

Chapter 5

Hypothesis Test of the Mean

Section 5.4

Power Analysis

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- Power Analysis and a Meaningful Effect
 - Managerial vs Statistical Decisions
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5.4a

Managerial vs Statistical Decisions

Managerial Decision versus Statistical Decision

Purpose of statistical analysis is to guide managerial decisions

- ▶ **Managerial decision:** Implement, or not, a specific action
- ▶ **Key Concept:** A technical decision regarding statistical inference provides information about reality that guides a subsequent managerial decision
- ▶ Statistical inference, confidence interval or hypothesis test, often focuses on a specific value in terms of the null hypothesis
- ▶ **Statistical decision** regarding a mean: Does a difference exist from the hypothesized value?
- ▶ The statistical decision is to either reject or not reject this reference value of interest, $\mu = \mu_0$
- ▶ Each outcome of the statistical decision, reject or not, may lead to a different managerial decision

Statistical Decision Guides the Managerial Decision

The statistical analysis informs the decision maker

- ▶ Consider the goal to assess a potential improved process with a trial version, implemented on a small scale, which management is considering to implement company wide
- ▶ To assess, compare a sample mean from the new process against the current mean, which is the null hypothesis
- ▶ One outcome is that the statistical analysis detects a difference from the null in the desired direction
 - Statistical decision → Reject the null hypothesis, so ...
 - typical Managerial decision → Implement the change
- ▶ Another outcome is that the statistical analysis does not detect a difference from the null
 - Statistical decision → Do not reject the null, so ...
 - typical Managerial decision → Retain the current process

Slippage Between Statistical and Managerial Decisions

Issue: These two decisions are not so tightly coupled

- ▶ However, this relation between statistical and managerial decisions must be qualified because the statistical and managerial decisions are distinct concepts
- ▶ **Key Concept:** The managerial decision does not necessarily follow directly from the statistical decision
- ▶ The statistical decision guides the managerial decision, but the statistical decision does not rigidly imply what action the manager should take
- ▶ In terms of deciding to retain or change to a new process
 - Rejection of the null hypothesis does not necessarily imply that the new process should be implemented
 - Failure to reject the null hypothesis does not necessarily imply that the current process should be retained
- ▶ This potential slippage is explored next

5.4b Type I and Type II Errors

Binary Decisions

Definition and Examples of Binary Decisions

Choose one alternative or the other

- ▶ **Binary decision:** Choose between two alternatives
- ▶ Professional and personal life presents a series of binary decisions, from the trivial to the profound
 - Cross the road now, do not cross now
 - Hire the applicant, do not hire the applicant
 - Invest in increased manufacturing capacity, do not invest
 - Marry the person, do not marry
 - Reject the null hypothesis, do not reject

Consequences of Binary Decisions

Four possible outcomes from one binary decision

- ▶ Each binary decision provides **two ways to be right** (opportunity) and two ways to be wrong (error)
- ▶ Each outcome implies different consequences
- 😊 **Correct:** Cross road now and do not get run over
- 😊 **Correct:** Do not cross road now and watch the car go by
- 😞 **Error:** Do not cross now and no car goes by – consequence is to lose a little bit of time
- 😞 **Error:** Cross now and get run over – consequence is death or severe injury
- ▶ These two types of errors have names, rather unimaginatively called **Type I and Type II errors**

Four Possible Outcomes for an Hypothesis Test

Four possible outcomes from one binary decision

- ▶ The manager's decision regards an **unknown reality**, which at some later time reveals itself
- ▶ **Statistical Decision Rule** for analysis of the mean: **If m is in the rejection region, decide H_0 is false**, otherwise the data is consistent with H_0
- 😊 **Correct:** Decide the null is true, and it really is
- 😊 **Correct:** Decide a false null is false, a real difference detected
- 😞 **Type I Error, False Positive:** An unusually deviant event leads to decide the null is false, but it really is true
- 😞 **Type II Error, False Negative:** Decide the null is true, but fail to detect a difference that actually exists

