Exploring Complexity

In Science and Technology

Nov. 17, 2010

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Logistics

- Go over
  - HW6 and Lab5
- Due Monday HW7; Wednesday Lab 6
- Check that Blackboard grades are accurate
- Final Paper Questions?
  - I need your paper topic proposals!
  - if I don't already have them
- Questions?

Analogy Making as Thinking

- "Analogy-making is the ability to perceive abstract similarity between two things in the face of superficial differences."
  - Melanie Mitchell
- "This ability pervades almost every aspect of what we call intelligence."
  - Melanie Mitchell
  - p. 187-8 examples
- "All perception of truth is the detection of an analogy."
  - Henry David Thoreau

Simplifying Analogy

- Strings of letters:
  - abc ➔ abd; kji ➔ ?
  - groups
- "Conceptual Slipping"
- Balance between exploration and exploitation
  - Combine randomness with highly directed response based on feedback
  - How do ants do this? The immune system?

Models in Science

- Mathematical Models
- Mechanistic Models
- Idea Models Examples
  - Maxwell's demon
  - Logistic model and logistic map
  - von Neumann's self-reproducing automata
  - Genetic algorithms
  - Cellular automata
  - Koch curve
  - Copycat

Idea Models

- Show that a proposed mechanism for a phenomenon is plausible (or implausible)
- Explore the effects of vartiations on a simple model and prime one's intuitions about a complex phenomenon
- Inspire new technologies
- Lead to mathematical theories

Exercise

- Each player starts with $100 and can choose to invest from $0 to their current balance.
- In each round, each member's investment is subtracted from each member's balance
- The total of all the investments from the current round are doubled and then divide evenly among all the players
- This gives each player a new balance to start a new round
- We will do several rounds—please do not get ahead of (or behind) me. I will announce when each round starts and stops
- Player with highest final score wins a prize
- There is no communication allowed between players

Main Theories for the Evolution of Altruism

- Multilevel Selection
  - Altruist dominated groups do better; altruists within groups do worse
  - $\Delta Q = \Delta Q_B + \Delta Q_W$
- Inclusive Fitness/Kin Selection
  - Gene self interest, Hamilton's rule ($\Delta Q > 0$ if $rb > c$)
  - $w_{inclusive} = w_{direct} + w_{indirect}$
- Reciprocal Altruism
  - Conditional behaviour, Iterated Prisoner's Dilemma (IPD), emphasis on non-relatives
  - Indirect reciprocity, strong reciprocity, reciprocity on graphs
- Others
  - By-product mutualism, conflict mediators, policing, social markets

Advances in Unification Program

- Unifying Multilevel Selection and Inclusive Fitness Theories
  - (Price 1970, Wade 1980, Breden 1990, Queller 1992, Frank 1998, Sober and Wilson 1998, Lehmann et al. 2007)
- Unifying Reciprocal Altruism and Inclusive Fitness Theories
  - (Queller 1985, Nee 1989, Frank 1994, 1998, Sober and Wilson 1998)
  - Less successful; less formal; less accepted

Prisoner's Dilemma (PD)

Actor's Fitness (Utility)

		opponent's behavior C	*opponent's behavior* D
actor's behavior	C	4	0
	D	5	1

- Individually rational to defect
- Collectively irrational

Additive Prisoner's Dilemma (PD)

Actor's Fitness (Utility)

		opponent's behaviour	
		C contributes *b*	**D** contributes 0
actor's behaviour	**C** sacrifices *c*	$w_0 + b - c$ 4	$w_0 - c$ 0
	D sacrifices 0	$w_0 + b$ 5	w_0 1

- $w_0 = 1$; $b = 4$; $c = 1$

Non-Additive PD

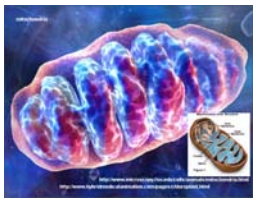
Actor's Fitness (Utility)

		opponent's behavior	
		C contributes *b*	**D** contributes 0
actor's behavior	**C** sacrifices *c*	$w_0 + b - c$ (+*d*) 3	$w_0 - c$ 0
	D sacrifices 0	$w_0 + b$ 5	w_0 1

