

Logarithms and Shannon Information

$\log_2 n$ (“log base 2 of n ”) is equal to the power of 2 that yields number n

Examples:

$$\log_2 1 = 0$$

$$\log_2 2 = 1$$

$$\log_2 4 = 2$$

$$\log_2 8 = 3$$

$$\log_2 16 = 4$$

$$\log_2 1/2 = -1$$

$$\log_2 1/4 = -2$$

To calculate $\log_2 n$ on a calculator, you can calculate

$$\log_{10} n / \log_{10} 2$$

Shannon entropy of a message source:

Suppose the message source can emit the symbols 0 and 1.

Let p_0 denote the probability of emitting a 0

Let p_1 denote the probability of emitting a 1

Then the Shannon entropy (or “Information Content”) of the message source is

$$H = -(p_0 \log_2 p_0 + p_1 \log_2 p_1) \text{ bits}$$

More generally, suppose the source can emit the symbols x_1, x_2, \dots, x_n , with probabilities p_1, p_2, \dots, p_n respectively.

Then the Shannon entropy (or “Information Content”) of the message source is

$$H = -\sum_{i=1}^n (p_i \log_2 p_i)$$