Multiple Regression

Template for Analysis

David Gerbing

Purpose of Regression Analysis

[Secs 1.7, 5.1.1]

The two primary purposes of regression analysis:

1. Understand the relations among the variables, especially as related to the target variable.
[via slope coefficients]

2. Predict the value of the target from the values of the predictor variables.
[from computing fitted values on new data, the predictions]

To accomplish these goals, estimate a model from the data to get the equation that provides the slope coefficients and the full equation (model) for computing predicted values.

Problem Definition

Three advertising campaigns across three different media — *TV*, *Radio*, and *Newspaper* — promoted a variety of products in 200 different markets.

Data: Spending for each media type in each market is recorded along with *Sales*

Purpose of the analysis:
How is spending in these markets related to Sales of the company's products?

Target Variable: Sales (number of units sold)
Predictor Variables: TV, Radio, Newspaper (spending in USD)

lessR Input: `reg(Sales ~ TV + Radio + Newspaper)`

The Data

TV	Radio	Newspaper	Sales
230.1	37.8	69.2	22.1
44.5	39.3	45.1	10.4
17.2	45.9	69.3	9.3
151.5	41.3	58.5	18.5
180.8	10.8	58.4	12.9
8.7	48.9	75	7.2
57.5	32.8	23.5	11.8
120.2	19.6	11.6	13.2
8.6	2.1	1	4.8
199.8	2.6	21.2	10.6

plus 190 more rows

Sales: in 1000 units

e.g., first row has recorded Sales of 22,100 units

Ad Budgets: in $1000 USD

e.g., first row has ad budgets of $230,100 and $37,800, and $69,200, respectively

Correlation Matrix

Develop Intuition for the Model

Relevance: Do the predictor variables relate to the target (response) variable?

[Sec 5.2]

Correlation Matrix

	Sales	TV	Radio	Newspaper
Sales	1.00	**0.78**	**0.58**	**0.23**
TV	**0.78**	1.00	0.05	0.06
Radio	**0.58**	0.05	1.00	0.35
Newspaper	**0.23**	0.06	0.35	1.00

TV ads correlates much with Sales, Radio ads also correlate well, though to a lesser extent

Newspaper ad spending does not correlate well with Sales

Uniqueness: Does each predictor provide unique info?

[Sec 5.3]

Correlation Matrix

	Sales	TV	Radio	Newspaper
Sales	1.00	0.78	0.58	0.23
TV	0.78	1.00	**0.05**	**0.06**
Radio	0.58	**0.05**	1.00	**0.35**
Newspaper	0.23	**0.06**	**0.35**	1.00

Newspaper ads has a slight correlation with Radio of 0.35

The two most relevant predictor variables, TV ads and Radio ads, do not correlate with each other, only 0.05

Conclude: Not much of a collinearity problem at least in terms of pairs of variables, each predictor appears to provide reasonably unique information

Model Selection: Given the correlations, what is the most likely candidate for the final model?

[Sec 5.2]

Correlation Matrix

	Sales	TV	Radio	Newspaper
Sales	1.00	0.78	0.58	0.23
TV	0.78	1.00	0.05	0.06
Radio	0.58	0.05	1.00	0.35
Newspaper	0.23	0.06	0.35	1.00

The model will clearly include TV ads because of its high correlation with Sales

Given the low correlation between TV ad spending and Radio ad spending, and the moderately high correlation of Radio ads with Sales, Radio ads adds new, useful information to the model beyond TV ads, so it also will likely be included in the model

Newspaper ad spending is not relevant

The Model

Relate the Response Variable to the Predictors

Write the estimated regression model

[Sec 2.3]

Estimated Model for Sales

	Estimate	Std Err	t-value	p-value	Lower 95%	Upper 95%
(Intercept)	**2.939**	0.312	9.422	0.000	2.324	3.554
TV	**0.046**	0.001	32.809	0.000	0.043	0.049
Radio	**0.189**	0.009	21.893	0.000	0.172	0.206
Newspaper	**-0.001**	0.006	-0.177	0.860	-0.013	0.011