Four Possible Outcomes for an Hypothesis Test

		Decision	
		Do Not Reject H_0	Reject H_0
Reality	H_0 is true	CORRECT: "accept" a true null	TYPE I ERROR: reject a true null
	H_0 is false	TYPE II ERROR: accept a false null	CORRECT: reject a false null

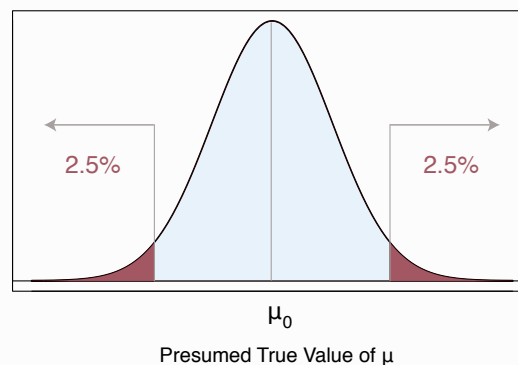
Type I Error

Null Hypothesis is True: Probability of a Type I Error

Consider the null hypothesis to be true

- ▶ Distribution of sample mean, m , centered over hypothesized value, μ_0 , assumed true *for purposes of the test*
- ▶ Presume the hypothesized value, μ_0 , *really is true*, not just for purposes of the test
- ▶ IF μ_0 is true, then ...
 - The applicable error is a Type I error, rejecting a true μ_0
 - A Type I Error occurs when m falls in the rejection region
 - What is the (conditional) probability that m falls in the rejection region given that the null hypothesis is true?

Null Hypothesis is True: Probability of a Type I Error



- ▶ Probability of a Type I Error for any one sample is not computed, but instead set at the probability of the complete rejection region, α , usually $\alpha = .05$ as shown here

Type II Error

More of the What-If Game

Assume $\mu = \mu_0$, but what if this assumption is false?

- ▶ To understand and estimate the value of the unknown value of μ , probe the consequences of assuming different values of μ
- ▶ The basis of analyzing Type II error is that the assumed null hypothesis, $\mu = \mu_0$ is wrong
- ▶ **Alternative value** of the mean: To presume $\mu \neq \mu_0$ means that some alternative value of μ , μ_{ALT} , is true, $\mu = \mu_{ALT}$
- ▶ **Type II Error Analysis:**
 - First, set up the hypothesis test by assuming
WHAT IF $\mu = \mu_0$?
 - Second, add a second assumption beyond the first,
WHAT IF we first assume $\mu = \mu_0$,
but then ask actually WHAT IF, instead, $\mu = \mu_{ALT}$?
- ▶ Will we properly reject our first assumption of $\mu = \mu_0$ if, instead, the truth is that $\mu = \mu_{ALT}$?

Concepts and Notation

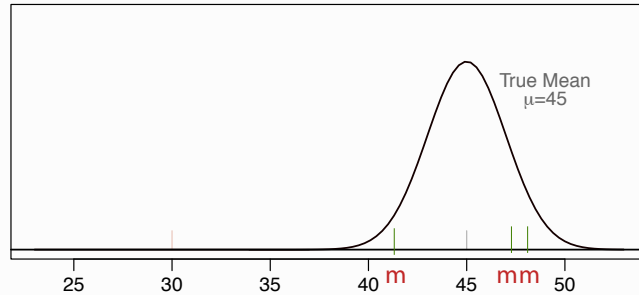
Many presumed values of μ when μ_0 is false

- ▶ Decision rule: Fail to reject μ_0 if m is reasonably close to μ_0 , as assessed by the corresponding hypothesis test
- ▶ The error here, a Type II error, is to “accept” $\mu = \mu_0$ when some alternative value is actually the true mean, $\mu = \mu_{ALT}$
- ▶ The probability of a Type II error is defined in terms of one corresponding specific value of μ_{ALT} relative to the specified value of μ_0
- ▶ There are many probabilities of a Type II error, each probability calculated relative to a specific value of the presumed true mean, $\mu = \mu_{ALT}$
- ▶ There are many possibilities for μ_{ALT} , which each can be systematically investigated, as shown next, beginning with a specific example where $\mu_{ALT} = 45$

