Spatial Interpolation

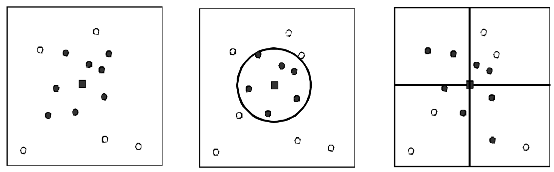
- What is spatial interpolation?
  - Estimate values
  - Converting point data to surface data
  - Converting line data to surface data (contours to DEM)
  - Converting area data to surface data (areal interpolation)
- Observations (control points) and interpolator
- Interpolators
  - Global / Local
  - Exact / Approximate
  - Stochastic / Deterministic
  - Geostatistical

Global/Local Methods

- Global methods
  - Trend surface analysis (Global polynomial interpolation)
- Local methods
  - IDW
  - Local polynomial interpolation

Local Method

- Neighbors
  - Distribution of control points
  - Extent of spatial autocorrelation



(a) (b) (c)

(a) find the closest points to the point to be estimated, (b) find points within a radius, and (c) find points within each of the four quadrants.

IDW

$$z = \frac{\sum_{1}^{s} z_i \frac{1}{d_i^k}}{\sum_{1}^{s} \frac{1}{d_i^k}}$$

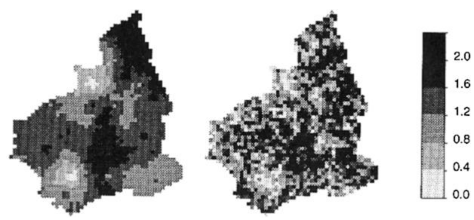
Point	Z	D	Z*1/D^k	1/D^k
A	10	8	1.25	0.125
B	5	2	2.5	0.5
		Sum =	3.75	0.625
k	1			
X		Z =	6	

Geostatistical / Simulation Interpolation

- Geostatistical estimation (Kriging)

$$\hat{Z}(\mathbf{s}_0) = \sum_{i=1}^{N} \lambda_i Z(\mathbf{s}_i)$$

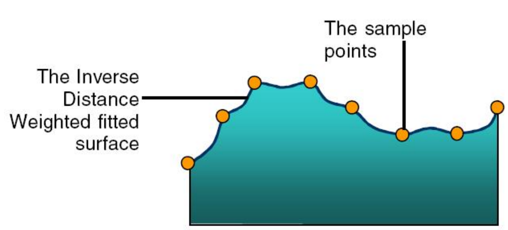
- Stochastic simulation, conditional to:
  1. Observed data values at their locations
  2. The histogram of observed data set
  3. The semivariance model of observed data set

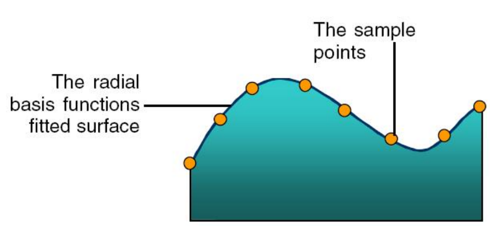


(Source: Goovaerts 1997)

Spline

Produces a continuous surface with minimum curvature.

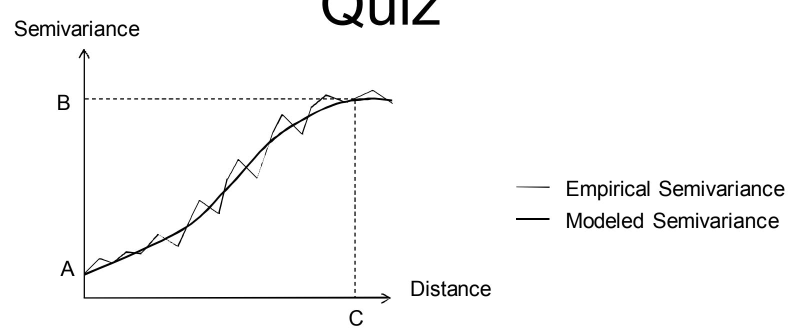




Steps of Geostatistical Interpolation

1. Calculating the empirical semivariogram
2. Fitting a model (modeled semivariogram)
3. Creating the (inverse) gamma matrix
4. Making a prediction
5. Repeat steps 3, 4 for each location to create a surface

