Exploring Complexity

In Science and Technology

Nov. 22, 2010

Jeff Fletcher

Logistics

- Due Today HW7; Wednesday Lab 6
- Check that Blackboard grades are accurate
- Final Paper Questions?
  - I need your paper topic proposals! (if I don’t already have them)
- Questions?
- Other readings
  - Baby names article:
    - http://sites.bio.indiana.edu/~hahnlab/MediaFiles/BabyMedia/BabyNature.html
  - The Power of Power Laws
    - http://edgeperspectives.typepad.com/edge_perspectives/2007/05/the_power_of_po.html

Exercise

- Each player starts with $100 and can choose to invest from $0 to their current balance.
- In each round, each member's investment is subtracted from each member's balance
- The total of all the investments from the current round are doubled and then divide evenly among all the players
- This gives each player a new balance to start a new round
- We will do several rounds—please do not get ahead of (or behind) me. I will announce when each round starts and stops
- Player with highest final score wins a prize
- There is no communication allowed between players

Main Theories for the Evolution of Altruism

- Multilevel Selection
  - Altruist dominated groups do better; altruists within groups do worse
  - $\Delta Q = \Delta Q_B + \Delta Q_W$
- Inclusive Fitness/Kin Selection
  - Gene self interest, Hamilton's rule ($\Delta Q > 0$ if $rb > c$)
  - $w_{inclusive} = w_{direct} + w_{indirect}$
- **Reciprocal Altruism**
  - Conditional behaviour, Iterated Prisoner's Dilemma (IPD), emphasis on non-relatives
  - Indirect reciprocity, strong reciprocity, reciprocity on graphs
- Others
  - By-product mutualism, conflict mediators, policing, social markets

Prisoner’s Dilemma (PD)

Actor's Fitness (Utility)

opponent’s behavior

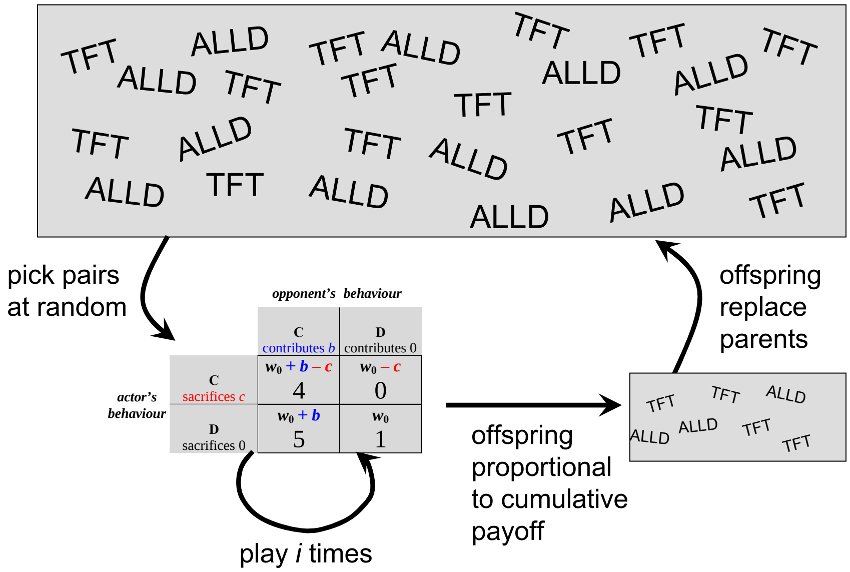
actor’s behavior	**C**	**D**
C	4	0
D	5	1