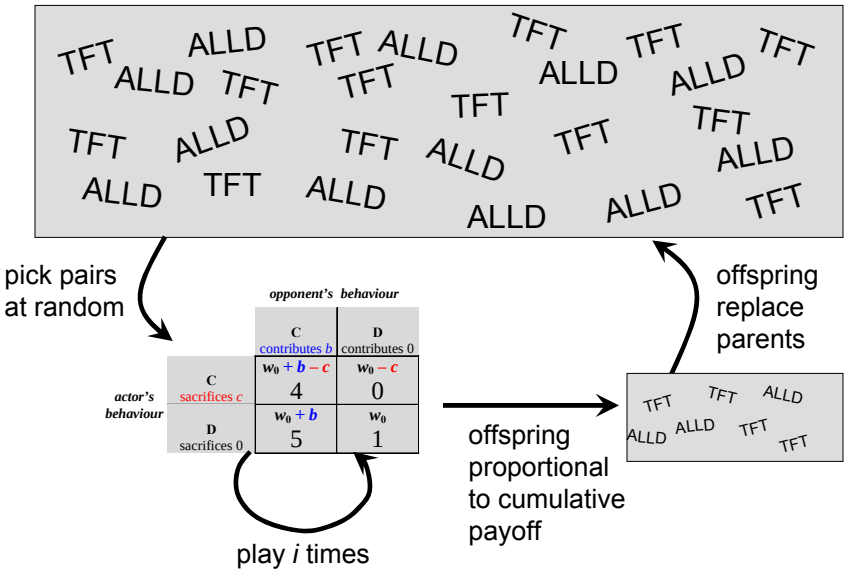
- $w_0 = 1$; $b = 4$; $c = 1$; $d = -1$

Reciprocal Altruism: Iterated Conditional Behaviours

- In randomly formed, single-generation pairings, D wins (Hamilton 1975)
- Axelrod's Tournaments (late 1970s on)
  - Tit-For-Tat (TFT)
    - Anatol Rapoport
  - Evolutionary experiments with iterated interactions within a generation
  - where offspring proportional to cumulative fitness payoffs



Simple Iterated PD Model



Can We Apply Hamilton's Rule?

- $rb > c$
- Start with the additive PD (no d term)

$$r = \frac{\mathrm{cov}(G_A, G_O)}{\mathrm{var}_t(G_A)}$$

- Hamilton's $r = 0$ for random pairing
- Hamilton's rule (focused on genetic similarity) does not predict TFT increase (given random interactions)
- Because it does not account for conditional behavior

Axelrod and Hamilton (1981)

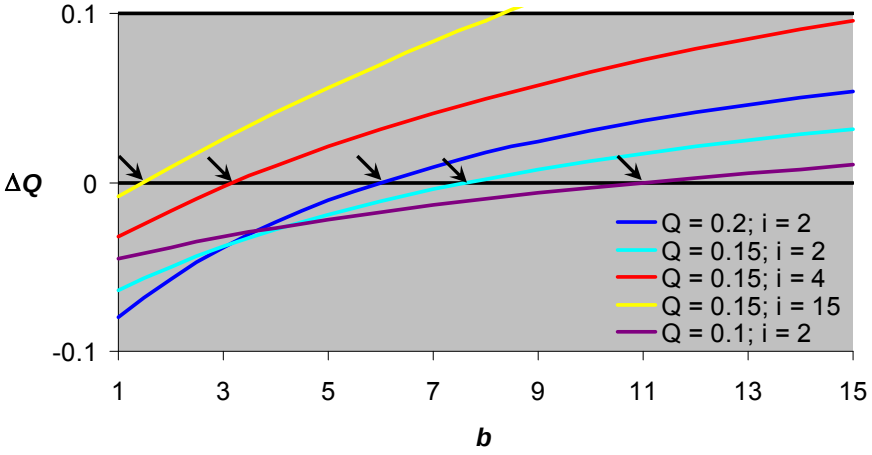
- Distinguished two mechanisms
  - Inclusive Fitness for relatives
  - Reciprocal Altruism for non-relatives
- Still current thinking
  - Sachs *et al* 2004 QRB
  - Many papers this last year
- Two obstacles for unification
  1. Phenotype/Genotype differences
  2. PD used is non-additive

Queller's Generalization

- To solve problem 1 (*G*/*P* difference)
  - Use *phenotypes* (behaviors), not just *genotypes,* in Hamilton's rule
  - Hamilton (1975) $r = \frac{\mathrm{cov}(G_A, G_O)}{\mathrm{var}_t(G_A)}$ Queller (1985) $r = \frac{\mathrm{cov}(G_A, P_O)}{\mathrm{cov}(G_A, P_A)}$
- To solve problem 2 (non-additivity)
  - Use an additional term to account for deviations from additivity (Queller 1985)

$$\frac{\mathrm{cov}(G_A, P_O)}{\mathrm{cov}(G_A, P_A)} b + \frac{\mathrm{cov}(G_A, P_A P_O)}{\mathrm{cov}(G_A, P_A)} d > c$$

Numerical Simulations of Iterated PD varying *Q*, *i*, and *b* (*c* = 1)



