Geometric Transformation

Affine Transformation

- aka linear or first-order transformation

Original Matrix	Translation	Scale	Shear/Rotation
$x_{predict} = (0) + (1)x + (0)y$	$x_{predict} = (1) + (1)x + (0)y$	$x_{predict} = (0) + (2)x + (0)y$	$x_{predict} = (0) + (1)x + (2)y$
$y_{predict} = (0) + (0)x + (1)y$	$y_{predict} = (1) + (0)x + (1)y$	$y_{predict} = (0) + (0)x + (2)y$	$y_{predict} = (0) + (2)x + (1)y$

$$x_o = a_1 + a_2 x_i + a_3 y_i$$
$$y_o = b_1 + b_2 x_i + b_3 y_i$$

x_i and y_i = source coordinates

$+x_o$ and y_o = rectified coordinates

Translation   Differential Scaling   Skew   Rotation

$$\begin{bmatrix} x_0 \\ y_0 \end{bmatrix} = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{bmatrix} \begin{bmatrix} 1 \\ x_i \\ y_i \end{bmatrix}$$

1

Rectification Steps

- Locate GCPs
- Calculate and evaluate a transformation
- Apply the transformation. Pixels must be resampled to conform to the grid.

Locate GCPs

- Ideal location: road intersections, corners of landscape objects, or single pixel objects dispersed evenly on the image.
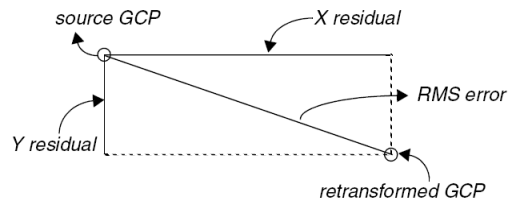- Source and reference coordinates.

of GCPs

- The more the better.
- 16 (Bernstein et al. 1983)

$$\frac{((t+1)(t+2))}{2}$$

Order of Transformation	Minimum GCPs Required
1	3
2	6
3	10
4	15
5	21
6	28
7	36
8	45
9	55
10	66

GCP Evaluation

- Residual
- Error per GCP (R)
- RMSE (T)
- Error contribution by point (R/T)

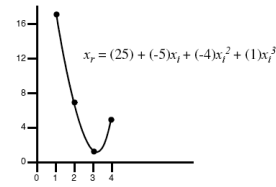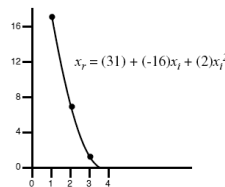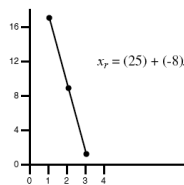# Transformation equations and coefficients

1st-order polynomial equations

$$x' \;=\; a_0 + a_1 x + a_2 y$$
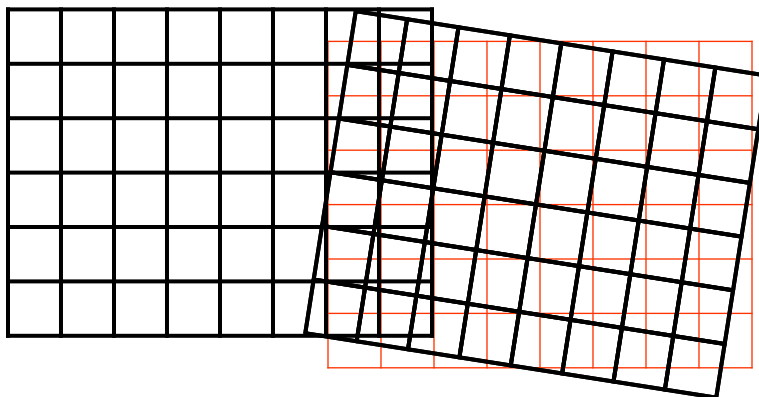
$$y' \;=\; b_0 + b_1 x + b_2 y$$

2nd-order polynomial equations

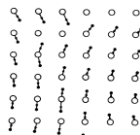$$x' \;=\; c_0 + c_1 x + c_2 y + c_3 xy + c_4 x^2 + c_5 y^2$$

$$y' \;=\; d_0 + d_1 x + d_2 y + d_3 xy + d_4 x^2 + d_5 y^2$$

$x_r = (25) + (-8)x_I$

$x_r = (31) + (-16)x_I + (2)x_I^2$

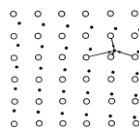$x_r = (25) + (-5)x_I + (-4)x_I^2 + (1)x_I^3$
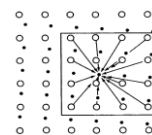
Transformation & Resampling
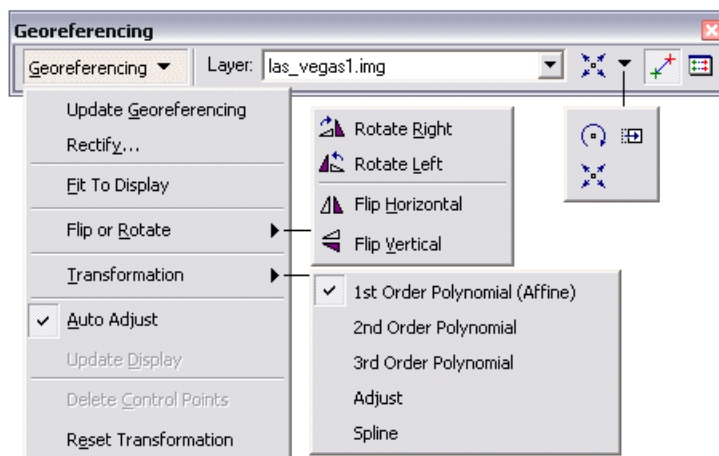
Nearest neighbor   Bilinear interpolation   Cubic convolution

Georeferencing in ArcGIS

- Spatial Adjustment tools (lab 5)
  - For vector data only
  - Transform (affine, similarity, projective)
  - Rubbersheet
  - Edgesnap
- Georeferencing tools (lab 6)
  - For raster data only
  - Flip or rotate
  - Transform

Georeferencing

Georeferencing ▼   Layer: las_vegas1.img

Update Georeferencing	
Rectify…	
Fit To Display	⌂ Rotate Right
Flip or Rotate ▶	⌂ Rotate Left
Transformation ▶	⌂ Flip Horizontal
✓ Auto Adjust	◁ Flip Vertical
Update Display	✓ 1st Order Polynomial (Affine)
Delete Control Points	2nd Order Polynomial
Reset Transformation	3rd Order Polynomial
	Adjust
	Spline

Update Georefencing vs Rectify in Raster Dataset

- Rectify: Permanently alters the spatial referencing information of a raster dataset by a transformation, which also alters the orientation of the pixels.

- Update Georeference: doesn't alter the orientation of pixels, instead it creates a "persisted transformation" with the raster dataset. The persisted transformation is stored within a geodatabase or as a .aux.xml and a .xfm file.