Sales_fitted = 2.939 + 0.046(TV) + 0.189(Radio) - 0.001(Newspaper)

Specify and interpret the sample slope coefficient

[Secs 2.1, 2.3, 3.3.2] Apply to the 1st predictor variable

Estimated Model for Sales

	Estimate	Std Err	t-value	p-value	Lower 95%	Upper 95%
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	**0.046**	0.001	32.809	0.000	0.043	0.049
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.001	0.006	-0.177	0.860	-0.013	0.011

$b_{TV} = 0.046$

Sales are expressed in terms of 1000 units sold, so 0.046 * 1000 = 46 individual units (products)

In this *particular* sample, on average, for each increase of $1000 of TV spending, Sales increase by 46 units — *holding the values of Radio and Newspaper constant*

Show the fitted value and its calculation

Use the advertising campaign of TV=232, Radio=8.6, Newspaper=8.7

[Sec 1.2.2, 2.4.1]

	Estimate	Std Err	t-value	p-value	Lower 95%	Upper 95%
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	0.046	0.001	32.809	0.000	0.043	0.049
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.001	0.006	-0.177	0.860	-0.013	0.011

	TV	Radio	Newspaper	Sales	fitted	resid	rstdnt	dffits	cooks
13	23.800	35.100	65.900	9.200	10.577	-1.377	-0.829	-0.147	0.005
53	216.400	41.700	39.600	22.600	20.663	1.937	1.159	0.146	0.005
200	**232.100**	**8.600**	**8.700**	13.400	**15.173**	-1.773	-1.062	-0.144	0.005
130	59.600	12.000	43.100	9.700	7.884	1.816	1.087	0.144	0.005

Sales_fitted = 2.939 + 0.046(TV) + 0.189(Radio) - 0.001(Newspaper)
= 2.939 + 0.046(232) + 0.189(8.6) - 0.001(8.7)
= 15.173

Show the associated residual, its calculation, interpret

Use the advertising campaign of TV=232, Radio=8.6, Newspaper=8.7

[Sec 2.4.1]

RESIDUALS AND INFLUENCE

	TV	Radio	Newspaper	Sales	fitted	resid	rstdnt	dffits	cooks
35	95.700	1.400	7.400	9.500	7.575	1.925	1.154	0.160	0.006
10	199.800	2.600	21.200	10.600	12.551	-1.951	-1.169	-0.154	0.006
1	230.100	37.800	69.200	22.100	20.524	1.576	0.947	0.152	0.006
13	23.800	35.100	65.900	9.200	10.577	-1.377	-0.829	-0.147	0.005
53	216.400	41.700	39.600	22.600	20.663	1.937	1.159	0.146	0.005
200	232.100	8.600	8.700	**13.400**	**15.173**	**-1.773**	-1.062	-0.144	0.005
130	59.600	12.000	43.100	9.700	7.884	1.816	1.087	0.144	0.005
34	265.600	20.000	0.300	17.400	18.864	-1.464	-0.880	-0.143	0.005

e_{200} = Sales - Sales_fitted = 13.400 – 15.173 = –1.773

The actual Sales for Sample 200 fell 1,773 units short of its fitted value

Hypothesis Test

Is a Slope Coefficient Different from Zero?

Specify the null hypothesis and its alternative for the hypothesis test of the slope coefficient

[Sec 3.2.1]

Apply to the 1st predictor variable

Estimated Model for Sales

	Estimate	Std Err	t-value	p-value	Lower 95%	Upper 95%
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	0.046	0.001	32.809	0.000	0.043	0.049
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.001	0.006	-0.177	0.860	-0.013	0.011

Null Hypothesis: Partial slope coefficient, the values of Radio and Newspaper held constant, of TV on Sales is 0. In symbols, $\beta_{TV} = 0$

Alternative to the Null: Holding the values of Radio and Newspaper constant, there is a relationship between TV advertising and Sales, $\beta_{TV} \neq 0$

Specify the calculation of the *t*-statistic by applying the relevant numbers from this specific analysis

[Secs 3.2.1, 3.3.2.1] Apply to the 1st predictor variable

Estimated Model for Sales

	Estimate	**Std Err**	**t-value**	p-value	Lower 95%	Upper 95%
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	**0.046**	**0.001**	**32.809**	0.000	0.043	0.049
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.001	0.006	-0.177	0.860	-0.013	0.011

How close is the estimate to what is hypothesized?

