Exploring Complexity

In Science and Technology

Nov. 29, 2010

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Logistics

- Due Today 8; Wednesday Lab 7
- Paper due Friday
- Check that Blackboard grades are accurate
- Final Paper Questions?
- Other Questions?
- Course Evaluations

The (Systems) Science of Networks

- Are there properties common to all complex networks?
- If so, why?
- Can we formulate a general theory of the structure, evolution, and dynamics of networks?
- Example of "stuff free" science
- Looking for commonalities vs. silos
  - Neural networks, social networks, web networks, etc.

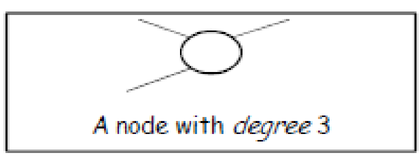
Network Terminology

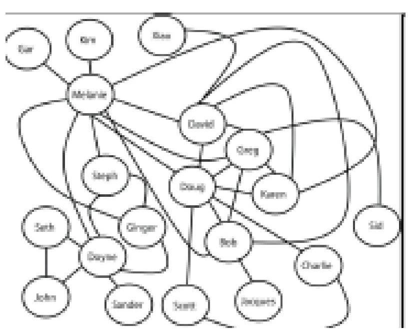
- Network = nodes + links (directional or not)
- Degree of a node
- Clustering (Global and Local)
- Degree distribution
- Hub
- Small World
- Scale Free

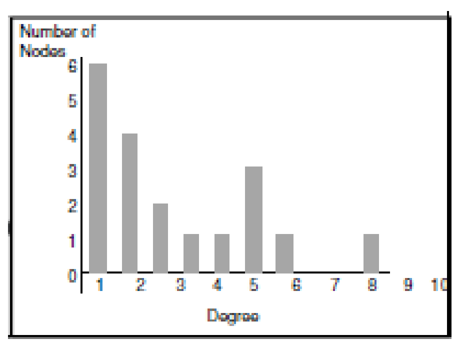
The Small-World Property (Watts and Strogatz)

- The network has relatively few “long-distance” links but there are short paths between most pairs of nodes, usually created by “hubs”.

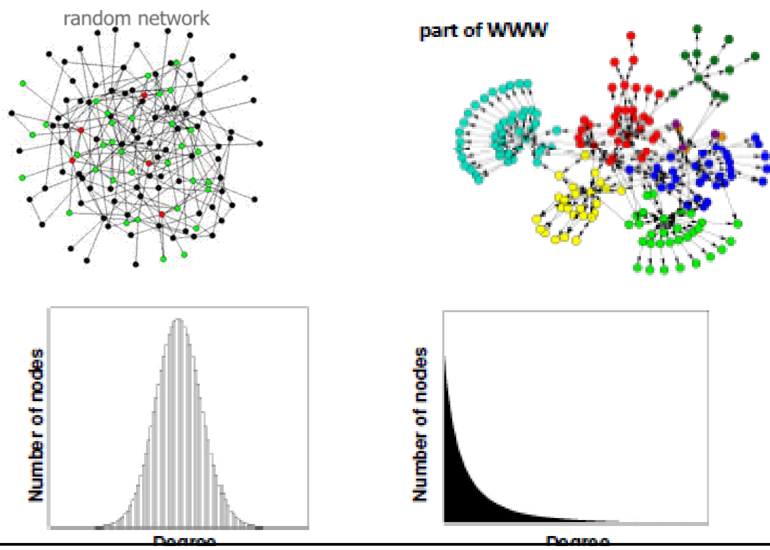
Concept of “Degree Distribution”





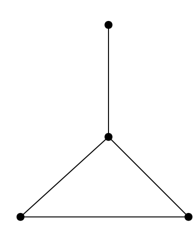


Scale-Free Structure (Albert and Barabási, 1998)



Three Important Measures of Network Topology

- Average path length
  - Average shortest distance between every pair of nodes
- Clustering coefficient
  - Global (about a graph)
    - C = # of closed triplets / number of connected triples of vertices
  - Local (about a node *n*)
    - C = # of *n*'s neighbors with pairwise links / # of potential links
- Degree distribution
- Answers:
  - 4/3
  - Global C = 3/5; for center node C = 1/3
  - 1 degree 1; 2 degree 2; 1 degree 3