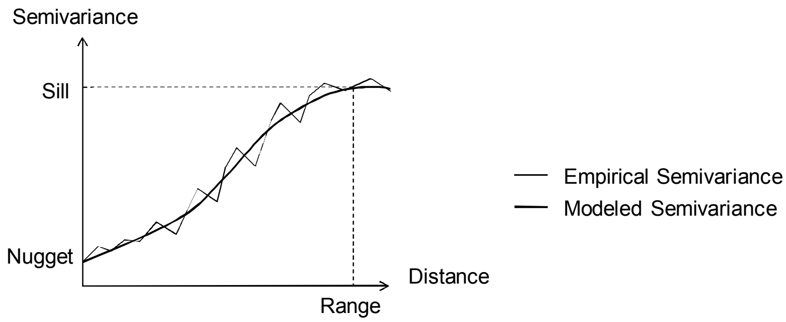
Quiz



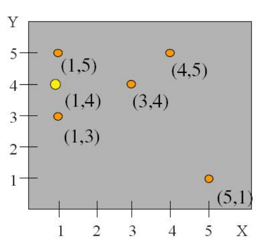
In the semivariogram above,
1. A is referred to as: a) cookie, b) sill, c) nugget, d) range.
2. B is referred to as: a) cookie, b) sill, c) nugget, d) range.
3. C is referred to as: a) cookie, b) sill, c) nugget, d) range.

Kriging

$$\hat{Z}(\mathbf{s}_0) = \sum_{i=1}^{N} \lambda_i Z(\mathbf{s}_i)$$



Empirical Semivariogram



Values:

(1,5) = 100

(3,4) = 105

(1,3) = 105

(4,5) = 100

(5,1) = 115

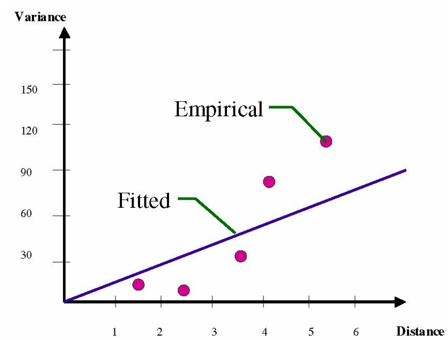
$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

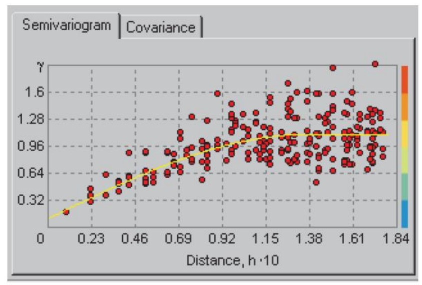
The empirical semivariance is
0.5 * average[(value at location *i* - value at location *j*)^2].