Probability of a Type II Error

Presume true mean is actually $\mu_{ALT} = 45$

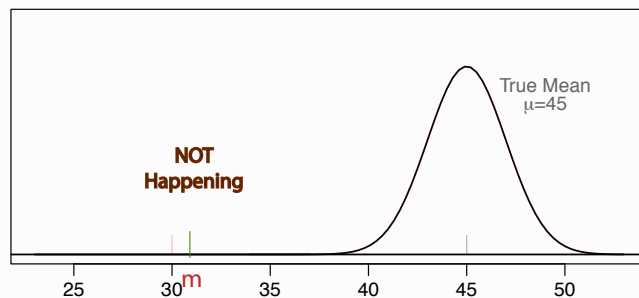
- ▶ Hypothetical distribution of m is centered over the presumed true mean of 45, with most values between 40 and 50
- ▶ For example, if a sample were taken, values might be obtained such as $m=48.8$, $m=47.1$ or $m=41.5$



Probability of a Type II Error

Consider an extremely deviant sample mean, $m = 31.0$

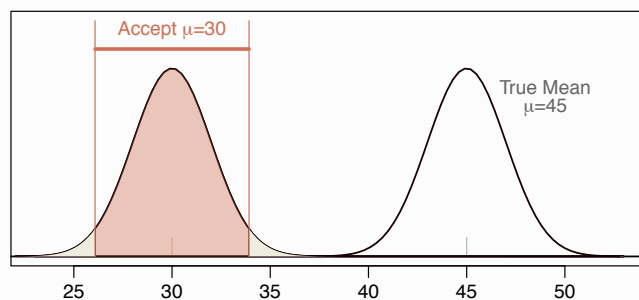
- ▶ A value of $m = 31.0$ would be highly unlikely to ever occur for this distribution with $\mu_{ALT} = 45$
- ▶ Although theoretically possible, a value of m this far below the mean of 45 would not happen in practice



Probability of a Type II Error

Hypothesized mean is $\mu_0 = 30$

- ▶ Now consider a hypothesis test with μ_0 set at 30, which, in this scenario is *false* as the true mean is presumed to be 45
- ▶ Around the value of the hypothesized mean of 30 is the "acceptance" region of the hypothesis test



Probability of a Type II Error

Presume true mean is actually $\mu_{ALT} = 45$

- ▶ $\mu = 30$ is false in this scenario, so a Type II error occurs when m falls in the “acceptance” region, “accepting” the false null
- ▶ But m will virtually never fall in the “acceptance” region
- ▶ The false $\mu_0 = 30$ would almost certainly be correctly rejected for any one sample because the sample mean, m , would be much higher than the hypothesized mean, μ_0
- ▶ The value of the one observed m is not known until the data are collected, but whatever value it assumes, a difference from the null value will be properly detected in this situation

Basis of a Type II Error

Focus on a specific alternative to μ

- ▶ The hypothesis test “accepts” the null hypothesis when m falls in the “acceptance” region, close to μ_0
- ▶ If μ_{ALT} is the true mean, then the sample mean m is sampled from the distribution centered about μ_{ALT}
- ▶ **Key Concept:** If μ_{ALT} is the true mean, m can still fall within the “acceptance” region defined by the null hypothesis
- ▶ A Type II Error occurs when the null is false, but m is in the “acceptance” region:
 - The difference from the null value μ_0 to the presumed reality μ_{ALT} exists, but the hypothesis test does not detect the difference
- ▶ What is the probability that a value of m is in the “acceptance” region given the value of μ_{ALT} ?

5.4c The Concept of Power

Power Defined and Computed

Probability of the Correct Choice: Power

Focus on a specific alternative to the hypothesized value of μ

- ▶ Typically prefer to state the probability of making the *correct* choice, of rejecting the null hypothesis when it is false
- ▶ **Power** of an Hypothesis Test: Probability of *correctly* detecting a real difference from a false null hypothesis
- ▶ When the null is false, so that μ is different from μ_0 ,
 - Either make the right choice, reject the false null, or
 - Make the wrong choice, do not reject the false null
- ▶ **Notation:** Probability of a Type II Error is β
- ▶ The probabilities of making the correct or incorrect choice, here for $\mu \neq \mu_0$, sum to 1,
$$\text{Power} + \beta = 1$$
or
$$\text{Power} = 1 - \beta$$

