow E_{F_1}[u(X)] \geqq E_{F_2}[u(X)]$ for every increasing function u. There are some other dominance criteria which have been studied by economists.

(ii) Second-order stochastic dominance (1)

$$F_1 \text{ SSD}_1 F_2 \quad \text{if} \int_0^x F_1(t)dt \leqq \int_0^x F_2(t)dt \quad \forall x.$$

It is seen that

F_1 SSD$_1$ $F_2 \Leftrightarrow E_{F_1}[u(X)] \geqq E_{F_2}[u(X)]$ for every increasing concave function u.

(ii)′ Second-order stochastic dominance (2)

$$F_1 \text{ SSD}_2 F_2 \quad \text{if} \int_x^\infty \bar{F}_1(t)dt \geqq \int_x^\infty \bar{F}_2(t)dt \quad \forall x.$$

Here it is seen that

$$F_1 \text{ SSD}_2 F_2 \Leftrightarrow E_{F_1}[u(X)] \geqq E_{F_2}[(u(X)]$$

for every increasing convex function u.

(iii) Third-order stochastic dominance (1)

$$F_1 \text{ TSD}_1 F_2 \quad \text{if} \int_0^y \int_0^x F(t)dtdx \leqq \int_0^y \int_0^x F(t)dtdx \quad \text{for } 0 \leqq y < \infty.$$

Here it is seen that F_1 TSD$_1$ $F_2 \Leftrightarrow E_{F_1}[u(X)] \geqq E_{F_2}[u(X)]$ for every $u > 0$, $u' < 0$ and $u'' > 0$.

(iii)′ Third-order stochastic dominance (2)

$$F_1 \text{ TSD}_2 F_2 \quad \text{if} \int_x^\infty \int_t^\infty \bar{F}_1(u)dudt \geqq \int_x^\infty \int_t^\infty \bar{F}_2(u)dudt \quad \text{for } 0 \leqq x < \infty.$$

It is obvious that

$$F_1 \text{ FSD } F_2 \Rightarrow \begin{cases} F_1 \text{ SSD}_1 F_2 \Rightarrow F_1 \text{ TSD}_1 F_2 \\ F_1 \text{ SSD}_2 F_2 \Rightarrow F_1 \text{ TSD}_2 F_2 \end{cases}$$

and if F_1 and F_2 have the same means then F_1 SSD$_1$ F_2 is equivalent to F_1 SSD$_2$ F_2.

The economic value of using a unit for time x may be represented by a function $u(x)$ called the utility function of the unit. On heuristic grounds one may specify some qualities of such functions. One may reasonably expect $u(x)$ to be an increasing function. Further specification, e.g., concavity or convexity, etc., can perhaps be made in special circumstances where the marginal behaviour of the function $u(x)$ (in terms of its derivative $u'(x)$) can be imagined. Since the lifetimes are random variables we look at the expected utility $E_F u(X)$ of $u(X)$ when X is assumed to have the life distribution F. Hence it is natural to prefer the unit with life distribution F_1 to the unit with life distribution F_2 if F_1

dominates F_2, in view of the definitions of the various kinds of stochastic dominance. It is realized that the higher the order of the stochastic dominance, the weaker is the comparison because the expectations of utility functions are then ordered for successively smaller classes of functions.

The above discussion clarifies why the comparisons between residual life distributions of different ages to describe the effect of age on the unit, in terms of the various stochastic dominance criteria, are meaningful. Obviously, if the residual life distribution of a unit of less age dominates that of a unit of greater age, it would indicate positive ageing; the reverse dominance would indicate negative ageing. Implicitly this has been recognized and made use of in defining various concepts of ageing. It will be seen that many of the definitions of positive ageing phenomena are in terms of one of these dominance criteria.

The above short discussion of stochastic dominance criteria and their applications is based on Hanoch and Levy (1969), Tesfatsion (1976), Deshpande and Singh (1984), Whitmore (1970), Fishburn (1977), Brown and Ge (1984), Stein et al. (1984), etc. Reference may be made to these and other papers in the area.

3. Existing positive ageing criteria

The following notions of positive ageing are current in reliability theory. For fuller accounts of these see, e.g., Bryson and Siddiqui (1969), Barlow and Proschan (1975), Klefsjö (1983), etc. In what follows, increasing and decreasing are used in the weak sense of non-decreasing and non-increasing. Also, by reversing the inequalities or by interchanging 'increasing' and 'decreasing', we describe the dual negative ageing concepts.