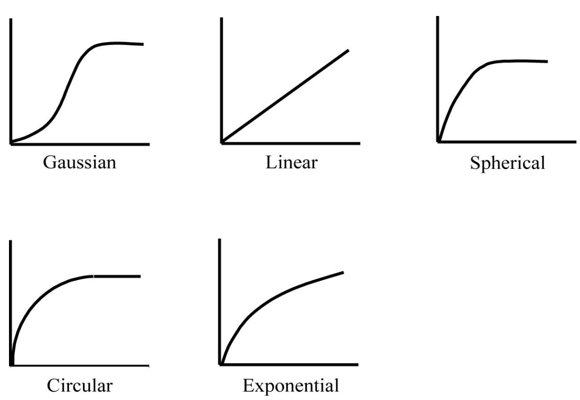
Locations	Distance Cal.	Distances	Difference^2	Semivariance
(1,5),(3,4)	sqrt[(1-3)^2 + (5-4)^2]	2.236	25	12.5
(1,5),(1,3)	sqrt[0^2 + 2^2]	2	25	12.5
(1,5),(4,5)	sqrt[3^2 + 0^2]	3	0	0
(1,5),(5,1)	sqrt[4^2 + 4^2]	5.657	225	112.5
(3,4),(1,3)	sqrt[2^2 + 1^2]	2.236	0	0
(3,4),(4,5)	sqrt[1^2 + 1^2]	1.414	25	12.5
(3,4),(5,1)	sqrt[2^2 + 3^2]	3.606	100	50
(1,3),(4,5)	sqrt[3^2 + 2^2]	3.606	25	12.5
(1,3),(5,1)	sqrt[4^2 + 2^2]	4.472	100	50
(4,5),(5,1)	sqrt[1^2 + 4^2]	4.123	225	112.5

	Binning the Empirical Semivariogram			
Lag Distance	Pairs Distance	Av. Distance	Semivariance	Average
1+-2	1.414, 2	1.707	12.5, 12.5	12.5
2+-3	2.236, 2.236, 3	2.491	12.5, 0, 0	4.167
3+-4	3.606, 3.606	3.606	50, 12.5	31.25
4+-5	4.472, 4.123	4.298	50, 112.5	81.25
5+	5.657	5.657	112.5	112.5

Fit a Model

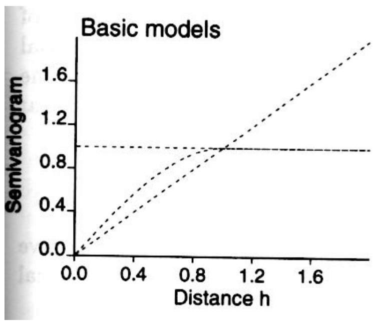


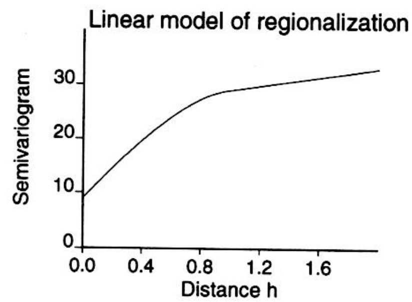




Some mathematical models for fitting semivariograms: Gaussian, linear, spherical, circular, and exponential.

Combining Variogram Models





Modeled Semivariogram

Spherical model

$$\gamma(\mathbf{h}) = \begin{cases} \theta_s \left[\frac{3}{2}\frac{h}{\theta_r} - \frac{1}{2}\left(\frac{h}{\theta_r}\right)^3 \right] & \text{for} \quad 0 \le h \le \theta_r \\ \theta_s & \text{for} \quad \theta_r < h \end{cases}$$

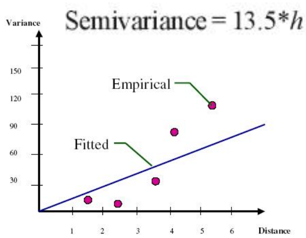
where

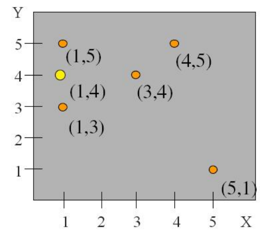
θ_s is the sill value,

h is the lag vector, and h is the length of **h** (distance between 2 locations),

θ_r is the range of the model.