Calculate Power and Probability of a Type II Error

Use R as an electronic table

- ▶ To use R to calculate Power is to use R as an electronic table of probabilities – no data file, no data analysis
- ▶ Provide a specific sample size, n , and standard deviation, s , such as from an initial data analysis
- ▶ To calculate Power and β , the actual values of the alternative and hypothesized values of μ per se are not of interest
- ▶ Rather, the focus is on the *difference* or “delta” between the hypothesized value of μ and the specific proposed alternative
$$\text{delta} = \mu_{ALT} - \mu_0$$
- ▶ For the same n and s , the power of detecting a real μ of 35 from a hypothesized value of 30 is *exactly the same* as detecting a real μ of 105 from a hypothesized value of 100

R Input: Power for a one-sample t -test

R function `power.t.test` calculates power

- ▶ Suppose for a specific data analysis:
 $n = 25$, $s = 10$, $\mu_0 = 30$ and $\mu_{ALT} = 35$
- ▶ R input:

```
> power.t.test(n=25, sd=10, delta=5,  
               type="one.sample")
```
- ▶ Required values:
 - `n`: sample size
 - `sd`: standard deviation of data
 - `delta`: change from μ_0 to the alternative value of μ
- ▶ Defaults:
 - `alternative="two.sided"`, i.e., two-tailed test
 - `type="two.sample"`, i.e., for a mean difference

R Output: Power for a one-sample t -test

One-sample t -test power analysis

```
> power.t.test(n=25, sd=10, delta=5,  
               type="one.sample")
```

```
One-sample t test power calculation  
  
      n = 25  
    delta = 5  
      sd = 10  
sig.level = 0.05  
  power = 0.6697014  
alternative = two.sided
```

- ▶ For a sample of size 25 with a standard deviation of 10, and a hypothesized value of 30, the **probability of correctly detecting a difference when $\mu = 35$** is 0.67, and so $\beta = 1 - 0.67 = 0.33$

Probability Distribution that Underlies a Type II Error

Technical Note

- ▶ The hypothesis test of the mean follows from **the distribution of t_m** based on the sample mean, **assuming the null hypothesized value, μ_0 , is true**
- ▶ The probability of a Type II Error follows from a **non-central t -distribution**, the distribution of the t -statistic, t_m , when μ_0 is assumed true but is actually false
- ▶ For **pedagogical simplicity**, the following illustrations are based on the sample mean, m , and the normal distribution of m instead of t_m and the non-central t -distribution
- ▶ The basic concepts remain the same, and **numerical computations only suffer when the sample size, n , is small**
- ▶ Fortunately, **Type II Error calculations with R are based the correct non-central t -distribution**, so actual applications of these probabilities from R are accurate for all values of n

Power: An Example Continued

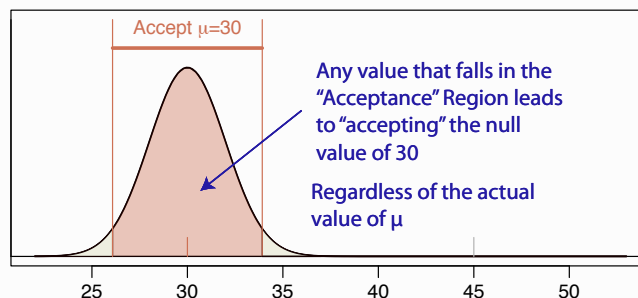
Illustrate Probability of Type II Error with Graphics

Consider a specific example

- ▶ Suppose the hypothesized value is $\mu_0 = 30$
- ▶ Collecting a sample of size $n = 25$ yielded a standard deviation of $s = 10$, for a standard error of the mean of $s_m = 2$
- ▶ Suppose the sample mean m was larger than 30, but close enough to $\mu_0 = 30$ that the null hypothesis was *not* rejected
- ▶ Although no difference from the null was detected, the analyst wonders if the test might have failed to detect a real difference with the true value of μ actually larger than 30
- ▶ What is the probability of detecting this difference if it actually exists?
- ▶ Power analysis provides the answer