(i) Increasing failure rate (IFR) distribution: F is IFR if $\bar{F}_{t_1}(x) \geqq \bar{F}_{t_2}(x)$ for $x \geqq 0$ and $0 \leqq t_1 \leqq t_2 < \infty$.

(ii) Increasing failure rate average (IFRA) distributions: F is IFRA if $-(1/x)\log \bar{F}(x)$ is increasing in x.

(iii) Decreasing mean residual life (DMRL) distribution: F is DMRL if $\int_0^\infty \bar{F}_t(x)dx$ is decreasing in t.

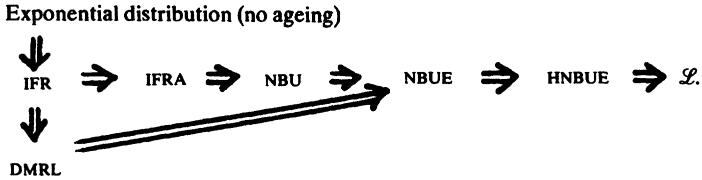
(iv) New better than used (NBU) distribution: F is NBU if $\bar{F}_t(x) \leqq \bar{F}(x)$ for $x \geqq 0$, $t \geqq 0$.

(v) New better than used in expectation (NBUE) distribution: F is NBUE if $\int_0^\infty \bar{F}_t(x)du \leqq \int_0^\infty \bar{F}(x)dx$ for $t \geqq 0$.

(vi) Harmonically new better than used in expectation (HNBUE) distribution: F is HNBUE if $\int_t^\infty \bar{F}(x)dx \leqq \mu \exp(-t/\mu)$ for $t > 0$, where μ is the mean of the distribution F.

(vii) $\mathscr{L}$-distribution: F is an $\mathscr{L}$-distribution if $\int_0^\infty \exp(-sx)\bar{F}(x)du \geqq \mu/(1+s)$ for $s \geqq 0$ where μ is the mean of F.

It is well known that among the above positive ageing properties the following, and only the following, implications hold.



There exists a vast amount of literature examining the relevance of the above notions in reliability theory/survival analysis. Closures of classes of distributions possessing the above properties under formation of coherent structures, under convolutions and under mixtures have been investigated. The evolution of distributions having the above properties through various shock models and on the basis of plausible assumptions regarding the life pattern of the unit, etc., have also been studied. In the remaining part of this section we focus on the use of stochastic dominance criteria in the above notions. From the above definitions the following proposition is obvious.

Proposition 3.1.

(i) F is IFR if and only if F_{t_1} FSD F_{t_2}, $0 \leqq t_1 < t_2 < \infty$.

(ii) F is NBU if and only if F FSD F_t, $0 \leqq t < \infty$.

(iii) F is HNBUE if and only if G SSD$_2$ F, where G is the exponential distribution with the same mean as F.

(iv) F is HNBUE if and only if $H_G(t)$ FSD $H_F(t)$ where G is again the exponential distribution with the same mean as F, and H_G and H_F are the corresponding equilibrium distributions.

It is seen that in (i) and (ii) the first-order stochastic dominance is applied between residual life distributions F_t of components of different ages corresponding to the same distribution function F, whereas (iii) is based on second-order stochastic dominance between F and the exponential distribution with the same mean. Similarly, (iv) is a first-order comparison between F and the exponential distribution with the same mean.

4. Some new positive ageing criteria

Once we recognize the role of stochastic dominance comparisons in the description of positive ageing, we can suggest new positive ageing criteria based on these. These comparisons may be, as above, either between two units of different ages or between units with exponential life distribution and another distribution which has some property in common with the exponential distribution (e.g., equal means, etc.). Three such definitions are introduced below.

Definition 4.1. F is called an increasing failure rate (of second order) distribution [IFR(2)] if F_{t_1} SSD$_1$ F_{t_2}, $0 \leqq t_1 < t_2 < \infty$.

Definition 4.2. F is called a new better than used (of second order) distribution [NBU(2)] if $F \text{ SSD}_1 F_t$, $0 \leqq t < \infty$.

Definition 4.3. F is called a harmonic new better than used in expectation (of third order) distribution [HNBUE(3)] if $G \text{ TSD}_2 F$ where G is the exponential distribution with the same mean as F.

The motivation for calling these IFR(2), NBU(2) and HNBUE(3) is, of course, the order of the stochastic dominance used. It was made clear in Proposition 3.1(iii) that the HNBUE criterion involves a second-order stochastic dominance criterion.