The *t*-statistic shows how many standard errors the sample estimate is from the null, hypothesize value, here 0

$t = (b_{TV} - 0) / 0.001 = 0.046 / 0.001 = 32.809$

Adapt the definition of the *p*-value to this analysis

[Sec 3.2.1, 3.3.2.1] Apply to the 1st predictor variable

Estimated Model for Sales

	Estimate	Std Err	**t-value**	**p-value**	Lower 95%	Upper 95%
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	0.046	0.001	**32.809**	**0.000**	0.043	0.049
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.001	0.006	-0.177	0.860	-0.013	0.011

Assuming the null hypothesis of no relation is true, the probability of getting a sample slope coefficient that is almost 33 standard errors from 0 is extremely small, less than 0.000

The usual 95% cutoff for a t-statistic is around 2.00, closer to 1.96 for large samples, and larger for very small samples.

Specify the basis for the statistical decision for the hypothesis test and the resulting statistical conclusion

[Sec 3.2.1, 3.3.2.1] Apply to the 1st predictor variable

Estimated Model for Sales

	Estimate	Std Err	t-value	**p-value**	Lower 95%	Upper 95%
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	0.046	0.001	32.809	**0.000**	0.043	0.049
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.001	0.006	-0.177	0.860	-0.013	0.011

The *p*-value=0.000 < alpha=0.05, so reject the null hypothesis of no relation between TV ad spending and Sales for any given levels of Radio and Newspaper advertising

Hypothesis Test: Interpretation, as an executive summary you would report to management

[Sec 3.2.1, 3.3.2.1] Apply to the 1st predictor variable

Estimated Model for Sales

	Estimate	Std Err	t-value	**p-value**	Lower 95%	Upper 95%
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	0.046	0.001	32.809	**0.000**	0.043	0.049
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.001	0.006	-0.177	0.860	-0.013	0.011

For any given level of Radio and Newspaper advertising, TV advertising is related to *increasing* Sales

Specify the null hypothesis and its alternative for the hypothesis test of the slope coefficient

[Sec 3.2.1]

Apply to the 3rd predictor variable

Estimated Model for Sales

	Estimate	Std Err	t-value	p-value	Lower 95%	Upper 95%
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	0.046	0.001	32.809	0.000	0.043	0.049
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.001	0.006	-0.177	0.860	-0.013	0.011

Null Hypothesis: Partial slope coefficient, that is, the values of Radio and TV held constant, of Newspaper ad spending on unit Sales is 0. In symbols, $\beta_{NP} = 0$

Alternative to the Null: Holding the values of Radio and TV ad spending constant, there is a relationship between Newspaper advertising and unit Sales, $\beta_{NP} \neq 0$

Specify the calculation of the *t*-statistic by applying the relevant numbers from this specific analysis

[Secs 3.2.1, 3.3.2.1] Apply to the 3rd predictor variable

Estimated Model for Sales

	Estimate	**Std Err**	**t-value**	p-value	Lower 95%	Upper 95%
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	0.046	0.001	32.809	0.000	0.043	0.049
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	**-0.001**	**0.006**	**-0.177**	0.860	-0.013	0.011

How close is the estimate to what is hypothesized? The *t*-statistic is how many standard errors the sample estimate is from the null, hypothesize value, here 0

$t = (b_{NP} - 0) / 0.006 = -0.001 / 0.006 = -0.177$

Adapt the definition of the *p*-value to this analysis

[Sec 3.2.1, 3.3.2.1] Apply to the 3rd predictor variable

Estimated Model for Sales

	Estimate	Std Err	**t-value**	**p-value**	Lower 95%	Upper 95%
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	0.046	0.001	32.809	0.000	0.043	0.049
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.001	0.006	**-0.177**	**0.860**	-0.013	0.011