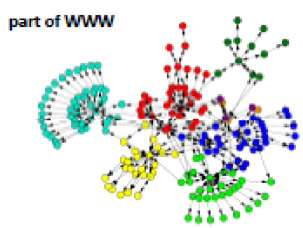


Other examples of power-laws in nature

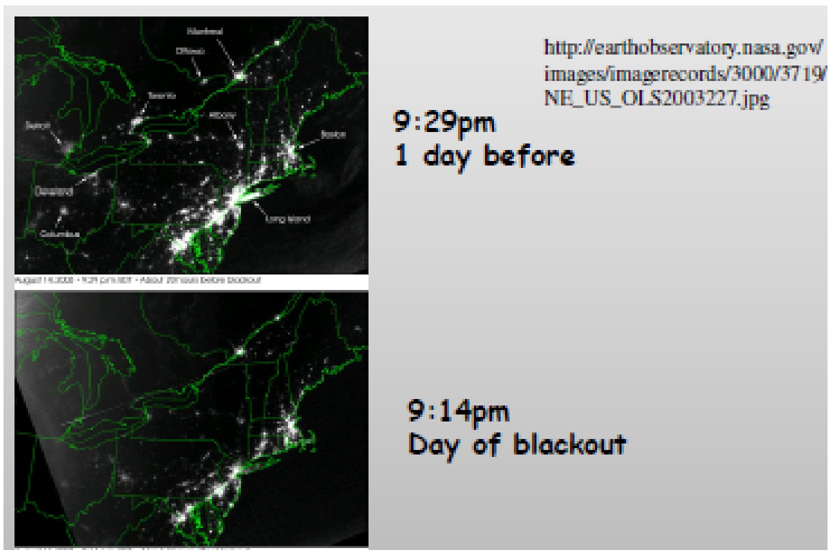
- Magnitude vs. frequency of earthquakes
  - Gutenberg-Richter Law
  - http://www.simscience.org/crackling/Advanced/Earthquakes/GutenbergRichter.html
- Magnitude vs. frequency of stock market crashes
- Income vs. frequency (of people with that income)
- Populations of cities vs. frequency (of cities with that population)
- Word rank vs. frequency in English text

Robustness of Scale-Free Networks

- **Vulnerable** to targeted "hub" failure
- **Robust** to random node failure
  - unless....
  - nodes can **cause** other nodes to fail
  - Can result in ***cascading failure***



August, 2003 electrical blackout in northeast US and Canada

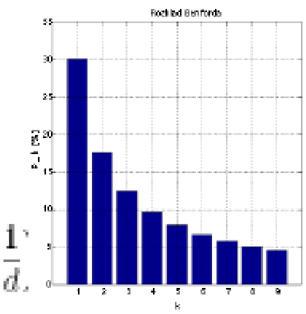


Scaling Laws

- Power law scaling in the Web:
  - Probability that a web page will have k in-links scales as $k^{-1/2}$
- Zipf's law 1:
  - In English text, word frequency scales as (word rank)$^{-1}$
  - Seems to work for other languages also!
  - Implies 2nd ranked word will be ?? times as frequent as 1st ranked word, 3rd ranked word will be ?? times as frequent as 1st ranked word, etc.

Other Scaling Laws

- Zipf's law 2: The population of cities in a country scales as (city rank)$^{-1}$
- Pareto distribution
  - Originally stated as "Number of people owning a fraction k of the total wealth in a country"
  - Also "80-20 rule": 20% of the population owns 80% of the wealth
- Benford's law:
  - in lists of numbers from many real-life sources of data, the probability of the leading digit being d (0 – 9) is



$$P(d) = \log_b(d+1) - \log_b(d) = \log_b\left(1 + \frac{1}{d}\right)$$

Scaling in Biology

- General Question:
  - How do properties of living systems change as their size is varied?
- Example:
  - How does basal metabolic rate (heat radiation) vary as a function of an animal's body mass?
  - Cells converting nutrients to energy give off heat, so heat radiation is a measure of metabolic rate

