NUMS 2024 Abstracts

Keynote Address

Speaker: Hannah Kravitz (Portland State University)

Title: Metric graphs: spectral methods, localization, and applications

Abstract: I study structures called "metric graphs," which are networks with a distance metric defined on their edges. This creates a 1D structure on which to solve partial differential equations (PDEs), with boundary conditions in the form of coupling conditions at the vertices. This seemingly simple setup (it is in one dimensional after all), leads to many interesting questions like:

1. What causes waves to get trapped in certain parts of the graph (localization)?
2. Which frequencies can exist on a graph (the eigenvalue problem)?
3. How can the research be applied in areas like optical science and modeling the spread of epidemics?
4. How can we utilize the graph's structure to develop efficient computational algorithms? Throughout this talk, I will explore these questions and share insights into how metric graphs offer new ways of thinking about wave behavior in a range of contexts.

Bio: Dr. Kravitz is an Assistant Professor of Mathematics at Portland State University. Her research focuses on numerical methods for partial differential equations on metric graphs, as well as applications of network science to epidemiology. This research combines many fields including numerical analysis, computational math and network science, and her work has several applications in public health and optical science.
Prior to graduate school, Dr. Kravitz worked as a data analyst for University of Washington's Institute for Health Metrics & Evaluation, conducting survey programming, census enumeration and data collection logistics, and verification and analysis of incoming data. In the past, she has also worked as an environmental engineer at a consulting firm and a governmental agency, performing air dispersion modeling, geographical data analysis, and environmental permitting.

Student Talks (alphabetical by last name of speaker)

Speaker: Alex Albors Juez (University of Washington)

Title: Graph-based methods for Semi-Supervised Learning

Abstract: Semi-Supervised Learning leverages the geometry of data to improve predictions with very few labeled examples. A prominent approach involves graph-based methods, where data is embedded into a graph and predictions are made by minimizing an energy function on this graph. I will survey two key methods, Laplace learning and Poisson learning, and discuss how tools from the calculus of variations and random walks can be used to study the behavior of these solutions.

Speaker: Greg Baimetov (University of Washington)

Title: A Decomposition Theorem on Balanced Measures

Abstract: Let $G = (V, E)$ be a connected graph. For a probability measure μ on V, let $T_\mu(v)$ be the "earth-mover's" cost of transporting the mass of μ to v. If all points in the support of μ are maxima of T_μ, then we call the measure v balanced. We prove that every graph G has balanced measures of a type which we call basic, such that any balanced measure on G is a convex combination of basic balanced measures, and provide a bound on the number of basic balanced measures on G.

Speaker: Gemma Bertain (University of Puget Sound)

Title: Reduced Trip Permutations of Plabic Graphs

Abstract: The decorated trip permutation of a reduced plabic graph tells us what cell in the Grassmannian the graph parameterizes. This research studies the trip permutations of unreduced plabic graphs and how they change when the graphs are reduced. If the output is a permutation with no fixed points, this tells us what cell in the Grassmannian an unreduced graph parameterizes without actually reducing the graph.

Speaker: Hunter Burr (Portland State University)

Title: Investigating MaxCut on Hypergraphs: Determining the Usage of Hypergraph-Specific and Graph-Based Methods

Abstract: We Investigated whether the MaxCut algorithm presented in recent research from UC San Diego (Distributed Constrained Combinatorial Optimization leveraging Hypergraph Neural Networks, Heydaribeni et. all) effectively utilizes hypergraph information, or applies graph-based methods after performing a projection. This was done by generating hypergraphs with varying edge configurations within and between clusters to assess performance differences. By systematically varying parameters P_{in} and Q_{in} from 0 to 1, we create heat maps identifying zones where hypergraph methods excel and graph methods falter. This

mapping was compared to the solution set we generated by running these graphs through the UC San Diego neural network to determine possibly what methods it was using.

Speaker: Hannah Dempsey (Oregon State University)

Title: Transcending the Alternating Harmonic Series

Abstract: In this presentation, we find a closed form expression for the sum of a general class of rearrangements of the alternating harmonic series. These expressions involve the subharmonic numbers, which explains the presences of radicals, π, and the logarithms of rational numbers. We also discuss the transcendence of the sums of these rearrangements.