Simple arguments bring out the following implications connecting these three new criteria with those of the previous section.

Proposition 4.1.

(i) IFR $\Rightarrow$ IFR(2) $\Rightarrow$ DMRL

(ii) NBU $\Rightarrow$ NBU(2) $\Rightarrow$ NBUE

(iii) HNBUE $\Rightarrow$ HNBUE(3)

(iv) IFR(2) $\Rightarrow$ NBU(2).

5. Comparison in terms of the equilibrium distribution

In earlier sections we have referred to

$$H_F(t) = \frac{1}{\mu} \int_0^t \bar{F}(x)dx, \qquad 0 \leqq t < \infty$$

the equilibrium distribution corresponding to the life distribution F. Let a system be of the following type. We have a unit in operation whose life distribution is F. As soon as this unit fails, another new unit, which acts independently of the first and has the same life distribution, is activated. This renewal of the system is continued indefinitely. Then the residual life of the unit under operation at time t as $t \to \infty$ is given by $H_F(t)$, the equilibrium distribution. Hence, obviously, stochastic comparisons between the equilibrium distributions corresponding to life distributions F and G or between F and its equilibrium distribution H_F are meaningful from the point of view of positive ageing. The life distribution of the unit which ages (adversely) more rapidly will come off worse in such a comparison. Proposition 3.1(iv) says exactly this where the equilibrium distribution of an HNBUE (positive ageing) distribution is seen to be stochastically dominated by the exponential distribution (no ageing) of the same mean.

Also, it is easily seen that the failure rate of the equilibrium distribution is the reciprocal of the mean residual life function of the original distribution, i.e. $r_{H_F}(t) = 1/e_F(t)$, $t > 0$. Hence we immediately have the following result.

Proposition 5.1. F is DMRL $\Leftrightarrow H_F$ is IFR.

This fact has been noted by Brown and Ge (1984). Since exponential H_F is equivalent to exponential F, it is observed here that IFR H_F meaning rather strong positive ageing of H_F is equivalent to somewhat mild positive ageing (DMRL) of F. We now put successively milder positive ageing conditions on H_F.

Definition 5.1. F is called a decreasing mean residual life in harmonic average (DMRLHA) if $[(1/t)\int_0^t (1/e_F(u))du]^{-1}$ is a decreasing function in t. By recognizing that the expression in square brackets is the failure rate average of H_F we have the following.

Proposition 5.2. F is DMRLHA $\Leftrightarrow H_F$ is IFRA.

Proposition 5.3. F is NBUE $\Leftrightarrow r_{H_F}(t) \geqq r_{H_F}(0)$ for $0 < t < \infty$.

Proof.

$$r_{H_F}(t) \geqq r_{H_F}(0) \qquad \forall t$$
$$\Leftrightarrow e_F(t) \leqq e_F(0) \qquad \forall t$$
$$\Leftrightarrow F \text{ is NBUE.}$$

Proposition 5.4. DMRL $\Rightarrow$ DMRLHA $\Rightarrow$ NBUE.

Proof. This proposition is just a restatement of the following chain of implications:

$$H_F \text{ is IFR} \Rightarrow H_F \text{ is IFRA} \Rightarrow r_{H_F}(t) \geqq r_{H_F}(0) \qquad \forall t.$$

Proposition 5.5. F is HNBUE $\Leftrightarrow (1/t)\int_0^t r_{H_F}(u)du \geqq r_{H_F}(0)$ for $0 < t < \infty$.

Proof. Substitute $1/e_F(t)$ by $r_{H_F}(t)$ and μ by $1/r_{H_F}(0)$ in the definition of HNBUE as given by Rolski (1975).

6. Comparison in terms of the failure rate

The most commonly applied concepts of positive ageing are in terms of the failure rate $r_F(t)$ of the distribution. The IFR and IFRA concepts are defined in Section 3. Here we provide two more criteria describing positive ageing in terms of the failure (hazard) rate.

Definition 6.1. F is called a new better than used in failure rate (NBUFR) distribution if $r_F(0) \leqq r_F(x)$, $0 \leqq x < \infty$.

Definition 6.2. F is called a new better than used in failure rate average (NBUFRA) distribution if $r_F(0) \leqq (1/x)\int_0^x r(u)du$, $0 < x < \infty$. This has recently been proposed by Loh (1984).