- Individually rational to defect
- Collectively irrational

Reciprocal Altruism: Iterated Conditional Behaviours

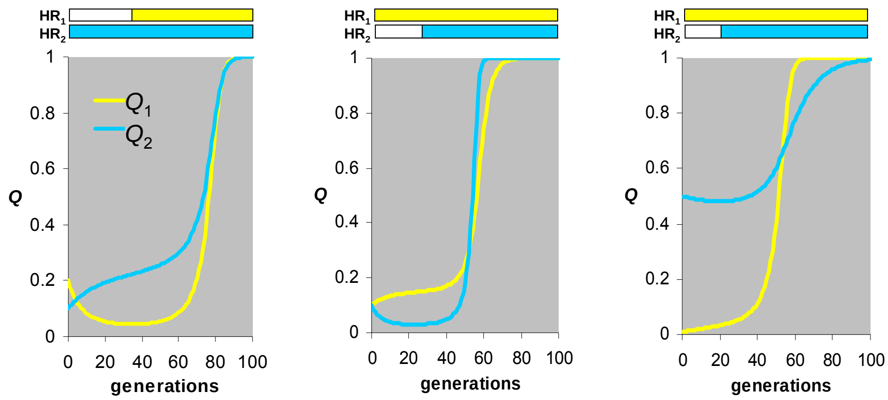
- In randomly formed, single-generation pairings, D wins (Hamilton 1975)
- Axelrod’s Tournaments (late 1970s on)
  - Tit-For-Tat (TFT)
    - Anatol Rapoport
  - Evolutionary experiments with iterated interactions within a generation
  - where offspring proportional to cumulative fitness payoffs

Simple Iterated PD Model



A Simple Mutualism Model

- Interactions are heterospecific and pair-wise
- Each species has two types
  - ALLD type
  - a cooperative type (e.g. TFT)
- *b, c, d,* and the cooperative strategy can all vary between species



	1	2
b	1.5	5
c	2.0	0.1
w_0	2	2
d	0	0
Str	TFT	TFT
i	4	

	1	2
b	4	4
c	1	1
w_0	1	1
d	0	0
Str	TF2T	Pavlov
i	80	

	1	2
b	2	2.2
c	1	0.1
w_0	1	1
d	0	1.3
Str	ALLC	TFT
i	100	

– Fletcher & Zwick, 2006. *The American Naturalist*

Games as Idea Models for Interactions

- Different order of payoffs define classic 2-player games
- N-player games model interactions involving multiple agents

Six Games of Altruism (Social Dilemmas)

Variant: Assurance

	C	D
C	1st, 1st	4th, 3rd
D	3rd, 4th	2nd, 2nd

Spite (no-conflict)
Weak altruism

	C	D
C	1st, 1st	3rd, 2nd
D	2nd, 3rd	4th, 4th

Chicken, Hawk-Dove
Snowdrift

	C	D
C	2nd, 2nd	3rd, 1st
D	1st, 3rd	4th, 4th

Assurance,
Stag Hunt,
Coordination

	C	D
C	1st, 1st	4th, 2nd
D	2nd, 4th	3rd, 3rd

PD, Strong Altruism
Tragedy of the Commons,
Public Goods Game

	C	D
C	2nd, 2nd	4th, 1st
D	1st, 4th	3rd, 3rd

Leader-Follower
Reluctant Hero

	C	D
C	3rd, 3rd	2nd, 1st
D	1st, 2nd	4th, 4th

Relative vs. Absolute fitness

- Selection Plays the Role of Rationality
  - But selection works on relative differences
  - For utility difference-maximizers, all 6 games are PDs
    - if absolute fitness breaks ties

	C	D
C	5, 5	0, 10
D	10, 0	-5, -5

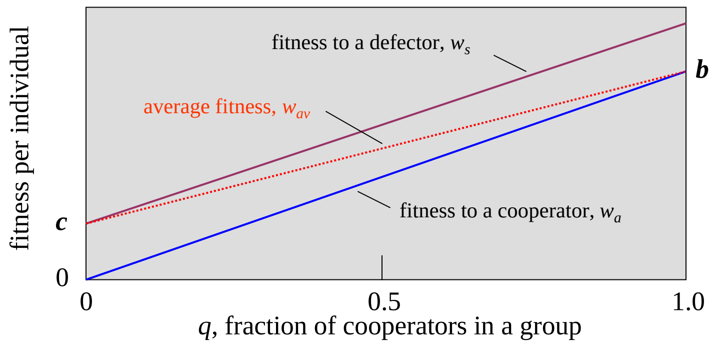
→

	C	D
C	$0+\varepsilon$, $0+\varepsilon$	-10, 10
D	10, -10	0, 0

- Hawk-Dove becomes PD
  - Note that except for ε this is zero-sum
  - All 6 games work this way—so can see why all can be used in study of evolution of cooperation
  - Note that this means the original payoffs were not the true payoffs

N-Player Prisoner's Dilemma

(Tragedy of the Commons)