Speaker: Mia Escobar (University of Washington, Tacoma)

Title: Dispersion reduction schemes for the three dimensional acoustic wave equation

Abstract: Finite-difference (FD) methods for the wave equation are flexible, robust and easy to implement. However, they in general suffer from numerical dispersion. FD methods based on accuracy give good dispersion at low frequencies, but waves tend to disperse for higher wave numbers. Moreover, waves in higher dimensions also suffer from dispersion errors in all propagation angles. In this work, we will explore a few different methodologies to derive dispersion reduction FD schemes for the three dimensional acoustic wave equation. This is an ongoing project.

Speaker: Lila Goldman (Western Washington University)

Title: Chutes and Ladders as a Markov Process

Abstract: Chutes and Ladders is a very simple board game with little strategy involved. The gameplay is dictated entirely by spinning a spinner and moving one's game piece accordingly. Because of this, players often want to know, "How long is this going to take?". As it turns out, we are able to answer that question with certainty because Chutes and Ladders is a fantastic example of a Markov process. In this talk, we will construct a Markov chain and use it to predict potential game lengths of Chutes and Ladders.

Speaker: Tamara Gratcheva (Portland State University)

Title: PT symmetry and eigenmodes

Abstract: Spectra of systems with balanced gain and loss, described by Hamiltonians with parity and time-reversal (PT) symmetry is a rich area of research. This work studies by means of numerical techniques, how eigenvalues and eigenfunctions of a Schrödinger operator change as a gain-loss parameter changes. Two cases on a disk with zero boundary conditions are considered. In the first case, within the enclosing disk, we place a parity (P) symmetric configuration of three smaller disks containing gain and loss media, which does not have PT -symmetry. In the second case, we study a PT-symmetric configuration of two smaller disks with gain and loss media. We find a rich variety of exceptional points, re-entrant PT-

symmetric phases, and a non-monotonic dependence of the PT-symmetry breaking threshold on the system parameters. Previous explorations of spectra of PT-symmetric systems have mainly been limited to finite discrete models or problems in one dimension. By leveraging systems on a two-dimensional continuum, we show how the complexity and variability of the spectral behavior increases. Finally, by considering small analytically computable examples, we study the concept of exceptional points and their relation to the PT-symmetry breaking threshold.

Speaker: Nathaniel Jett (Pacific University)

Title: Properties of nonzero component graphs of vector spaces over finite fields

Abstract: In 2016, Angsuman Das defined a graph over a finite-dimensional vector space called the nonzero component graph. Each vector in the vector space is a vertex in the graph, and two vertices are connected if the basis expansions of their associated vectors share a basis vector with nonzero coefficient. One major conclusion of this paper was that nonzero component graphs of isomorphic vector spaces are themselves isomorphic. A follow-up paper published in 2017 restricts these graphs to vector spaces over finite fields and explores consequential properties. In this talk, I restrict the graphs even further to fields with only two elements and discuss consequential properties. I will also extend this work and present two similarly-constructed graphs with different properties: the equal component graph and the complement graph of the nonzero component graph and show some examples.

Speaker: Warren Johnson (Oregon State University)

Title: A Journey Through Recursion: Multivariate Generating Functions

Abstract: Many problems in math require iteration, be it of derivatives, integrals, summations, or root-finding methods. I will discuss techniques and strategies I have learned while trying to unravel a system of differential difference equations, their solutions, and methods for "detangling" the solution set. This system of equations arises when trying to find the nth derivative of $\cos(x^2/2)$ with respect to x. General observations will be made, behaviors exhibited, representation in the language of operator theory, and then generalizations to non-polynomial arguments will be given. Steady state solutions are shown to exist for the system, and a generating function solution for the solution is provided.

Speaker: Aleister Jones (University of Washington)

Title: On Triangulations of Order Polytopes for Snake Posets

Abstract: Motivated by recent work by von Bell et al., we study triangulations of the order polytopes of generalized snake posets. We impose a partial ordering on the regular triangulations of this order polytope, and conjecture that it is a lattice and that it induces a good orientation as defined by Kalai. We explore the geometry of the secondary polytope of the order polytope and make progress towards proving that all two-dimensional faces are quadrilaterals, pentagons, and hexagons. We also investigate the "twist group", as

introduced by von Bell et al., which acts on the set of regular triangulations. We prove the twist action is free and give an eigenbasis for the action of the twist group on the secondary polytope. Lastly, we show that a subset of the triangulations admit the same number of bistellar flips. Our results make partial progress towards proving certain conjectures posed by von Bell et al.

