
Section 2.13: 1, A (optional)

1. Let X and Y be normed vector spaces over the same field, and let $A : X \rightarrow Y$ be a compact linear operator. Show that $\text{Im } A$ is separable.
- A. **(Optional)** Let X be a normed vector space, $A : X \rightarrow X$ be a compact linear operator, and $\lambda \in \mathbb{K}$ be non-zero. Show that $\lambda I - A$ is injective if and only if $\|(\lambda I - A)x_n\| \rightarrow \infty$ for every sequence $(x_n)_{n \in \mathbb{N}}$ in X satisfying $\|x_n\| \rightarrow \infty$.