- **Hamster**
  - Radius = 2 × Mouse radius
  - Mass $\propto$ 8 × Mouse radius
  - Surface area $\propto$ 4 × Mouse radius
- **Hippo**
  - Radius = 50 × Mouse radius
  - Mass $\propto$ 125K × Mouse radius
  - Surface area $\propto$ 2,500 × Mouse radius
- Volume of a Sphere $= 4/3\ \pi\ r^3$
- Surface area of a Sphere $= 4\ \pi\ r^2$
- Hypothesis 1: metabolic rate $\propto$ body mass
- Problem: mass is proportional to volume of animal but heat can radiate only from surface of animal

- Volume of a Sphere = $4/3 \pi r^3$
  - "Volume of a sphere scales as the radius cubed"
- Surface area of a Sphere = $4 \pi r^2$
  - "Surface area of a sphere scales as the radius squared"
- Surface area scales with volume to the 2/3 power
- Hypothesis 2 (): metabolic rate $\propto$ $mass^{2/3}$
  - Max Rubner 1883 the "surface hypothesis"

Mouse

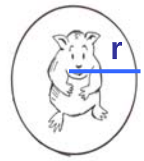
Hamster
(8 x mouse mass)

Hippo
(125,000 x mouse mass)

Metabolic Rate

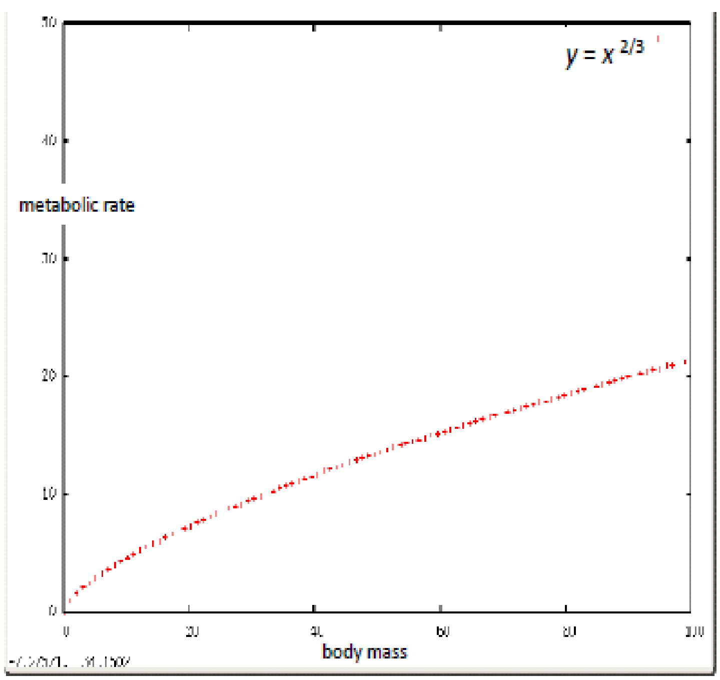
- Metabolic rate: "the process by which energy and materials are transformed within an organism and exchanged between the organism and its environment."
- Metabolic rate = ΣR_i
  - i.e., the sum of energy production (R_i) of all metabolic chemical reactions.
- This rate depends on rate of supply of substrates to cells, and rate of removal of products of reactions.
- The higher the metabolic rate as a function of body mass, the more efficient the organism.

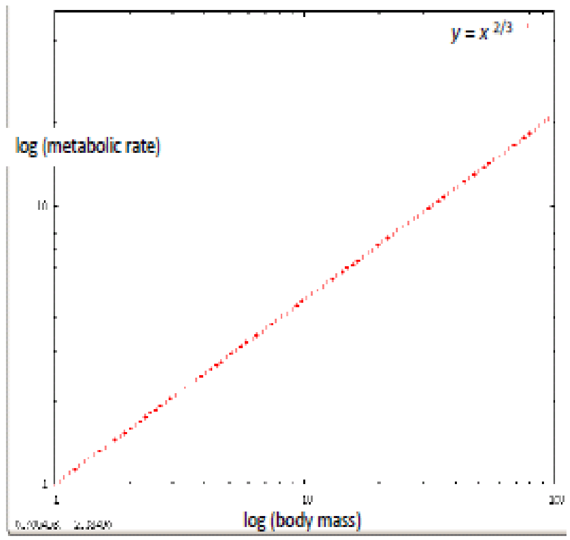
Metabolic Scaling



- Surface hypothesis:
  - Body is made of cells, in which metabolic reactions take place.
  - Can “approximate” body mass by a sphere of cells with radius r.
  - Can approximate metabolic rate by surface area
- Body mass $\propto r^3$
- Metabolic rate $\propto r^2$