Assuming the null hypothesis of no relation is true, the probability of getting a sample slope coefficient that is only 0.177 or more standard errors from 0 is high, 0.860

This sample slope coefficient of -0.001 is consistent with the null hypothesized value of 0

Specify the basis for the statistical decision for the hypothesis test and the resulting statistical conclusion

[Sec 3.2.1, 3.3.2.1]

Apply to the 3rd predictor variable

Estimated Model for Sales

	Estimate	Std Err	t-value	**p-value**	Lower 95%	Upper 95%
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	0.046	0.001	32.809	0.000	0.043	0.049
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.001	0.006	-0.177	**0.860**	-0.013	0.011

The *p*-value=0.860 > alpha=0.05, so cannot reject the null hypothesis of no relation between Newspaper ad spending and Sales for any given levels of Radio and TV advertising

Hypothesis Test: Interpretation, as an executive summary you would report to management

[Sec 3.2.1, 3.3.2.1] Apply to the 3rd predictor variable

Estimated Model for Sales

	Estimate	Std Err	t-value	p-value	Lower 95%	Upper 95%
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	0.046	0.001	32.809	0.000	0.043	0.049
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.001	0.006	-0.177	**0.860**	-0.013	0.011

For any given level of Radio and TV advertising, no relationship between Newspaper ad spending and unit Sales is detected

Confidence Interval

What is the True Value of a Slope Coefficient?

Specify the value the confidence interval estimates

[Secs 3.2.2, 3.3.2.2] Apply to the 1st predictor variable

Estimated Model for Sales

	Estimate	Std Err	t-value	p-value	Lower 95%	Upper 95%
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	0.046	0.001	32.809	0.000	0.043	0.049
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.001	0.006	-0.177	0.860	-0.013	0.011

The confidence interval estimates the true (population) slope coefficient that relates TV advertising to Sales with given values of Radio and Newspaper advertising

Specify the computation of the margin of error by applying the relevant numbers of this analysis
(can assume 2 for the *t*-cutoff)

[Secs 3.2.2, 3.3.2.2] Apply to the 1st predictor variable

Estimated Model for Sales

	Estimate	**Std Err**	t-value	p-value	Lower 95%	Upper 95%
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	0.046	**0.001**	32.809	0.000	0.043	0.049
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.001	0.006	-0.177	0.860	-0.013	0.011

The 95% margin of error is about plus or minus 2 standard errors, depending on the exact value of the *t*-cutoff value that cuts off the upper and lower 2.5% of the *t*-distribution over many hypothetical repeated samples

E = (2)(0.001)

Show the computations of the confidence interval illustrated with the specific numbers from this analysis

[Secs 3.2.2, 3.3.2.2] Apply to the 1st predictor variable

Estimated Model for Sales

	Estimate	Std Err	t-value	p-value	**Lower 95%**	**Upper 95%**
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	**0.046**	0.001	32.809	0.000	**0.043**	**0.049**
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.001	0.006	-0.177	0.860	-0.013	0.011

The 95% confidence interval is plus and minus the margin of error about the sample value, here for the sample estimate $b_{TV} = 0.046$

Lower Bound of CI: $b_{TV} - E = 0.046 - E = 0.043$

Upper Bound of CI: $b_{TV} + E = 0.046 + E = 0.049$

Confidence Interval: Interpretation, as an executive summary you would report to management

[Secs 3.2.2, 3.3.2.2] Apply to the 1st predictor variable

Estimated Model for Sales

	Estimate	Std Err	t-value	p-value	Lower 95%	Upper 95%
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	0.046	0.001	32.809	0.000	**0.043**	**0.049**
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.001	0.006	-0.177	0.860	-0.013	0.011

For any given level of Radio and Newspaper advertising, each additional $1000 of TV advertising relates to an average increase of unit Sales likely somewhere between 43 to 49

Demonstrate the consistency of the confidence interval and hypothesis test using the specific numbers for this analysis for both results

