
Section 3.2: 2 (optional), 3

2. (Optional) Let Ω be a bounded open set, and let $p \in [1, \infty)$. Show that the space $(C(\overline{\Omega}), \|\cdot\|_{L^p(\Omega)})$ is not complete. Hint: Construct a Cauchy sequence that does not converge.
3. Suppose that the space $\mathcal{P}$ of all polynomials $p : \mathbb{R} \rightarrow \mathbb{R}$ is equipped with the norm $\|p\| \doteq \sup_{x \in [0,1]} |p(x)|$. Show that $(\mathcal{P}, \|\cdot\|)$ is not complete.

Hint: You might want to consider $f(x) = e^x$ and its (Maclaurin series) partial sum approximations. Dini's Theorem (Exercise 2.4.1) may be relevant in such an argument.