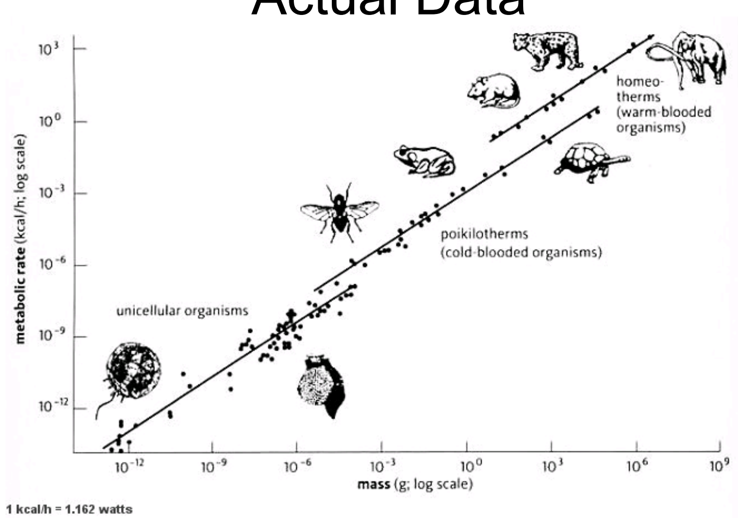
- Thus, metabolic rate $\propto (r^3)^{2/3} \propto$ body mass$^{2/3}$





- On log-log plot, a power law is a straight line whose slope is the exponent of power law:
  - log (metabolic rate) $\propto$ log (body mass$^{2/3}$) =2/3 log(body mass)

Actual Data



- Kleiber's law (1930's): metabolic rate $\propto$ body mass$^{3/4}$

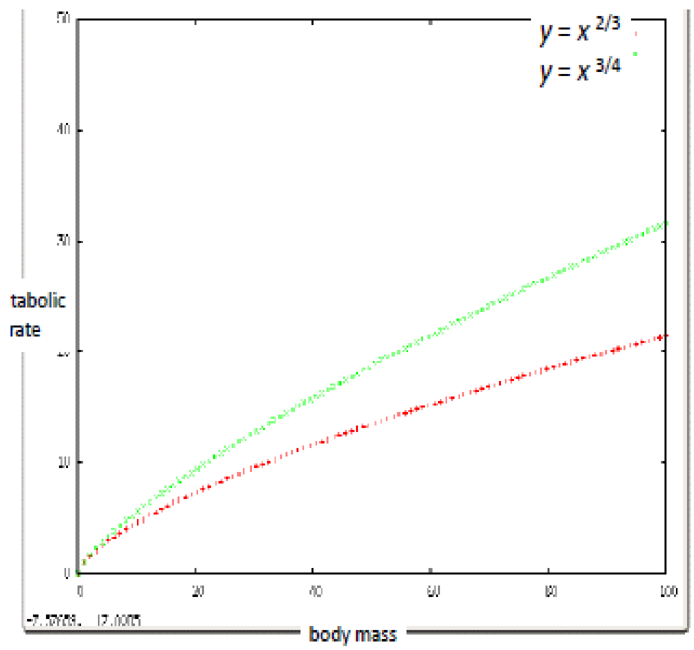
- http://universe-review.ca/I10-83-metabolic.jpg

Question

- Suppose you have a mouse that weighs 20 grams and an elephant that weighs 4,000 kilograms. What is the ratio of the elephant's metabolic rate to the mouse's metabolic rate, according to
  - (a) the Surface Hypothesis?
  - (b) Kleiber's law?
- Answers
  - (a) 3,420
  - (b) 9,457
  - Ratio of mass is 200,000

Cause of Kleiber's law?

