
Section 3.6: A, 3

- A. Let $(X, \|\cdot\|_X)$ and $(Y, \|\cdot\|_Y)$ be normed spaces, and suppose that $A \in \mathcal{L}(X, Y)$, $B, C \in \mathcal{L}(Y, X)$ and $M, N \in \mathcal{L}(X)$. Show the following:
- (a) If $AB = I_Y$ (the identity map on Y), then A is surjective (onto Y).
 - (b) If $CA = I_X$ (the identity map on X), then A is injective.
 - (c) If $AB = I_Y$ and $CA = I_X$, then A is invertible, with $A^{-1} = B = C$.
 - (d) If $MN = NM$ and N is invertible, then $MN^{-1} = N^{-1}M$.
3. Let $(X, \|\cdot\|)$ be a Banach space, and let $A \in \mathcal{L}(X)$ be such that $\|A^p\| < 1$ for some integer $p \geq 2$. Show that $I - A \in \mathcal{L}(X)$ is bijective, and that its inverse $(I - A)^{-1} : X \rightarrow X$ is also in $\mathcal{L}(X)$.