- $a_i' = a_i\,[1 + w_a(q_i)]$ $\quad s_i' = s_i\,[1 + w_s(q_i)]$
- $w_a(q_i) = bq_i - c + w_0$ $\quad w_s(q_i) = bq_i + w_0$

– Fletcher & Zwick, 2007. *Journal of Theoretical Biology*

Complex Network Examples

- Neural Network (*C. Elegans*)
  - http://gephi.org/wp-content/uploads/2008/12/screenshot-celegans.png
- Food Web
  - http://1.bp.blogspot.com/_vIFBm3t8boU/SBhzqbchIeI/AAAAAAAAAXk/RsC-Pj45Avc/s400/food%2Bweb.bmp
- Social Networks
  - http://ucsdnews.ucsd.edu/graphics/images/2007/07-07socialnetworkmapLG.jpg
- Airline Routes
  - http://www.continental.com/CMS/ContinentalDocuments/pdfs/route-maps/co-us_201010.pdf
- US Power Grid
  - http://www.npr.org/templates/story/story.php?storyId=110997398

Metabolic Networks

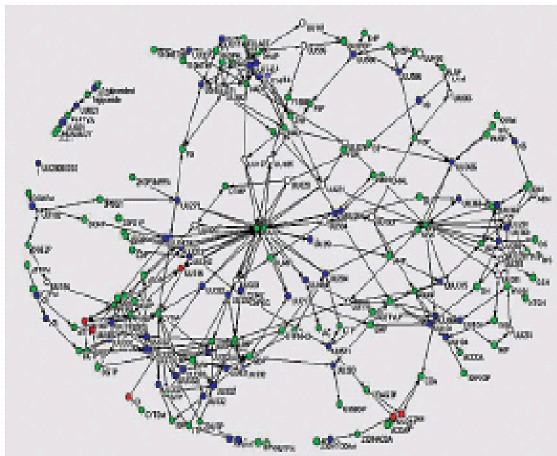
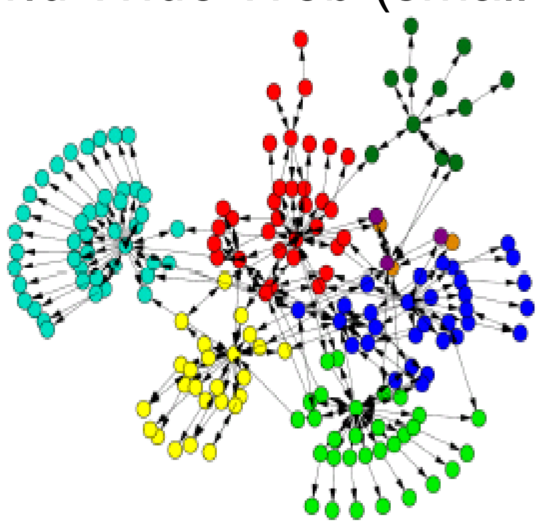


Figure 2. Bipartite graph of the metabolic network of *Ureaplasma urealyticum*. Dark gray and white nodes represent enzymes and light gray nodes represent metabolites (Lemke et al., 2004).

- http://www.funpecrp.com.br/gmr/year2005/vol3-4/wob01_full_text.htm

World Wide Web (small part)



From M. E. J. Newman and M. Girvin, Physical Review Letters E, 69, 026113, 2004.

More Examples

- Genetic Regulatory Networks
  - http://expertvoices.nsdl.org/cornell-info204/files/2009/03/figure-3.jpeg
- Internet
  - http://www.visualcomplexity.com/vc/images/270_big01.jpg
- World Wide Web
  - http://www.vlib.us/web/worldwideweb3d.html
- Movie co-stars
  - http://oracleofbacon.org/

The (Systems) Science of Networks

- Are there properties common to all complex networks?
- If so, why?
- Can we formulate a general theory of the structure, evolution, and dynamics of networks?
- Example of "stuff free" science
- Looking for commonalities vs. silos
  - Neural networks, social networks, web networks, etc.

