
Section 2.19 4, 6

4. Show that the convex hull of an open set A in a topological vector space X is open.

Note: Recall that X being a topological vector space over the field $\mathbb{K}$ requires that vector addition, $(x, y) \in X \times X \mapsto x + y \in X$, is continuous with respect to the product topology on $X \times X$; and that scalar multiplication, $(\alpha, x) \in \mathbb{K} \times X \mapsto \alpha x \in X$ is continuous with respect to the product topology on $\mathbb{K} \times X$. Some implications of this are that, if $U, V \subset X$ are open, and $\alpha \in \mathbb{K}$ is non-zero, then $U + V$ and αU are also open.

6. (a) (Carathéodory's Theorem) Let $A \subset \mathbb{R}^n$. Show that any point $x \in \text{co}(A)$ can be expressed as a convex combination of at most $n + 1$ points.
- (b) Using (a), show that the convex hull of a compact subset of $\mathbb{R}^n$ is also compact.