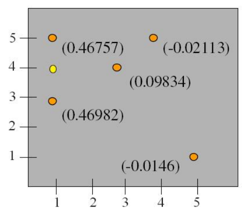
Making a Prediction





Point	Distance	g Vector for (1,4)
(1,5)	1	13.5
(3,4)	2	27.0
(1,3)	1	13.5
(4,5)	3.162	42.69
(5,1)	5	67.5
		1

Kriging Weights = g * Inverse of Distance Matrix



Values:

(1,5) = 100
(3,4) = 105
(1,3) = 105
(4,5) = 100
(5,1) = 115

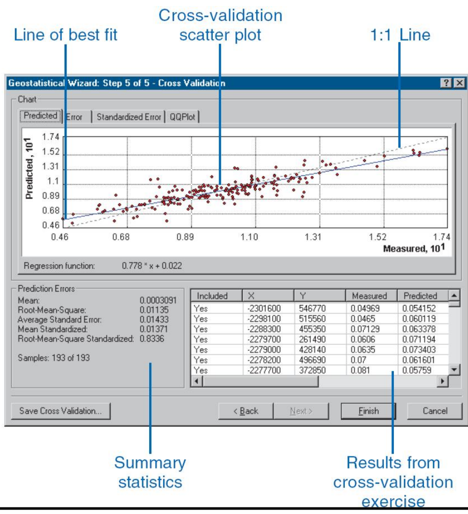
Weights	Values	Product	
0.46757	100	46.757	
0.09834	105	10.3257	
0.46982	105	49.3311	
-0.02113	100	-2.113	
-0.0146	115	-1.679	
-0.18281		102.6218	Kriging Predictor

Kriging Variance

G Vector	Weights (λ)	g Vector Times Weights
13.5	0.46757	6.312195
27.0	0.09834	2.65518
13.5	0.46982	6.34257
42.69	-0.02113	-0.90204
67.5	-0.0146	-0.9855
1	-0.18281	-0.18281
	Kriging Variance	13.2396
	Kriging Std Error	3.6386

Cross-Validation

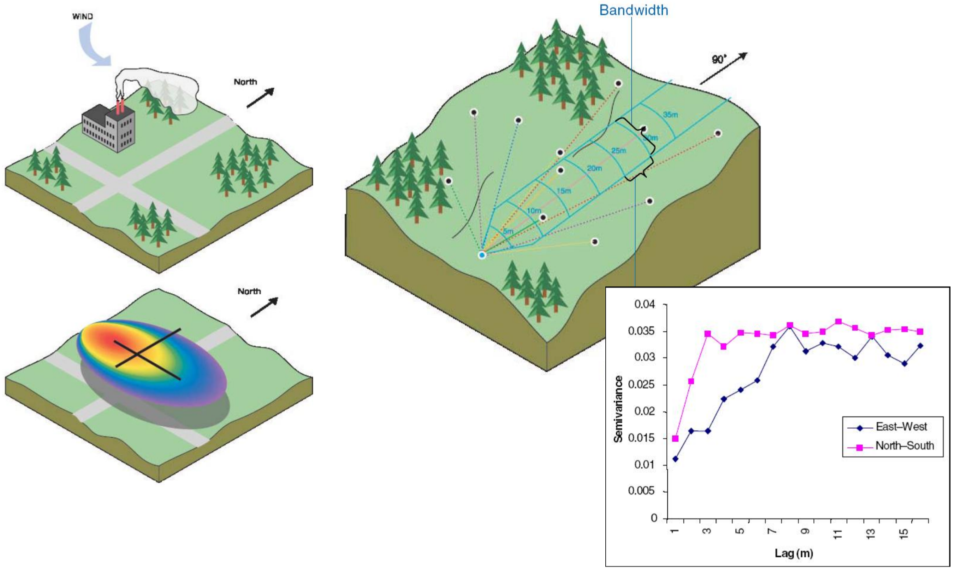
For all points, cross-validation sequentially omits a point, predicts its value using the rest of the data, and then compares the measured and predicted values.



