Exploring Complexity

In Science and Technology

Oct. 20, 2010

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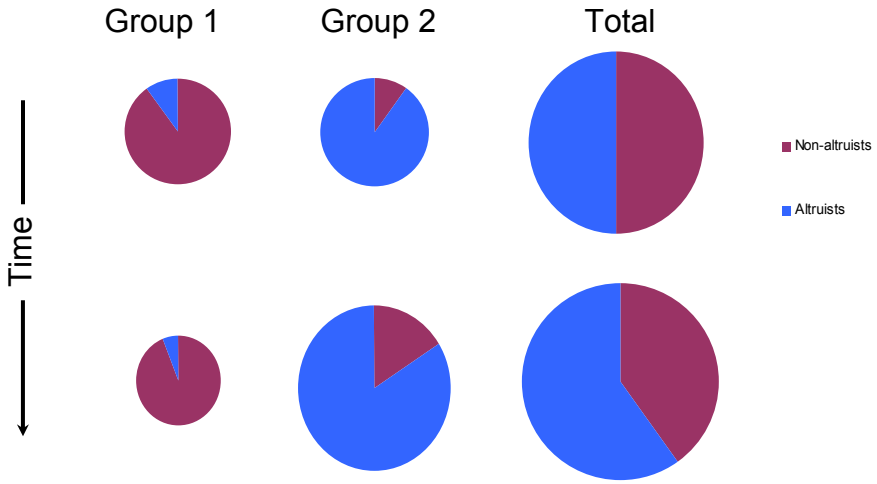
Logistics

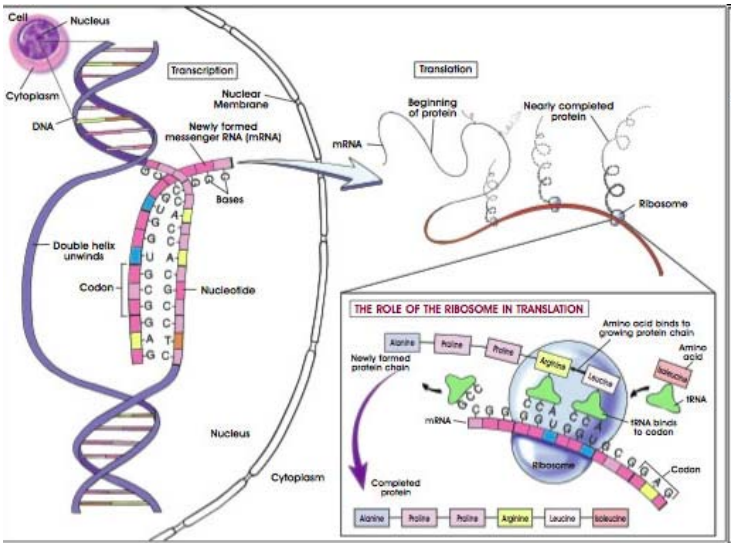
- Due today
  - HW3 due in class
  - Lab2 (Information) due on Blackboard
- Questions?
- If can't upload to Blackboard, please see OIT help desk (Smith basement) or me

Simpson's Paradox & Multilevel Selection



- Fraction altruists (*q*) decreases in each group, but increases overall

Genetics Basics



Complexity measures discussed in book

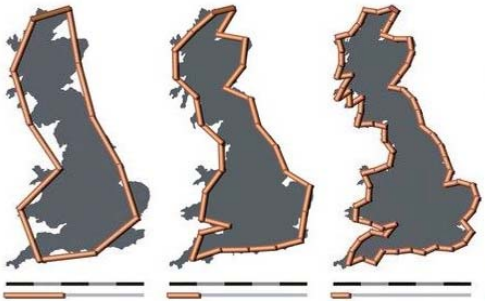
- Complexity as size
- Complexity as entropy
- Complexity as algorithmic information content
- Complexity as logical depth
- Complexity as thermodynamic depth
- Statistical complexity
- Complexity as fractal dimension
- Complexity as hierarchy