[Sec 3.3.2.3] Apply to the 1st predictor variable

Estimated Model for Sales

	Estimate	Std Err	t-value	**p-value**	**Lower 95%**	**Upper 95%**
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	0.046	0.001	32.809	**0.000**	**0.043**	**0.049**
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.001	0.006	-0.177	0.860	-0.013	0.011

The *p*-value for the hypothesis test of a 0 partial population slope coefficient for TV is less than 0.05, so 0 was rejected as a plausible value

The confidence interval contains the range of plausible values for the slope coefficient, which does not contain 0, again rendering 0 as implausible

Model Selection and Fit

Choose the Most Useful Predictors and Assess Fit

Are any predictor variables much less useful for predicting the response variable? Why or why not?

[Secs 5.2, 5.4]

Estimated Model for Sales

	Estimate	Std Err	t-value	**p-value**	**Lower 95%**	**Upper 95%**
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	0.046	0.001	32.809	0.000	0.043	0.049
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.001	0.006	-0.177	**0.860**	**-0.013**	**0.011**

The *p*-value for the hypothesis test of a 0 partial population slope coefficient for Newspaper ad spending is much greater than 0.05 so 0 cannot be rejected, and the confidence interval contains 0 and both - and + values, all rendered as plausible

No relationship is detected for Newspaper ad spending and Sales given a set of values for TV and Radio ad spending

Evaluate fit with the standard deviation of residuals

[Secs 2.4, 3.3.3]

```
Sales:  Mean = 14.022    SD = 5.217
        Min = 1.600    Max = 27.000


Model Fit

Standard deviation of residuals:  1.686 for 196 degrees of freedom
95% range of residual variation:  6.648    [2 * 1.972 * 1.686]

R-squared:  0.897    Adjusted R-squared:  0.896    PRESS R-squared:  0.891
```

The standard deviation of unit Sales is 5.217, yet the standard deviation of the residuals about the regression line reduces down to 1.686, a considerable reduction

The model fits at least reasonably well, though ultimately fit depends on the value judgement of the impact of the size of the 95% range of variation of the residuals, 6.648

Evaluate fit with R^2 and PRESS R^2, including their comparison

[Secs 2.5.2, 3.3.3, 6.3.3]

```
Sales:  Mean = 14.022    SD = 5.217
        Min = 1.600    Max = 27.000


Model Fit

Standard deviation of residuals:  1.686 for 196 degrees of freedom
95% range of residual variation:  6.648    [2 * 1.972 * 1.686]

R-squared:  0.897    Adjusted R-squared:  0.896    PRESS R-squared:  0.891
```

R-sq above 0.6 is usually considered a good fit to the data, a descriptive statistic

PRESS R-sq generalizes to new data and so usually is lower

Here PRESS R-sq is almost as large as R-sq, indicating no practical overfitting and excellent fit to new data as well

Any collinearity problems? Why or why not?

[Sec 5.3]

Collinearity

	Tolerance	VIF
TV	**0.995**	1.005
Radio	**0.873**	1.145
Newspaper	**0.873**	1.145

Tolerance below 0.2 usually is considered to indicate linear dependence of the predictor variable on the others, which corresponds to VIF > 5

Clearly no collinearity with these predictor variables

Any potential outliers? Why or why not?

[Secs 6.2, 6.3]

RESIDUALS AND INFLUENCE

	TV	Radio	Newspaper	Sales	fitted	resid	rstdnt	dffits	cooks
131	0.700	39.600	8.700	**1.600**	**10.428**	-8.828	**-5.758**	-1.127	**0.273**
6	8.700	48.900	75.000	**7.200**	**12.478**	-5.278	**-3.288**	-0.734	**0.128**
76	16.900	43.700	89.400	8.700	11.858	-3.158	-1.943	-0.478	0.056
36	290.700	4.100	8.500	12.800	17.007	-4.207	-2.571	-0.459	0.051
179	276.700	2.300	23.700	11.800	16.011	-4.211	-2.570	-0.435	0.046
127	7.800	38.900	50.600	6.600	10.577	-3.977	-2.422	-0.403	0.040
166	234.500	3.400	84.800	11.900	14.224	-2.324	-1.433	-0.391	0.038

Two samples have a much larger Cook's Distance values and Studentized residuals than the others, and so are likely outliers

Both samples have negative residuals, which means that Sales was drastically over predicted

For example, for the first sample, unit Sales was 1600 and fitted Sales is 10,428

From this analysis, what model do you recommend? Why?