Network Terminology

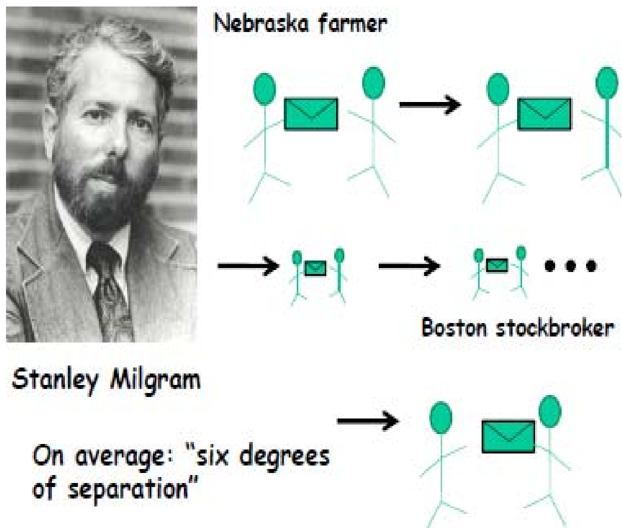
- Network = nodes + links
- Degree of a node
- Clustering
- Degree distribution
- Hub

Common Properties

(of biological, social, technological networks)

- Small world property
- Scale-free structure
- Clustering and community structure
- Robustness to random node failure
- Vulnerability to targeted hub attacks
- Vulnerability to cascading failures

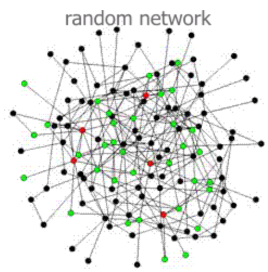
"Six degrees of separation"

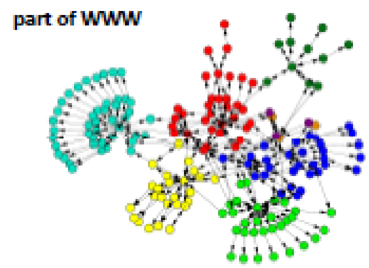


The Small-World Property (Watts and Strogatz)

- The network has relatively few "long-distance" links but there are short paths between most pairs of nodes, usually created by "hubs".

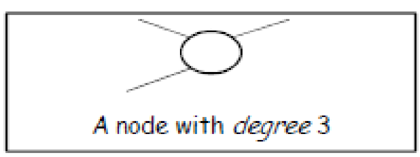
Scale-Free Structure (Albert and Barabási, 1998)

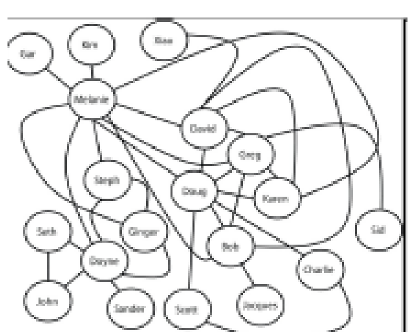


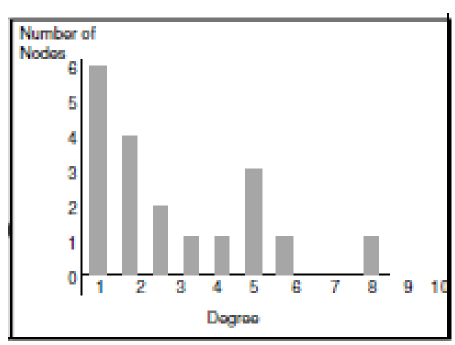


- Typical structure of a randomly connected network
- Typical structure of World Wide Web (nodes = web pages, links = links between pages)

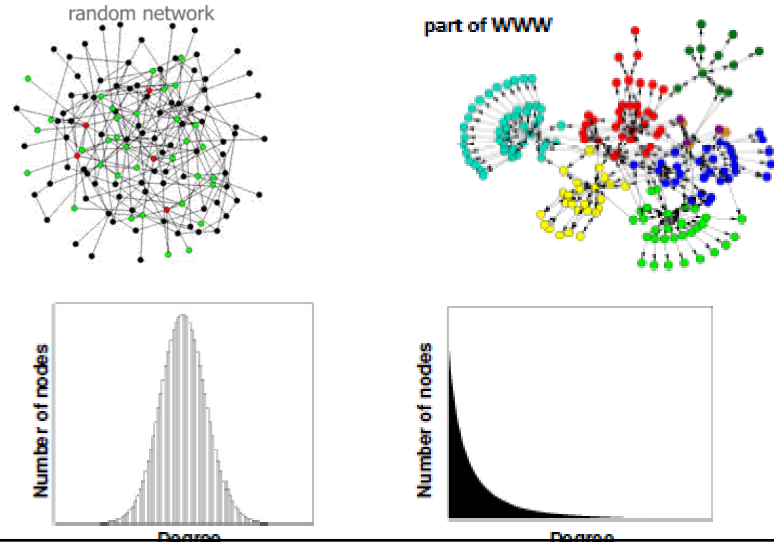
Concept of “Degree Distribution”







