
Section 2.20: A, 1, 6

- A. Let A be a convex set in a real vector space. Let $f : A \rightarrow \mathbb{R}$ be a convex function, and let $g : f(A) \rightarrow \mathbb{R}$ be a convex and increasing function. Show that $h = g \circ f : A \rightarrow \mathbb{R}$ is a convex function.
1. Let $(X, \|\cdot\|)$ be a normed vector space, and let $p \geq 1$. Show that $h : X \rightarrow \mathbb{R}$ defined by $h(x) = \|x\|^p$ is a convex function.
6. Show that a strictly convex finite-dimensional normed vector space is uniformly convex.
Hint: You may wish to argue that

$$K(\varepsilon) \doteq \{(x, y) \in X \times X : \|x\| = \|y\| = 1 \text{ and } \|x - y\| \geq \varepsilon\}$$

is compact in $X \times X$ (under the standard product topology).