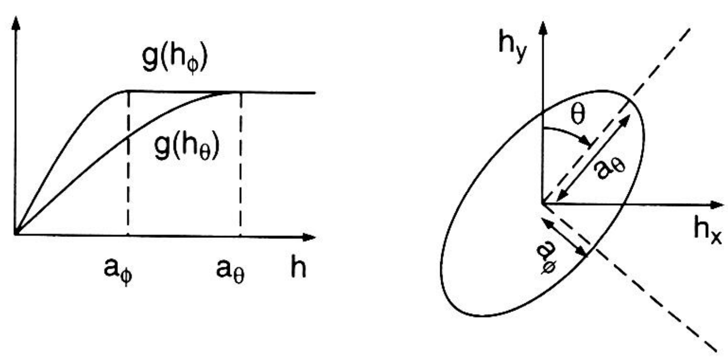
Kriging Methods

- Simple Kriging (surface with a constant mean)
- Ordinary Kriging (surface with local means)
- Universal Kriging (surface with a trend)
- Indicator Kriging (categorical surface)
- Co-Kriging (Kriging with a secondary variable)

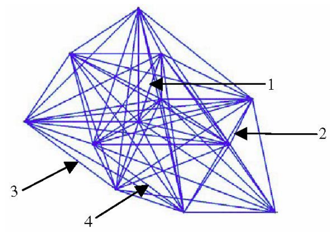
Directional Semivariogram

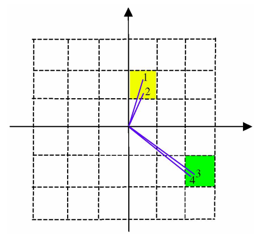


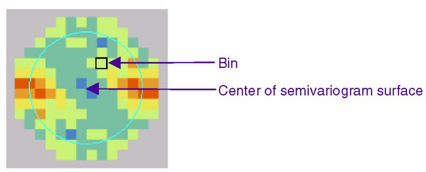
Anisotropy and Directional Semivariograms



Semivariogram Surface



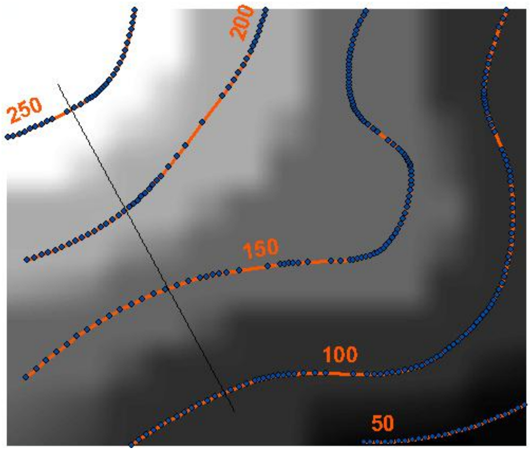


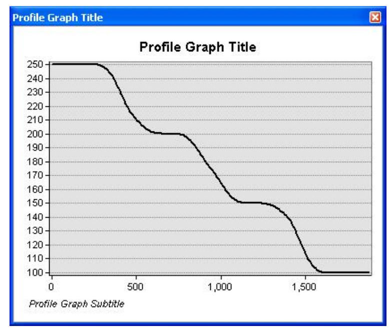


Spatial Interpolation with Sparse Sample Points

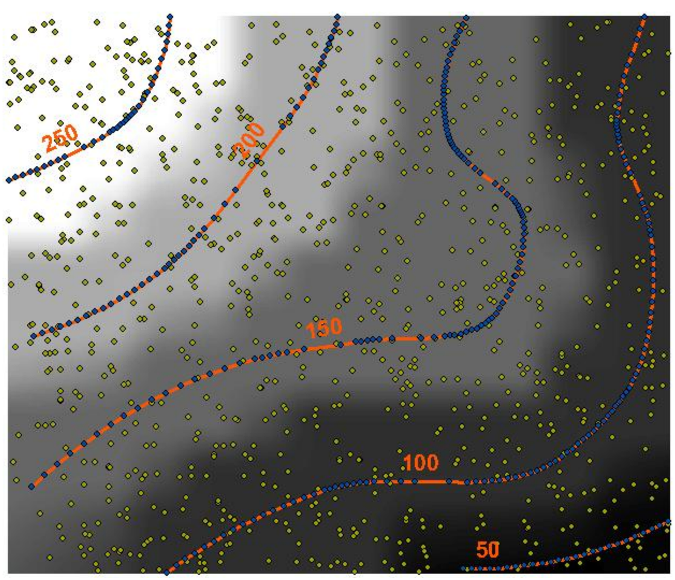
- Convert contours to DEM
- Generate DEM from transects