Speaker: Emerson Konkol (Boise State University)

Title: Distributions of Graph Primes

Abstract: In analogy to the addition and multiplication defined on the positive integers, one can define operations of addition and multiplication on graphs. The accompanying algebraic structure then admits an analogous notion of 'primeness', which is quite rich. However, due to the difficulty of verifying whether a graph is prime or composite, the properties and distribution of these primes is not fully understood in the literature.
Through a mixed strategy of computational and analytical work, we've collected data which demonstrates the distribution of graph primes and composites with respect to 3 different products and the relationships between these distributions. From this data, we formulate conjectures to provide motivation for future research.

Speaker: Shosuke Kiami (University of Washington)

Title: Stochastic Dynamical System

Abstract: In their recent work, Steinerberger-Zeng introduced a class of dynamical systems in the complex plane that take on the form $z_n = z_{n-1} \pm e^{ia_n}$ where a_n is some sequence of reals and the sign of "+/-" is chosen so as to minimize $|z_n|$. In our work, we study the case where a_n is chosen uniformly at random from $[0, 2\pi)$ which yields a stochastic process with a peculiar looking stationary distribution. This stationary distribution has yet eluded a closed form, however, we prove various properties about the stationary distribution including its asymptotic behavior in higher dimensions.

Speaker: Malyna Malinov (Portland State University)

Title: Several Results Concerning Projectors and Their Norms

Abstract: A brief introduction is given for projectors and norms of projectors in finite dimensional Hilbert spaces, with an explanation and proof of a property of elements of magnitude 1 related to the norm of a given projector. Namely, given a projector P and an element u of magnitude 1 in a finite dimensional Hilbert space, with the property that $\|Pu\| = \|P\|$, we show that u is orthogonal to the nullspace of the given projector.

Speaker: Ishan Panpaliya (Seattle University)

Title: On the Atomicity of One-dimensional Monoid Algebras

Abstract: An integral domain is said to satisfy the ascending chain condition on principal

ideals (ACCP) if every ascending chain of principal ideals stabilizes. The ACCP is almost always complementary to atomicity within the context of integral domains: so much so, in fact, that Cohn himself stated without proof that these two conditions are equivalent. This assertion has since been shown to be false, however most known counterexamples utilize technical algebraic constructions. In 2017, Gotti conjectured that for every $q > 0$, atomicity ascends from the exponentially cyclic Puiseux monoid M_q to its monoid algebra over the field of rationals. It can be shown quite easily that the previous conjecture holds when q is not contained in the set $S := [(0,1) \cap \mathbb{Q}] \setminus \mathbb{N}_{\geq 2}^{-1}$. If this conjecture were true when $q \in S$, it would provide an extremely wide class of atomic integral domains of Krull dimension one which do not satisfy the ACCP, and so would be perhaps the simplest possible such class of counterexamples. In a recent preprint, Bu et al. proved the atomicity of the monoid algebra $\mathbb{Q}\left[M_{3/4}\right]$, establishing the first step towards settling this conjecture. Here, we prove an analogous result for all $q \in S$ having denominator an odd prime power.

Speaker: Jinha Park (Boise State University)

Title: Leading Digits of the p-adic Sequences

Abstract: The *first digit problem* has been actively studied in $\mathbf{R}$ and has the famous, and rather counterintuitive, result called Benford's law. Instead of sequences in $\mathbf{R}$, we investigate the p-adic leading digits of some sequences in $\mathbf{Q}_p$, the p-adic numbers. We find some p-adic sequences whose p-adic leading digits have repeating behavior. In addition, we show some sequences that experimentally seem to have Benford distribution.

Speaker: Anna Singley (Los Alamos National Laboratory)

Title: Network Analysis of Collaboration in Mathematics

Abstract: Complex networks can be used to explore a plethora of biological, social, and ecological systems. Visualization of complex data with a network allows for more intuitive analysis and realization of underlying structure in a dataset. Tools from network topology and graph theory allow for a more thorough study of the evolution of a network over time. Using a repository of open access journal articles organized by math subject code (MSC), we can visualize the evolution of a field over with the use of networks. Further, we can conduct a process similar to contact tracing to attempt to study the spread of an idea over a collaborative network - helping us to visualize the evolution of a mathematical subfield.