Proposition 6.1. NBU $\Rightarrow$ NBUFR $\Rightarrow$ NBUFRA.

Proof.

$$F \text{ is NBU} \Leftrightarrow \frac{\bar{F}(t+x)}{\bar{F}(t)} \leqq \bar{F}(x) \qquad \forall t, x > 0$$

$$\Leftrightarrow \frac{\bar{F}(t+x)-\bar{F}(t)}{\bar{F}(t)} \leqq -F(x)$$

$$\Rightarrow \lim_{x\to 0} \frac{F(x)}{x} \leqq \lim_{x\to\infty} \frac{F(t+x)-F(t)}{x\bar{F}(t)}$$

$$\Leftrightarrow f(0) \leqq \frac{f(t)}{\bar{F}(t)} \quad \text{or} \quad r_F(0) \leqq \frac{f(t)}{\bar{F}(t)} \qquad \forall t$$

$$\Leftrightarrow F \text{ is NBUFR.}$$

The second implication is obvious.

Proposition 6.2. Let G_0 be the exponential distribution with the constant failure rate $r_F(0)$. Then F NBUFRA $\Leftrightarrow G_0$ FSD F.

Proof.

$$F \text{ NBUFRA} \Leftrightarrow r_F(0) \leqq \frac{1}{x}\int_0^x r_F(u)du \qquad \forall x$$

$$\Leftrightarrow -xr(0) \geqq -\int_0^x r_F(u)du \qquad \forall x$$

$$\Leftrightarrow \exp\{-xr(0)\} \geqq \exp\left\{-\int_0^x r(u)du\right\} \qquad \forall x$$

$$\Leftrightarrow G_0(x) \leqq F(x) \qquad \forall x.$$

This proposition shows that NBUFRA can be said to be comparing the distribution F to the exponential distribution G_0 which has the same failure rate as the initial failure rate $r(0)$ of F, in the first-order stochastic dominance comparison. And F, being a positive ageing distribution, comes off worse in this comparison. The definition (vi) of Section 3 of HNBUE distributions may be recalled wherein a second-order stochastic dominance comparison between the distribution F and the exponential distribution with the same mean is carried out.

At this stage we also recall that $[e_F(t)]^{-1}$, the reciprocal of the mean residual life function of the distribution F, is the same as $r_{H_F}(t)$, the failure rate of the equilibrium distribution H_F corresponding to F. Hence we have the following results.

Proposition 6.3. H_F is NBUFR $\Leftrightarrow F$ is NBUE.

Proposition 6.4. H_F is NBUFRA $\Leftrightarrow F$ is HNBUE.

Proof.

$$H_F \text{ is NBUFRA} \Leftrightarrow r_{H_F}(0) \leqq \frac{1}{x}\int_0^x r_{H_F}(u)du, \qquad 0<x<\infty$$

$$\Leftrightarrow \frac{1}{e_F(0)} \leqq \frac{1}{x}\int_0^x \frac{1}{e_F(u)}du, \qquad 0<x<\infty$$

$$\Leftrightarrow e_F(0) \leqq x \Big/ \int_0^x \frac{1}{e_F(u)}du, \qquad 0<x<\infty$$

$$\Leftrightarrow F \text{ is HNBUE.}$$

7. Remarks

(i) Collecting the results proved in Sections 5 and 6 the following interesting correspondence between the positive ageing properties of a probability distribution F and those of its equilibrium distribution H_F is noticed:

F is exponential $\Leftrightarrow$ H_F is exponential

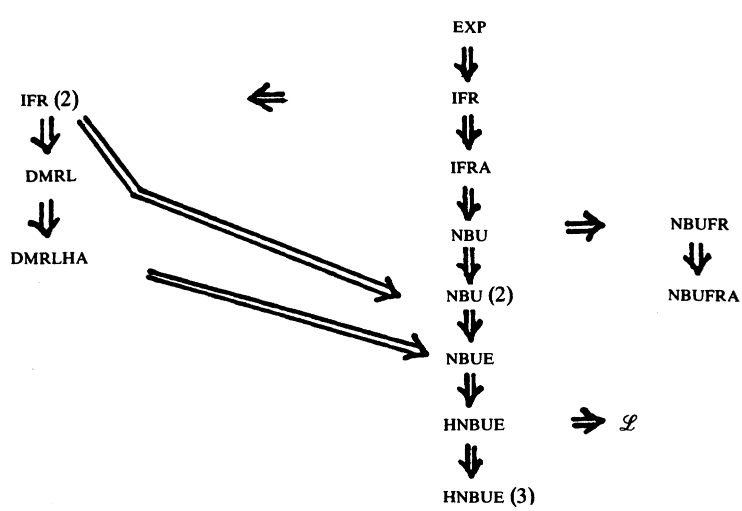
F is DMRL $\Leftrightarrow$ H_F is IFR

F is DMRLHA $\Leftrightarrow$ H_F is IFRA

F is NBUE $\Leftrightarrow$ H_F is NBUFR

F is HNBUE $\Leftrightarrow$ H_F is NBUFRA.