- "What common aspect of nearly all organisms could give rise to this simple, elegant law?" (p. 261)
- For sixty years, no explanation



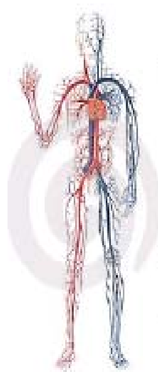
Other Observed Biological Scaling Laws

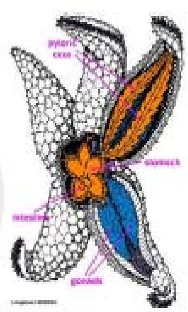
- Heart rate ∝ body mass$^{-1/4}$
- Blood circulation time ∝ body mass$^{1/4}$
- Life span ∝ body mass$^{1/4}$
- Growth rate ∝ body mass $^{-1/4}$
- Heights of trees ∝ tree mass$^{1/4}$
- Sap circulation time in trees ∝ tree mass$^{1/4}$
- **Why quarter-power scaling laws?**

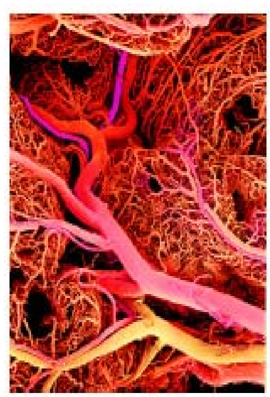
West, Brown, and Enquist's Theory (1990s)

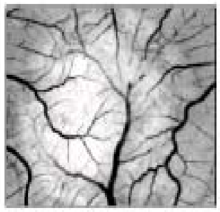
- General idea: "metabolic scaling rates (and other biological rates) are limited not by surface area but by rates at which energy and materials can be distributed between surfaces where they are exchanged and the tissues where they are used."
- How are energy and materials distributed?
  - Depends on delivery system (e.g. circulatory system)

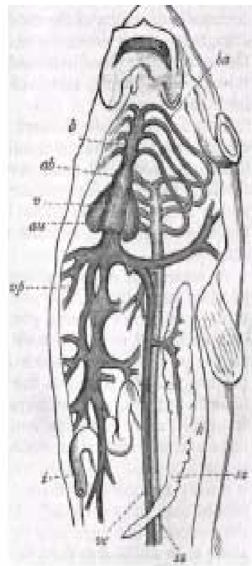
Distribution Systems

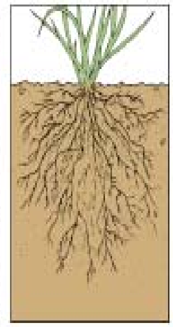


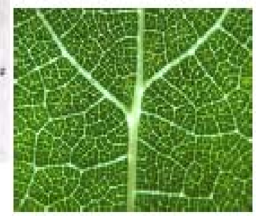






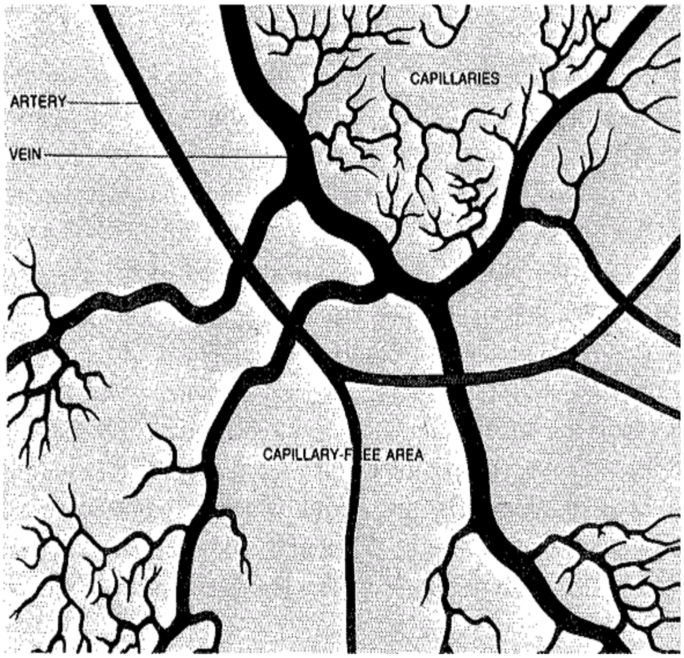






West, Brown, and Enquist’s Theory (1990s)

- Assumptions about distribution network:
  - branches to reach all parts of three-dimensional organism (i.e., needs to be as “space-filling” as possible)
  - has terminal units (e.g., capillaries) that do not vary with size among organisms
    (cells also don’t vary with organism size)
  - evolved to minimize total energy required to distribute resources