Scale-Free Structure (Albert and Barabási, 1998)

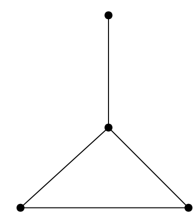


The Web's approximate Degree Distribution

- "Scale-free" distribution
- The probability that a node will have degree k is proportional to $1/k^2$
- "Scale-free" distribution = "power law" distribution

Three Important Measures of Network Topology

- Average path length
  - Average shortest distance between every pair of nodes
- Clustering coefficient
  - Global (about a graph)
    - C = # of closed triplets / number of connected triples of vertices
  - Local (about a node *n*)
    - C = # of *n*'s neighbors with pairwise links / # of potential links
- Degree distribution
- Answers:
  - 4/3
  - Global C = 3/5; for center node C = 1/3
  - 1 degree 1; 2 degree 2; 1 degree 3

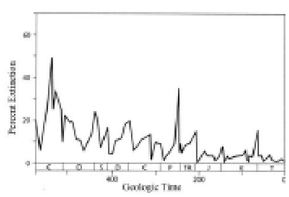


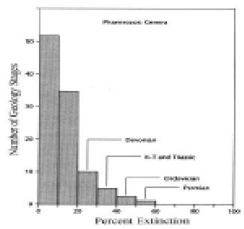
Other examples of power-laws in nature

- Magnitude vs. frequency of earthquakes
  - Gutenberg-Richter Law
  - http://www.simscience.org/crackling/Advanced/Earthquakes/GutenbergRichter.html
- Magnitude vs. frequency of stock market crashes
- Income vs. frequency (of people with that income)
- Populations of cities vs. frequency (of cities with that population)
- Word rank vs. frequency in English text

How are scale-free networks created?

- Barabàsi and Albert: **Preferential attachment**
- Netlogo demos
- More examples of scale-free networks
  - http://www.orgnet.com/cases.html
- Regularity of Biological Extinctions

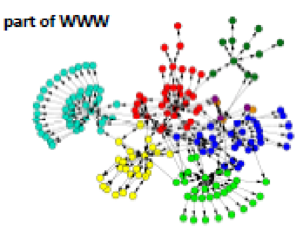




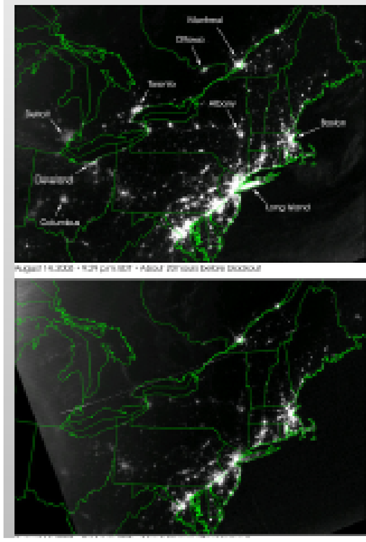
Robustness of Scale-Free Networks

- **Vulnerable** to targeted “hub” failure
- **Robust** to random node failure
  - unless....
  - nodes can **cause** other nodes to fail
  - Can result in ***cascading failure***

part of WWW



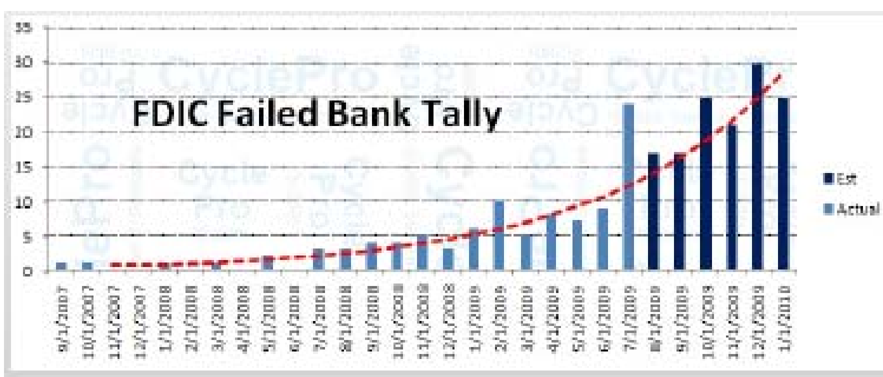
August, 2003 electrical blackout in northeast US and Canada



http://earthobservatory.nasa.gov/images/imagerecords/3000/3719/NE_US_OLS2003227.jpg