Contours to DEM

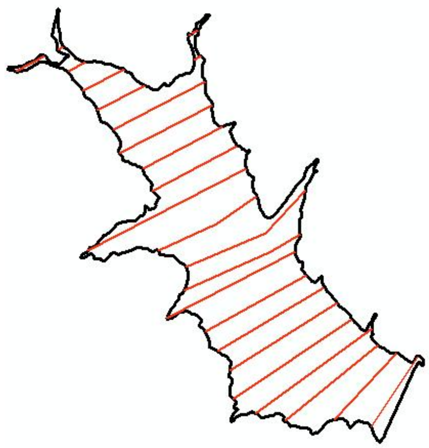


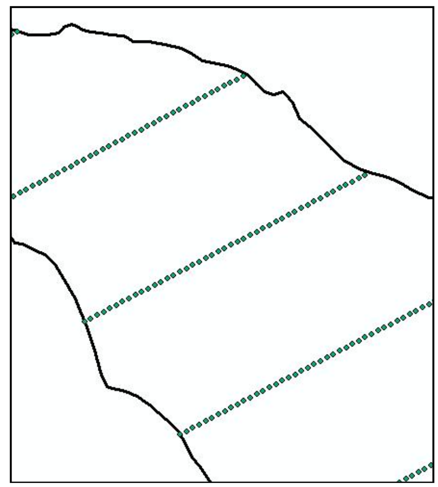


Densification of Sample Points

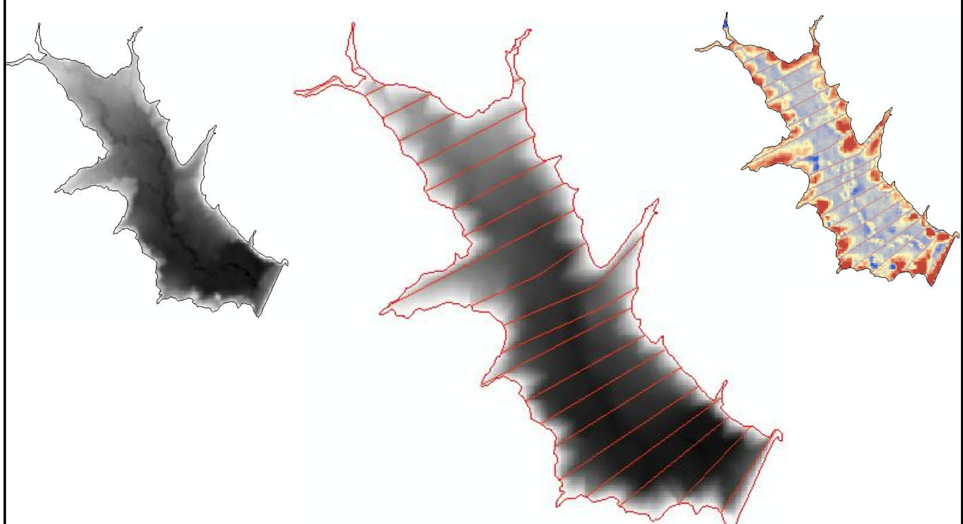


Lake Bathymetry





Experiments



Densification of Sample Points

3D Kriging

- 3D data sources (x, y, z and value)
- Multiple semivariograms are needed
- Anisotropy: azimuth and dip
- Different data resolutions (z usually has a higher resolution)
- Visualization of results (e.g., slicing)
- GSLib (http://www.gslib.com/)