– Fletcher & Zwick, 2006. *The American Naturalist*

Implications for Indirect Fitness

- Queller's version emphasizes direct fitness (there is no G_O term)

$$\frac{\mathrm{cov}(G_A, P_O)}{\mathrm{cov}(G_A, P_A)} b + \frac{\mathrm{cov}(G_A, P_A P_O)}{\mathrm{cov}(G_A, P_A)} d > c$$

- More intuitive form

$$\mathrm{cov}(G_A, P_O) b \; + \mathrm{cov}(G_A, P_A P_O) d > \mathrm{cov}(G_A, P_A) c$$

- An even simpler form

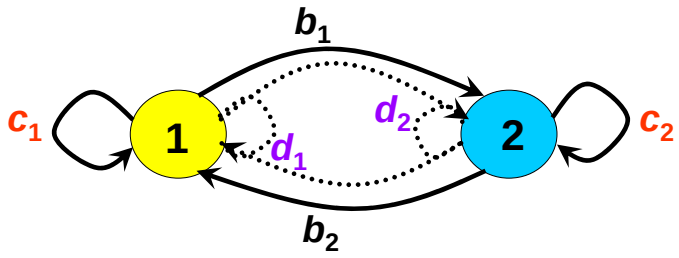
$$\mathrm{cov}(G_A,\; P_O b + P_A P_O d - P_A c) \; > 0$$

$$\mathrm{cov}(G_A,\; \textit{net fitness benefits to } A) \; > 0$$

A Simple Mutualism Model

- Interactions are heterospecific and pair-wise
- Each species has two types
  - ALLD type
  - a cooperative type (e.g. TFT)
- *b, c, d,* and the cooperative strategy can all vary between species

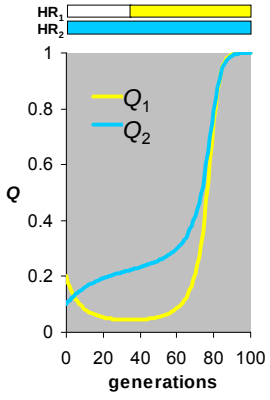
A Simple Mutualism Model



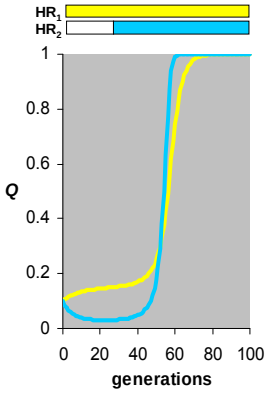
$$r_1 = \frac{\mathrm{cov}(G_1, P_2)}{\mathrm{cov}(G_1, P_1)} \qquad r_2 = \frac{\mathrm{cov}(G_2, P_1)}{\mathrm{cov}(G_2, P_2)}$$

HR_1: $r_1 b_2 > c_1$ HR_2: $r_2 b_1 > c_2$

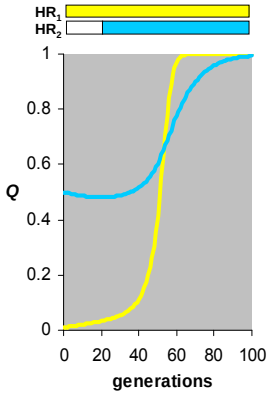
– Fletcher & Zwick, 2006. *The American Naturalist*



	1	2
b	1.5	5
c	2.0	0.1
w_0	2	2
d	0	0
Str	TFT	TFT
i	4	



	1	2
b	4	4
c	1	1
w_0	1	1
d	0	0
Str	TF2T	Pavlov
i	80	



	1	2
b	2	2.2
c	1	0.1
w_0	1	1
d	0	1.3
Str	ALLC	TFT
i	100	

– Fletcher & Zwick, 2006. *The American Naturalist*

There is no general theory of mutualism that approaches the explanatory power that 'Hamilton's Rule' appears to hold for the understanding of within-species interactions.

– Herre et al. 1999, *TREE* 14:49-53

Games as Idea Models for Interactions

- Different order of payoffs define classic 2-player games
- N-player games model interactions involving multiple agents

Ritualized Fighting

		opponent's behaviour	
		Dove	**Hawk**
actor's behaviour	**Dove**	$V/2$ 5	0
	Hawk	V 10	$(V-c)/2$ -5

- $V = 10$; $c = 20$
- The rare strategy has an advantage (i.e. frequency dependent selection)
- Hawk-Dove, Chicken, Snowdrift, Brinkmanship
- If $0 < c < V$, then game is PD instead

Evolutionarily Social Dilemma Games

- What features do Hawk-Dove and the PD have in common?
  - Cs do better in CC pairs than Ds do in DD pairs
  - Ds do better than Cs in mixed pairs
- Given 4 utility levels (1st, 2nd, 3rd, 4th) how many 2-player, symmetric games are there that capture this idea of "social dilemma"?
- Can you name them?