9:29pm
1 day before

9:14pm
Day of blackout



- “Few economists saw our current crisis coming, but this predictive failure was the least of the field’s problems. More important was the profession’s blindness to the very possibility of catastrophic failures in a market economy.”
  – Paul Krugman, New York Times, September 6, 2009
- We see similar patterns of cascading failure in biological systems, ecological systems, computer and communication networks, wars, etc

Scaling Laws

- Power law scaling in the Web:
  – Probability that a web page will have k in-links scales as $k^{-1/2}$
- Zipf’s law 1:
  – In English text, word frequency scales as (word rank)$^{-1}$
  – Seems to work for other languages also!
  – Implies 2nd ranked word will be ?? times as frequent as 1st ranked word, 3rd ranked word will be ?? times as frequent as 1st ranked word, etc.

Word frequencies from 423 Time Magazine articles

the	15861	0.065	it	1290	0.095	week	793	0.113
of	7239	0.059	from	1228	0.095	they	697	0.102
to	6331	0.077	but	1138	0.093	govern	687	0.104
a	5878	0.096	u	955	0.082	all	672	0.104
and	5614	0.114	had	940	0.084	year	672	0.107
in	5294	0.129	last	910	0.087	its	620	0.101
that	2507	0.072	be	915	0.089	britain	589	0.098
for	2228	0.073	have	914	0.093	when	579	0.099
was	2149	0.079	who	894	0.095	out	577	0.101
with	1839	0.075	not	882	0.097	would	577	0.103
his	1815	0.081	has	880	0.100	new	572	0.105
is	1810	0.089	an	873	0.103	up	559	0.105
he	1700	0.090	s	865	0.106	been	554	0.106
as	1581	0.090	were	848	0.107	more	540	0.106
on	1551	0.095	their	815	0.106	which	539	0.108
by	1467	0.096	are	812	0.109	into	518	0.106
at	1333	0.092	one	811	0.112			

Word frequencies from 46,449 Wall Street Journal articles

Word	f_r	r_f*p_r	Word	f_r	r_f*p_r	Word	f_r	r_f*p_r
the	1130021	0.059	from	96900	0.092	or	54958	0.101
of	547311	0.058	he	94585	0.095	about	53713	0.102
to	516635	0.082	million	93515	0.098	market	52110	0.101
a	464736	0.098	year	90104	0.100	they	51359	0.103
in	390819	0.103	its	86774	0.100	this	50933	0.105
and	387703	0.112	be	85588	0.104	would	50828	0.107
that	204351	0.075	was	83398	0.105	u	49281	0.106
for	199340	0.084	company	83070	0.109	which	48273	0.107
is	152483	0.072	an	76974	0.105	bank	47940	0.109
said	148302	0.078	has	74405	0.106	stock	47401	0.110
it	134323	0.078	are	74097	0.109	trade	47310	0.112
on	121173	0.077	have	73132	0.112	his	47116	0.114
by	118863	0.081	but	71887	0.114	more	46244	0.114
as	109135	0.080	will	71494	0.117	who	42142	0.106
at	101779	0.080	say	66807	0.113	one	41635	0.107
mr	101679	0.086	new	64456	0.112	their	40910	0.108
with	101210	0.091	share	63925	0.114			

Other Scaling Laws

- Zipf's law 2: The population of cities in a country scales as (city rank)$^{-1}$
- Pareto distribution
  - Originally stated as "Number of people owning a fraction k of the total wealth in a country"
  - Also "80-20 rule": 20% of the population owns 80% of the wealth
- Benford's law:
  - in lists of numbers from many real-life sources of data, the probability of the leading digit being d (0 – 9) is

$$P(d) = \log_b(d+1) - \log_b(d) = \log_b\left(1 + \frac{1}{d}\right)$$