Homework 2: Complexity Chapters 3-4

DUE: Wednesday, October 13, start of class.

1. Consider the slot machine in Figure 3.2 of the textbook.
    a. How many microstates correspond to the macrostate “pictures contain at least one lemon”? Please show your work
    b. How many microstates correspond to the macrostate “two pictures the same”?

2. Explain in your own words (one paragraph) how Boltzmann’s definition of entropy, $S = k \log W$, captures the notions of “order” and “disorder”.

3. Explain in your own words (one paragraph) how the second law of thermodynamics defines the “arrow of time”.

4. Suppose two-year-old Finn has a vocabulary of 403 words. When talking, he will say the word “car” one-tenth of the time, the word “train” one twentieth of the time, the word “Daddy” also one twentieth of the time, and the rest of the time all his other words will be used equally often. What is the average Shannon information per word of his side of a conversation? Show how you calculated this.

5. Devise a set of rules for a Turing machine that reads a tape containing all blank cells except for some number of 1s (possibly zero) sandwiched between exactly two 0s, and determines whether or not there are more than three 1s. If so, the final output of the machine will be a single 1 with all other cells blank; if not, the final output will be a single 0 with all other cells blank. Assume the tape reader starts on the leftmost 0 symbol. Your rules should be of the form:
   –Current state—Current Symbol—New state—New Symbol—Motion—

6. Consider a Turing machine M whose tape starts with a integer in binary notation (and all other cells blank), and which decides whether or not that integer is prime.
    a. Is it possible to construct such a machine that could do this task in finite time for any finite integer? Why or why not?
    b. Supposing the answer to part (a) is “yes”, will this machine halt when run on its own code (given that this code is represented in binary form as explained at the bottom of page 64)? Why or why not?

7. In your own words, explain (in one paragraph) what Turing’s halting problem says about the limits of computation using Turing machines.