Probability of a Type II Error

Null hypothesis is $\mu = 30$



- ▶ Here the null hypothesized value, μ_0 , is 30
- ▶ The hypothesis test sets up an "acceptance" region around 30
- ▶ Whatever the actual value of μ , a m that falls in the "acceptance" region leads to "accepting" the null

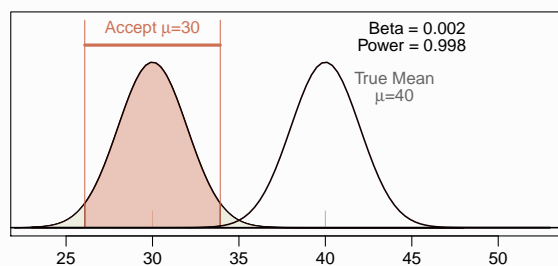
Probability of a Type II Error

Probability depends on the true value of μ

- ▶ So the computation of the probability of a Type II Error, β , depends on the true value of μ
- ▶ Presume the value of the null hypothesis, μ_0 , is false, so what is the alternative value of μ , μ_{ALT} to use for the probability calculation?
- ▶ We do not know the true value of μ , which is the whole point of statistical inference, to estimate μ
- ▶ In absence of the actual value of μ , explore what happens for a range of potential different alternative values of μ , from values far away from μ_0 to values that are close to μ_0
- ▶ What are Power and β ... IF the actual $\mu = \mu_{ALT}$ was equal to a value just one unit above or below μ_0 , or two units, or ...

Probability of a Type II Error

Hypothesize 30; Presume true mean is actually $\mu_{ALT} = 40$

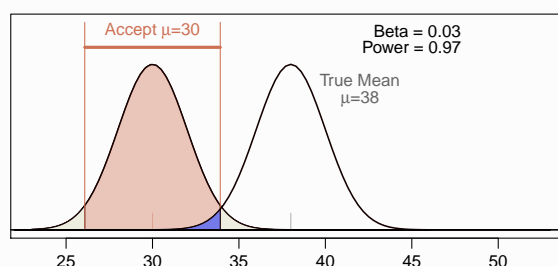


```
> power.t.test(n=25, sd=10, delta=10, type="one.sample")
```

- ▶ For $\mu_{ALT} = 40$, m likely varies somewhere between 34 and 46
- ▶ Rarely a m may just cross the line into the "acceptance" region, resulting in a Type II Error

Probability of a Type II Error

Hypothesize 30; Presume true mean is actually $\mu_{ALT} = 38$

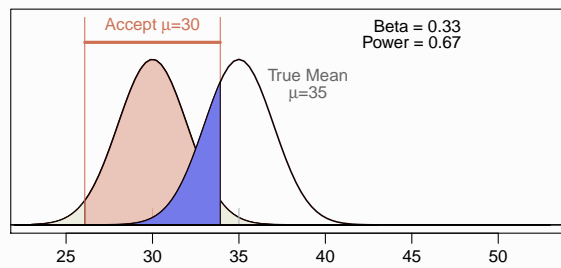


```
> power.t.test(n=25, sd=10, delta=8, type="one.sample")
```

- ▶ For $\mu_{ALT} = 38$, m likely varies somewhere between 32 and 44
- ▶ Here m occasionally lands in the "acceptance" region, as a Type II Error has a probability of only 0.03 (blue area)

Probability of a Type II Error

Hypothesize 30; Presume true mean is actually $\mu_{ALT} = 35$

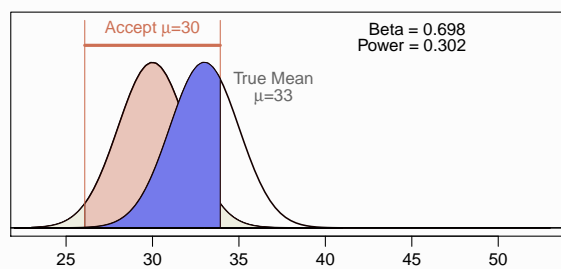


```
> power.t.test(n=25, sd=10, delta=5, type="one.sample")
```