Algorithmic Information Content (also called “Kolmogorov Complexity”)

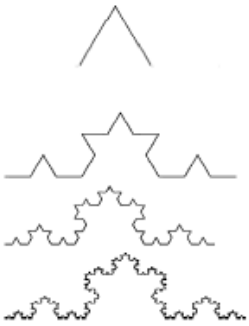
- Let K(*s*) denote the algorithmic information content of string *s*.
- It can be proved that K(s) is incomputabl
  - that is, there is no Turing machine that can compute K(*s*) for any *s*.
- Related paradox (due to Barry):
  - "Let *n* be the smallest positive integer that cannot be defined in fewer than twenty English words."

Fractals and Fractal Dimension

- http://webecoist.com/2008/09/07/17-amazing-examples-offractals-in-nature/
- Measuring the Coastline of Great Britain



Koch curve



- How long is it?
- What is its dimension?

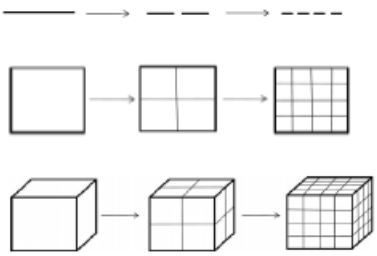
Definition of dimension

- Generalized definition of dimension:
- Let M be the magnification factor of side to get from level n+1 to level n.
- Let N be the number of copies at level n+1 of each object in level n.
- Then

