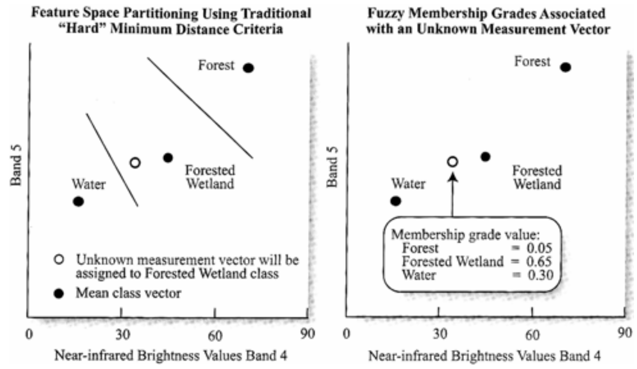
Fuzzy Classification

Hard- versus soft-classifiers



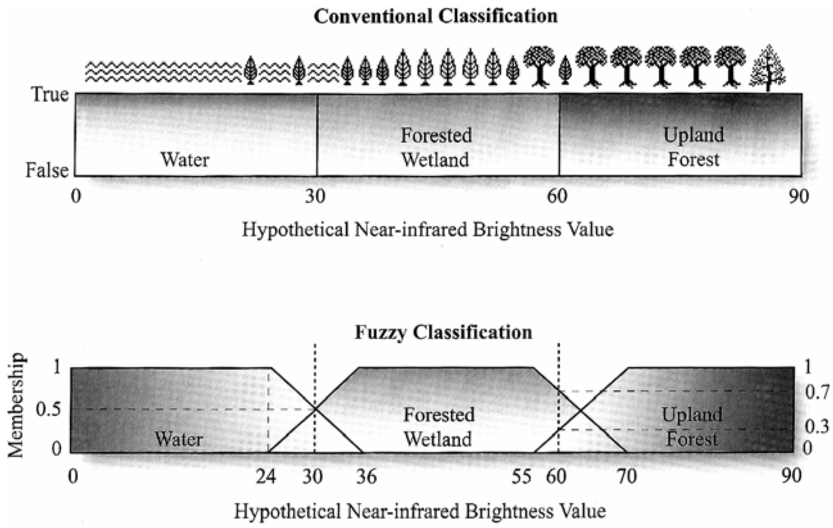
Why use soft-classifiers?

- Sub-pixel classification
- Uncertainty of classification/scheme
- Incorporating ancillary data (hardeners)

Fuzzy Classification Steps

- Classification scheme
- (Fuzzy) signatures
- Fuzzy classifiers
- Hardener (defuzzification)
- Classification uncertainty
- Classification accuracy

Fuzzy Classification Scheme

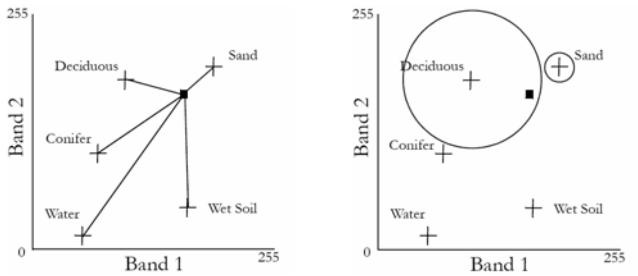


(Fuzzy) Signatures

- Training sites (homogeneous vs. fuzzy)

	Water	Forested Wetland	Upland Forest	Sum
Site#1	0.7	0.2	0.1	1
Site#2	0.2	0.2	0.6	1
Site#3	0.5	0.1	0.4	1
…	…			

Fuzzy Classifiers



- Bayesian Probability Theory (BAYCLASS)
- Dempster-Shafer Theory (BELCLASS)
- Fuzzy Set Theory (FUZCLASS)
- Linear Mixture Model (UNMIX)

BAYCLASS

- Based on the Bayesian prob. of the hypothesis being true given the evidence.
- Hard classification scheme and hard signatures

BAYCLASS and MAXLIKE:

- Maximum likelihood classifier (hard-classifier)
- BAYCLASS classifier (soft-classifier)
- MAXLIKE = Hardened BAYCLASS

BELCLASS & Dempster-Shafer Theory

- Dempster-Shafer Theory: Dealing with decisions made under partial information (i.e., the presence of unknown classes).
- Belief: the degree to which evidence provides concrete support for an hypothesis
- Plausibility: the degree to which the evidence does not refute that hypothesis
  PL(A) = 1 – BEL(Ā); where (Ā) is not (A)
- Belief interval = ABS(Belief – Plausibility)
  - High uncertainty: Belief Interval -> 1
  - Low uncertainty: Belief Interval -> 0

Fuzzy Set Theory Classifier

An image with *m* classes and *n* pixels, its fuzzy partition matrix is:

$$\begin{bmatrix} f_{F_1}(x_1) & f_{F_1}(x_2) & \cdots & f_{F_1}(x_n) \\ f_{F_2}(x_1) & f_{F_2}(x_2) & \cdots & f_{F_2}(x_n) \\ \vdots & \vdots & \vdots & \vdots \\ f_{F_m}(x_1) & f_{F_m}(x_2) & \cdots & f_{F_m}(x_n) \end{bmatrix}$$

$$0 \le f_{F_i}(x) \le 1$$

$$\sum_{x \in X} f_{F_i}(x) > 0$$

$$\sum_{i=1}^{m} f_{F_i}(x) = 1$$

Fuzzy signature components: mean and covariance matrix

$$\mu_c^* = \frac{\sum_{i=1}^{n} f_c(x_i) x_i}{\sum_{i=1}^{n} f_c(x_i)}$$

$$V_c^* = \frac{\sum_{i=1}^{n} f_c(x_i)(x_i - \mu_c^*)(x_i - \mu_c^*)^T}{\sum_{i=1}^{n} f_c(x_i)}$$

$$f_c(x) = \frac{P_c^*(x)}{\sum_{i=1}^{m} P_i^*(x)}$$

where

$$P_i^*(x) = \frac{1}{(2\pi)^{N/2} |V_i^*|^{1/2}} \times \exp[-0.5(x - \mu_i^*)^T (V_i^*)^{-1} (x - \mu_i^*)]$$