[Sec 5.4]

Best Subset Regression Models

TV	Radio	Newspaper	R2adj	X's
1	**1**	**0**	**0.896**	**2**
1	1	1	0.896	3
1	0	1	0.642	2
1	0	0	0.610	1
0	1	0	0.329	1
0	1	1	0.326	2
0	0	1	0.047	1

The most parsimonious model is the model with TV and Radio advertising as the predictor variables

The fit of R2-adjusted is as large as the full three variable model, 0.896, and noticeably improved over the just the single predictor model with TV as the predictor, 0.610

Prediction Intervals

How Much Error is Expected from Predictions of y?

Show the calculation of the 95% prediction interval approximated with the t-cutoff of 2

Use the advertising campaign of TV=232, Radio=8.6, Newspaper=8.7

[Sec 4.3.4.2]

```
PREDICTION ERROR

            TV  Radio Newspaper  Sales   pred s_pred pi.lwr pi.upr width
 97 197.600  3.500     5.900 11.700 12.636  1.701  9.281 15.991 6.710
 17  67.800 36.600   114.000 12.500 12.824  1.757  9.359 16.288 6.929
 58 136.200 19.200    16.600 13.200 12.775  1.691  9.439 16.110 6.672
...
200 232.100  8.600     8.700 13.400 15.173 1.701 11.819 18.527 6.708
 46 175.100 22.500    31.500 14.900 15.162 1.690 11.828 18.495 6.667
```

Margin of Error: The standard error of prediction (forecast), s_pred, multiplied by about 2

95% Prediction Interval: Plus and minus the margin of error about the predicted value, Sales_pred=15.173

Lower Bound of PI: Sales_fitted – E = 15.173 – E = 11.819
Upper Bound of PI: Sales_fitted + E = 15.173 + E = 18.527

Interpret the prediction interval

Use the advertising campaign of TV=232, Radio=8.6, Newspaper=8.7

[Sec 4.3.4.3]

```
PREDICTION ERROR

            TV  Radio Newspaper  Sales   pred    sf pi.lwr pi.upr width
    97 197.600  3.500     5.900 11.700 12.636 1.701  9.281 15.991 6.710
    17  67.800 36.600   114.000 12.500 12.824 1.757  9.359 16.288 6.929
    58 136.200 19.200    16.600 13.200 12.775 1.691  9.439 16.110 6.672
...
   200 232.100  8.600     8.700 13.400 15.173 1.701 11.819 18.527 6.708
    46 175.100 22.500    31.500 14.900 15.162 1.690 11.828 18.495 6.667
   147 240.100  7.300     8.700 13.200 15.294 1.702 11.937 18.651 6.714
```

With 95% confidence, the actual value of Sales -- for a sample with TV ad spending of 232, Radio of 8.6, and Newspaper of 8.7 -- will lie between 11,819 and 18,527 units

Conclusion

Management Recommendation

What decision do you recommend to management?

	Estimate	Std Err	t-value	p-value	Lower 95%	Upper 95%
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	0.046	0.001	32.809	**0.000**	0.043	0.049
Radio	0.189	0.009	21.893	**0.000**	0.172	0.206
Newspaper	-0.001	0.006	-0.177	**0.860**	-0.013	0.011

TV	Radio	Newspaper	R2adj	X's
1	**1**	**0**	**0.896**	**2**
1	1	1	0.896	3
1	0	1	0.642	2
1	0	0	0.610	1
0	1	0	0.329	1
0	1	1	0.326	2
0	0	1	0.047	1

Controlling the values of TV and Radio ad spending, Newspaper ad spending contributes little to Sales

Maybe by itself Newspaper ad spending would be useful, but that hypothesis is not evaluated in this analysis. Instead, implement TV and Radio ads, dropping Newspaper ads

The End