Six Games of Altruism

	C	D
C	1st, 1st	4th, 3rd
D	3rd, 4th	2nd, 2nd

Variant: Assurance

	C	D
C	1st, 1st	3rd, 2nd
D	2nd, 3rd	4th, 4th

Spite (no-conflict)
Weak altruism

	C	D
C	2nd, 2nd	3rd, 1st
D	1st, 3rd	4th, 4th

Chicken, Hawk-Dove
Snowdrift

	C	D
C	1st, 1st	4th, 2nd
D	2nd, 4th	3rd, 3rd

Assurance,
Stag Hunt,
Coordination

	C	D
C	2nd, 2nd	4th, 1st
D	1st, 4th	3rd, 3rd

PD, Strong Altruism
Tragedy of the Commons,
Public Goods Game

	C	D
C	3rd, 3rd	2nd, 1st
D	1st, 2nd	4th, 4th

Leader-Follower
Reluctant Hero

Different Games

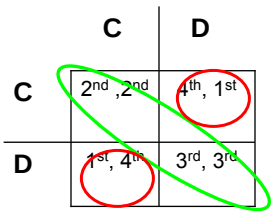
- Six games that can be used to model the evolution of cooperation (or altruism)
  - Two have dominant strategies, but…
    - One dominant for D (PD or Strong Altruism)
    - One dominant for C (Spite or Weak Altruism)
  - Four have no dominant strategy and best strategy is frequency dependent, but …
    - In two it is best to be rare (Hawk-Dove, Leader-Follower)
    - In two it is best to be common (Assurance, Assurance variant)
- In terms of game theory, these games are all over the map!

Similarities Under Selection

- All 6 games are “Prisoner’s Dilemma like” under selection
  - At the individual (within-group) level defection is always favored
  - At the collective level (between-group) level cooperation is always favored
- What actually evolves in any of these games depends on both
  - The relative magnitude of the fitness differences at each level
  - The relative frequencies with which different types of interactions occur

Strength of Selection at Different Levels

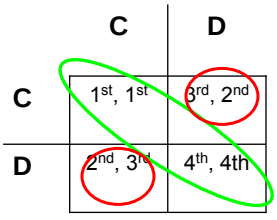
Prisoner’s Dilemma
Strong Altruism



$2^{nd} - 3^{rd}$

$1^{st} - 4^{th}$

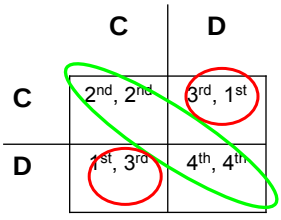
Spite (No-Conflict Game)
Weak Altruism



$1^{st} - 4^{th}$

$2^{nd} - 3^{rd}$

Hawk-Dove



$2^{nd} - 4^{th}$

1^{st} 1^{st} — 3^{rd} 3^{rd}

- Which type increases depends on:
  - Relative magnitudes of selective forces
  - Relative frequency of interaction types

Relative vs. Absolute fitness

- Selection Plays the Role of Rationality
  - But selection works on relative differences
  - For utility difference-maximizers, all 6 games are PDs
    - if absolute fitness breaks ties

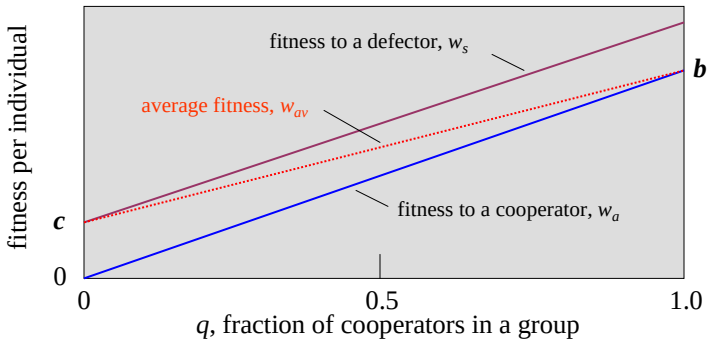
	C	D
C	5, 5	0, 10
D	10, 0	-5, -5

→

	C	D
C	0+ε, 0+ε	-10, 10
D	10, -10	0, 0

- Hawk-Dove becomes PD
  - Note that except for ε this is zero-sum
  - All 6 games work this way—so can see why all can be used in study of evolution of cooperation
  - Note that this means the original payoffs were not the true payoffs

N-Player Prisoner's Dilemma
(Tragedy of the Commons)



- $a_i' = a_i [1 + w_a(q_i)]$ $\quad s_i' = s_i [1 + w_s(q_i)]$
- $w_a(q_i) = bq_i - c + w_0$ $\quad w_s(q_i) = bq_i + w_0$

– Fletcher & Zwick, 2007. *Journal of Theoretical Biology*