(ii) The complete chain of implications among the positive ageing criteria now introduced and those already existing in the literature is given below.



(iii) Looking at the correspondences in (i) above and since it is known that IFR $\Rightarrow$ IFRA, IFRA $\Rightarrow$ NBU, NBUE $\Rightarrow$ HNBUE are strictly one-way implications, it follows that DMRL $\Rightarrow$ DMRLHA $\Rightarrow$ NBUE and NBUFR $\Rightarrow$ NBUFRA are also strictly one-way implications. Using the well-known properties of stochastic dominances of the different orders we know that IFR $\Rightarrow$ IFR (2) $\Rightarrow$ DMRL, NBU $\Rightarrow$ NBU (2) $\Rightarrow$ NBUE and HNBUE = HNBUE (3) too are only one-way implications.

(iv) Harshinder Singh and Deshpande (1985) have proposed some additional positive ageing criteria which have not been included in this study. They use Laplace transforms of distributions.

(v) The closure of the positive ageing classes defined by the new criteria under formation of coherent structures, convolutions and mixtures has not yet been completely studied. This study, when complete, will form a separate communication. Also, the use of this study for the purposes of statistical inference is yet to be made.

(vi) A similar study of the corresponding negative ageing criteria and the closure properties of the consequent classes of probability distributions can also be carried out.

(vii) There are some points of contact between this study and the first chapter of Stoyan (1983) and the last chapter of Ross (1983). Our presentation is essentially from the stochastic dominance point of view, and aims to be self-contained.

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References

BARLOW, R. E. AND PROSCHAN, F. (1975) *Statistical Theory of Reliability and Life Testing.* Holt Rinehart and Winston, New York.

BROWN, M. AND GE, G. (1984) Exponential approximations for two classes of ageing distributions. *Ann. Prob.* **12**, 869–875.

BRYSON, M. C. AND SIDDIQUI, M. M. (1969) Some criteria for aging. *J. Amer. Statist. Assoc.* **64**, 1472–1483.

DESHPANDE, J. V. AND SINGH, S. P. (1984) Ordering of probability distributions in the context of economics. *Proc. Ann. Conf. Indian Soc. Theory Probability and Applications* (1981 session).

FISHBURN, P. C. (1977) Mean-risk analysis with risk associated with below-target returns, *Amer. Econom. Rev.* **67**, 116–126.

HANOCH, G. AND LEVY, H. (1969) The efficiency analysis of choices involving risk. *Rev. Econom. Stud.* **36**, 335–346.

HARSHINDER SINGH AND DESHPANDE, J. V. (1985) On some new aging properties. *Scand. J. Statist.*

KLEFSJÖ, B. (1983) A useful aging property based on the Laplace transform. *J. Appl. Prob.* **20**, 615–626.

KLEFSJÖ, B. (1984) Reliability interpretations of some concepts from economics. *Naval Res. Logist. Quart.* **31**, 301–308.

LOH, W. Y. (1984) Bounds on ARE's for restricted classes of distributions defined via tail orderings. *Ann. Statist.* **12**, 685–701.

ROLSKI, T. (1975) Mean residual life. *Internat. Statist. Inst.* **46**, 266–270.

ROSS, S. M. (1983) *Stochastic Processes.* Wiley, New York.

STEIN, W. E., DATTERO, R. AND PFAFFENBERGER, R. C. (1984) Inequalities involving the lifetime of series and parallel systems. *Naval Res. Logist. Quart.* **31**, 647–651.

STOYAN, D. (1983) *Comparison Methods for Queues and Other Stochastic Models.* Wiley, New York.

TESFATSION, L. (1976) Stochastic dominance and the maximization of expected utility. *Rev. Econom. Stud.* **43**, 301–314.

WHITMORE, G. A. (1970) Third order stochastic dominance, *Amer. Econom. Rev.* **50**, 457–459.