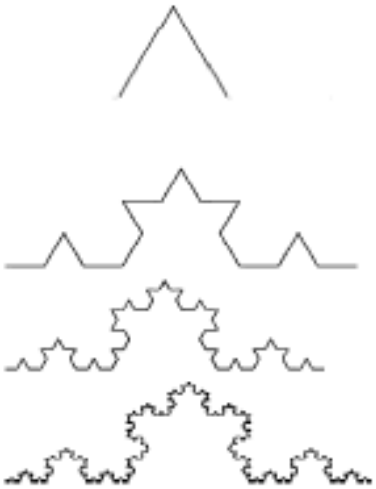
  $M^{Dimension} = N$

  or

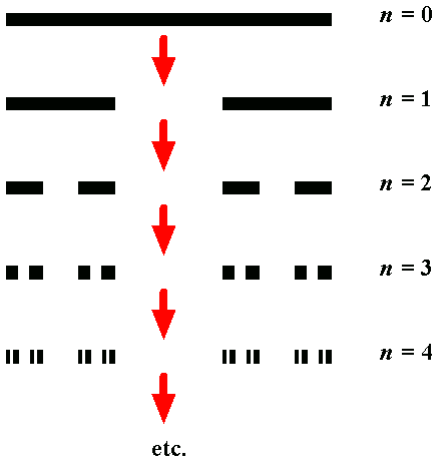
  Dimension = log N / log M



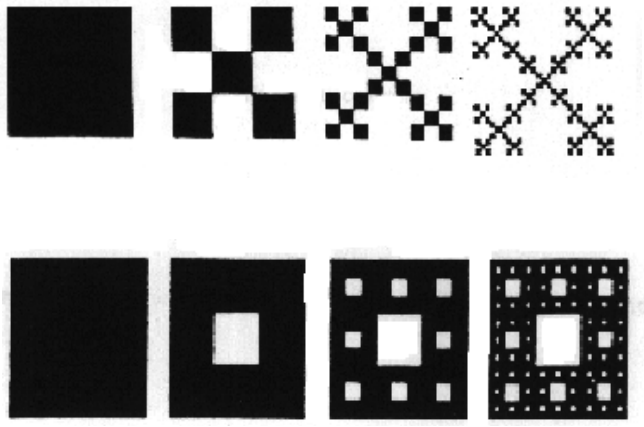
Fractal dimension of Koch curve



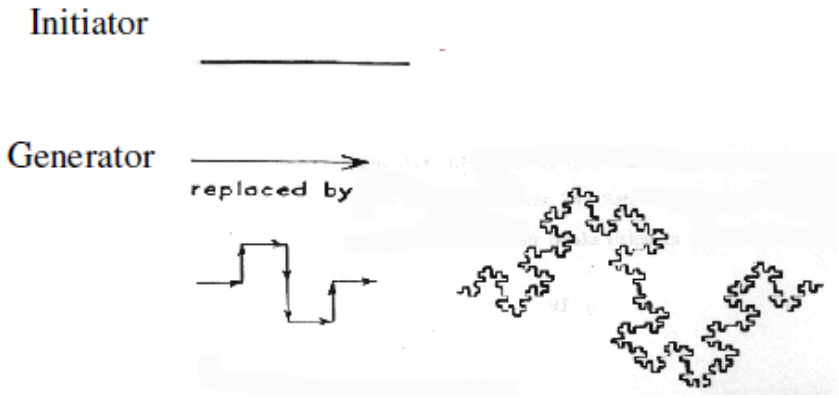
Fractal dimension of Cantor set



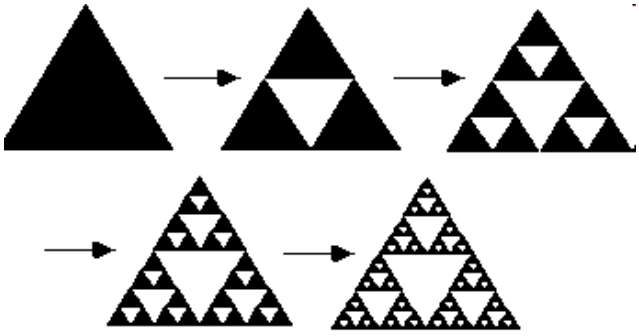
Fractal dimension of box fractals



Another example



Fractal dimension of Sierpinski triangle



- An Exercise!
- Algorithm for building up

Approximate dimension of coastlines (Shelberg, Moellering, and Lam)

Curve	Slope (β)	D (1-β)	New D
West Coast of Great Britain	-.25	1.25	1.2671
Coast of Australia	-.13	1.13	1.1490
Coast of South Africa	-.02	1.02	1.0356
Land-frontier between Spain and Portugal	-.14	1.14	1.1014

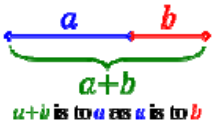
- See Google Maps

Approximate dimension of Cauliflower

- Each branch
  - contains ~13 subbranches,
  - each three times smaller

Fractals and the Golden Ratio

- **Golden ratio**



a+b is to *a* as *a* is to *b*

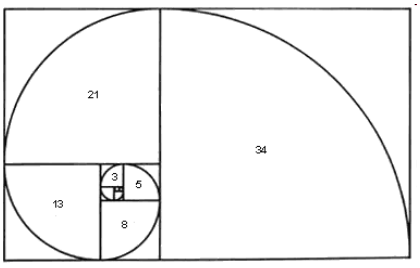
$$\frac{a+b}{a} = \frac{a}{b} = \varphi\,.$$

$$\varphi = \frac{1+\sqrt{5}}{2} \approx 1.6180339887\ldots$$

- **Fibonacci sequence**

  0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987...

Fibonacci spiral



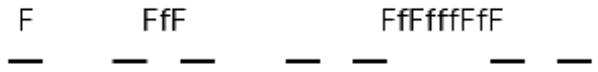
Other Fractals

L systems
(Aristid Lindenmayer)

- Grammars for generating fractals (and other shapes)
- Need “axiom” and “grammar rules”
- Alphabet for rules: {F,f,+, −}
  - − Turn counterclockwise by a specified angle q
  - \+ Turn clockwise by a specified angle q
  - F Move forward one step while drawing a line
  - f Move forward one step without drawing a line

Example 1: Cantor middle thirds set

- **axiom:**
  - **S = F**
- **rules:**
  - **F → FfF**
  - **f → fff**



Example 2: Koch curve

- **axiom:**
  - **S = F**
- **rules:**
  - **F → F − F++F − F**
  - **+ → +**
  - **− → −**
- **q = 60 degrees**