Speaker: Clare Strode (Willamette University)

Title: Subdivided Icosahedron Coloring

Abstract: While playing around with geometry toys, I had an idea- I wanted to build an icosahedron where each triangular face was subdivided into smaller, differently colored triangles. The obstacle I quickly ran into was that all of the configurations I tried were aesthetically ugly. I wanted my icosahedron to be pretty- but how does one mathematically define pretty? I came up with a definition that worked for me: A graph P is prettily colored

if it meets the following criteria:

1. P is properly colored using 4 colors
2. Every 5-cycle contains all 4 colors

With this definition, I began the daunting task of trying to make my shape pretty. In this talk I will explore the theorems I proved about icosahedron coloring as well as how they apply to coloring my subdivided icosahedron.

Speaker: Esther Toobian (Portland State University)

Title: Multi-Fidelity Data Fusion for Model Optimization: From Simple Data Combination to Advanced Feature Integration

Abstract: This project investigates innovative methods for integrating high-fidelity (hi-fid) and low-fidelity (lo-fid) data to optimize model training in scenarios with limited hi-fid data availability. Beginning with simple data integration methods, we explore increasingly sophisticated techniques, including various methods for combining features extracted from hi-fid and lo-fid models trained independently. By examining how lo-fid data can enhance hi-fid models and, conversely, how hi-fid data can augment models built predominantly with lo-fid data, our work aims to identify optimal fusion techniques that maximize model performance with constrained resources. This presentation will showcase the progression from foundational data combination to advanced feature fusion, along with key insights and planned future directions in multi-fidelity model training.

Speaker: Austin Ulrigg (University of Washington)

Title: A Practical Genus Algorithm

Abstract: The problem of determining the minimal orientable (or nonorientable) genus of a given finite connected graph has been the subject of extensive and intriguing study. In this paper for an arbitrary graph G with V vertices, an $O\left(\frac{2^{\left(V^2+3V\right)}}{V^{(V+1)}}\right)$ time algorithm is presented that determines the orientable genus of G. This algorithm avoids the difficulties that many other genus algorithms have with handling bridge placements and is able to quickly find the genus of graphs with high complexity. Additionally, a method is presented via an example of Balaban's (3,10) cage which presents a method for determining the genus of even larger graphs that would be infeasible to compute using an algorithm alone, by exploiting their automorphism group. Further, we classify the genus of all known $(3, g)$ cage graphs, and other graphs and include this in the appendix. The results and methods of this paper can be used to classify the genus of many graphs and could be a valuable tool for solving open-problems such as completing the list of forbidden toroidal minors.

Speaker: Alice Williams (Central Washington University)

Title: Optimizing Feature Projection with Gradient Descent for Improved Multivariate Classification and Lossless Variate Tracing

Abstract: This work introduces a novel technique for optimizing class separation in multivariate data using gradient descent to identify linear combination coefficients that minimize inter-class variance and maximize intra-class variance. Unlike popular projection methods such as Principal Component Analysis (PCA), T-Stochastic Neighbor Embedding (T-SNE), Multi-Dimensional Scaling (MDS), and Uniform Manifold Approximation and Projection (UMAP), this approach directly operates on scaled data attributes, resulting in more interpretable outcomes. By assigning coefficients as feature importances in a linear combination, the method provides a clearer and more meaningful interpretation compared to common model explanation techniques like LIME and SHAP, which may produce counterfactual explanations. Negative coefficients indicate a retraction along an attribute, while positive coefficients denote an extension. Currently applied to cancer classification and health data analytics, this technique has demonstrated superior performance over all popular methods on benchmark datasets, improving classification accuracy and interpretability. The technique leverages gradient descent to iteratively minimize a variance-based score function, enabling optimal separation for n-D multi-class data. The resulting feature is visualized in $(n+1)$-D space, offering a quasi-white box model construction approach with greater interpretability. This method is applicable across varying dimensionalities and can account for non-linear trends by incorporating trigonometric transformations into the linear combinations, providing a scalable solution for complex data classification tasks. Future work includes applying this technique in conjunction with visual analytical boosting and set-based data slicing, combining methodologies from grey and white box model construction.

Speaker: Ihlara Williamson (University of Portland)

Title: Modeling Insurance Impact on Opioid Addiction

Abstract: In a 2009 study, the Cambridge Health Alliance found unequivocally that "uninsurance is associated with mortality". Specifically, people without insurance who have opioid use disorder are susceptible to higher fatality rates than their insured counterparts. We created a compartmental model that predicts the progression of opioid use and addiction through uninsured and insured populations. We created distinct pathways for insured and uninsured, with the potential for an individual to travel between categories. We were able to predict addiction and fatality rates for the United States as we changed proportion insured, quantifying the impact of the flaws in the current healthcare system.