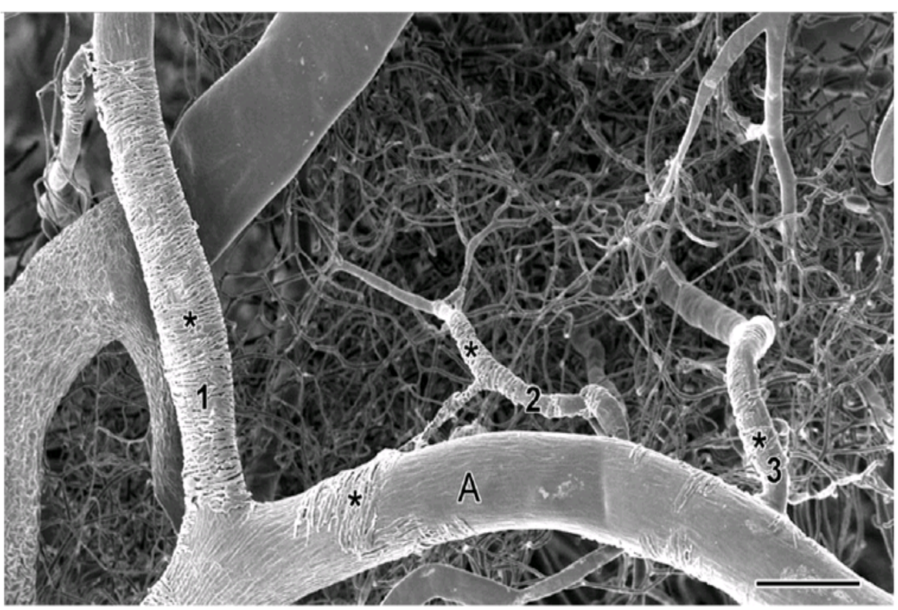


Figure 6. Scanning electron micrograph revealing vasculature within the area corresponding to the maximum acoustically evoked intrinsic signal. The arteries (A) and vein (V) can be clearly distinguished. 1, 2, 3: three types of arterial collateral vessels (see text). Note evidence of smooth muscle banding (asterisk symbols) on arteriole walls. Bar = 100 μm.

Prediction of Theory

- Distribution network will have fractal branching structure, and will be similar in all / most organisms (i.e., evolution did not optimize distribution networks of each species independently)
- Therefore, Euclidean geometry is the wrong way to view scaling; one should use fractal geometry instead!
- With detailed mathematical model using three assumptions, they derived:

  metabolic rate $\propto$ body mass$^{3/4}$

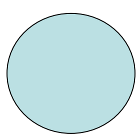
Their interpretation of their model

- Metabolic rate scales with body mass like surface area scales with volume...

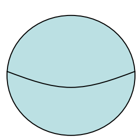
  but in four dimensions!

Recall Relationship of Fractals to Power Laws

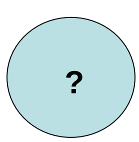
- Power law distributions are fractals
  - Self-similarity at different scales
- “The take-home message is that fractal structure is one way to generate a power-law distribution; and if you happen to see that some quantity (such as metabolic rate) follows a power-law distribution, then you can hypothesize that there is something about the underlying system that is self-similar or ‘fractal-like’. (p. 265)



- Circle
  - circumference = 2π
  - area = πr^2
  - circumference scales as area to the ???? power?



- Sphere
  - surface area = $4\pi r^2$
  - volume = $4/3\ \pi r^3$
  - surface area scales as volume to the ???? power?



- Hypersphere
  - surface volume (hyperarea) = $2\pi^2 r^3$
  - hypervolume = $\frac{1}{2}\ \pi^2 r^4$
  - surface volume scales as hypervolume to the ???? power?

- “Although living things occupy a three-dimensional space, their internal physiology and anatomy operate as if they were four-dimensional. . . Fractal geometry has literally given life an added dimension.”
  – West, Brown, and Enquist

Critiques of their model

- Quarter-power scaling laws are not as universal as the theory claims
- The Kleiber scaling law is valid but the metabolic scaling theory is wrong
- The unsolved mystery of Power Laws
  – “Scientists have a pretty good handle on what gives rise to bell curve distributions in nature, but power laws are something of a mystery. As we have seen, there are many different explanations for the power laws observed in nature (e.g. preferential attachment, fractal structure, self-organized criticality, highly optimized tolerance, among others), and little agreement on which observed power laws are caused by which mechanisms.” (p. 269)