Classification Uncertainty

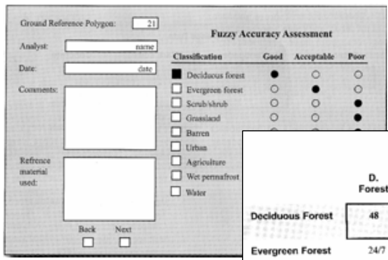
$$\text{ClassificationUncertainty} = 1 - \frac{\max - \frac{\text{sum}}{n}}{1 - \frac{1}{n}}$$

where

max	=	the maximum set membership value for that pixel
sum	=	the sum of the set membership values for that pixel
n	=	the number of classes (signatures) considered

(0.0 0.0 0.0)	Classification Uncertainty = 1.00
(0.0 0.0 0.1)	Classification Uncertainty = 0.90
(0.1 0.1 0.1)	Classification Uncertainty = 1.00
(0.3 0.3 0.3)	Classification Uncertainty = 1.00
(0.6 0.3 0.0)	Classification Uncertainty = 0.55
(0.6 0.3 0.1)	Classification Uncertainty = 0.60
(0.9 0.1 0.0)	Classification Uncertainty = 0.15
(0.9 0.05 0.05)	Classification Uncertainty = 0.15
(1.0 0.0 0.0)	Classification Uncertainty = 0.00

Fuzzy Accuracy Assessment (Green & Congalton 2003)



Reference Data

	D. Forest	E. Forest	Scrub/ Shrub	Grass	Barren	Urban	Ag.	Wet Perma	Water	User's Row Total Deterministic	User's Row Total Fuzzy
Deciduous Forest	**48**	4/0	2/0	0/1	0/0	0/1	0/1	0/0	0/0	48/56 (85.7%)	54/56 (96.4%)
Evergreen Forest	24/7	**17**	0/1	0/0	0/0	0/0	0/1	0/0	0/0	17/50 (34%)	41/50 (82%)
Scrub/shrub	6/1	0/1	**15**	5/1	0/2	0/0	7/15	0/0	0/0	15/57 (26.3%	33/57 (57.9%)
Grass	0/3	0/0	8/1	**14**	0/0	0/0	18/6	0/0	0/0	14/50 (28%)	40/50 (80%)
Barren	0/0	0/0	0/0	0/0	**0**	0/0	0/0	0/0	0/0	na	na
Urban	0/1	0/0	0/0	0/0	0/0	**20**	2/0	0/1	0/0	20/24 (83.3%)	22/24 (91.7%)
Agriculture	0/11	0/1	2/2	3/0	0/1	2/0	**29**	0/0	0/0	29/51 (56.9%)	36/51 (70.6%)
Wet permafrost	0/0	0/0	0/0	0/0	0/0	0/0	0/0	**0**	0/0	na	na
Water	0/18	0/3	0/0	0/0	0/0	0/0	1/2	1/0	**8**	8/33	10/33
Producer's Column Total Deterministic	48/123 (39%)	17/26 (65.4%)	15/31 (48.4%)	14/24 58.3%)	0/3 (0%)	20/22 (90.9%)	29/82 (35.4%)	0/2 (0%)	8/8 (100%)		
Producer's Column Total Fuzzy	78/123 (63.4%)	21/26 (80.8%)	27/31 (87.1%)	22/24 (91.7%)	0/3 (0%)	22/22 (100%)	57/82 (69.5%)	1/2 (50%)	8/8 (100%)		

Overall deterministic accuracy = 151/321 = 47%.
Overall fuzzy accuracy = 236/321 = 73.5%.

Fuzzy Error Matrix (Binaghi et al. 1999)

Table 1
Error matrix with p_{mn} representing the cardinality of the intersection between classification data (rows) and reference data (columns)

	Reference data				Total assignment
	1	2	…	Q	
Classification data					
1	p_{11}	p_{12}	…	p_{1q}	p_{1+}
2	p_{21}	p_{22}	…	p_{2q}	p_{2+}
⋮	⋮	⋮	…	⋮	⋮
Q	p_{q1}	p_{q2}	…	p_{qq}	p_{q+}
Total assignments					
	p_{+1}	p_{+2}	…	p_{+q}	

Hard Error Matrix

$$M(m,n) = |C_m \cap R_n| = \sum_{x \in X} \mu_{C_m \cap R_n}(x)$$

$$\mu_{C_m \cap R_n}(x) = \begin{cases} 1 & \text{iff } x \in C_m \wedge x \in R_n \\ 0 & \text{otherwise.} \end{cases}$$

Fuzzy Error Matrix

$$\tilde{M}(m,n) = \left|\tilde{C}_m \cap \tilde{R}_n\right| = \sum_{x \in X} \mu_{\tilde{C}_m \cap \tilde{R}_n}(x)$$

$$\mu_{\tilde{C}_m \cap \tilde{R}_n}(x) = \min(\mu_{\tilde{C}_m}(x), \mu_{\tilde{R}_n}(x))$$