- ▶ For $\mu_{ALT} = 35$, m likely varies somewhere between 29 and 41
- ▶ Here m lands in the "acceptance" region with reasonable frequency, as a Type II Error has probability of 0.330

Probability of a Type II Error

Hypothesize 30; Presume true mean is actually $\mu_{ALT} = 33$

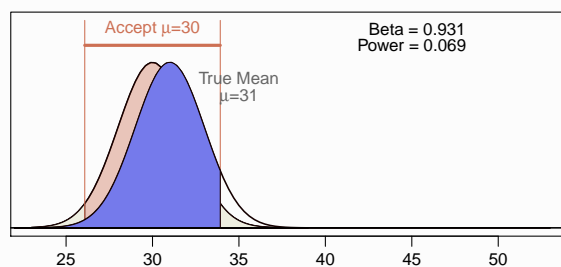


```
> power.t.test(n=25, sd=10, delta=3, type="one.sample")
```

- ▶ For $\mu_{ALT} = 33$, m likely varies somewhere between 27 and 39
- ▶ Now m lands in the "acceptance" region more often than not, as the probability of a Type II Error is 0.698

Probability of a Type II Error

Hypothesize 30; Presume true mean is actually $\mu = 31$



```
> power.t.test(n=25, sd=10, delta=1, type="one.sample")
```

- ▶ For $\mu_{ALT} = 31$, m likely varies somewhere between 25 and 37
- ▶ Now m lands in the "acceptance" region most of the time, with a probability of a Type II Error at 0.931

5.4d

The Power Curve

Power Over a Range of Alternative Values of μ

Simultaneously Consider Many Alternative Values of μ

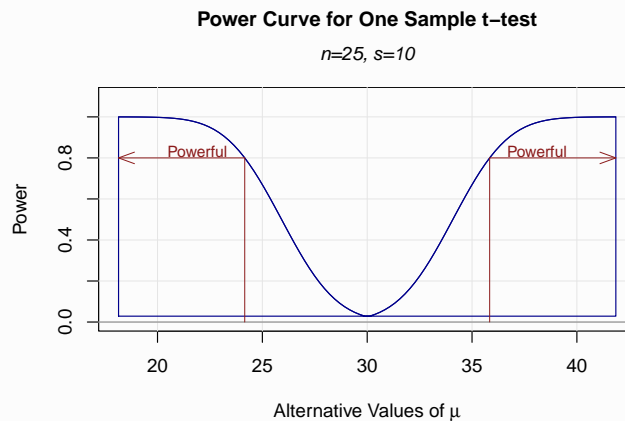
Only one μ_0 , but many possible alternative values of μ

- ▶ Organize this information regarding power for multiple alternative values of μ into a single graph
- ▶ **Power curve:** Plot the power relative to a specified null value, μ_0 , across a range of alternative values of μ , μ_{ALT}
- ▶ By analyzing power over a range of possible values of μ , an interesting pattern emerges that can provide the analyst much information for the interpretation of a specific hypothesis test
 - Values of μ far from μ_0 are almost assuredly detected
 - Values of μ close to μ_0 are almost assuredly *not* detected
- ▶ Discover “far from” and “close to” for a specific analysis
- ▶ One tradition is to establish a minimum threshold of desirable power of 0.8 in general, which may be adjusted up or down for a specific situation

Power Curve for Previous Example with Sample Size of 25

- ▶ lessR `ttestPower` function provides the power curve

```
> ttestPower(n=25, s=10, mu=30)
```



Power Curve Interpretation

What ranges of μ have high and low power, respectively?

- ▶ Small changes more difficult to detect, so **always have low power against values close to the hypothesized value**
- ▶ **Minimum desired power is usually set at 0.8, here not obtained until real $\mu > 36$ or $\mu < 24$**
- ▶ Probability of detecting a real change from 30 is **less than 50% for values of μ all the way up to 34 and down to 26**
- ▶ So what if the true mean is $\mu = 33$, which management decides is **a sufficiently large increase from $\mu_0 = 30$ to indicate a meaningful, interesting difference from 30?**
- ▶ If $\mu = 33$, then such a value is **too close to the hypothesized value of 30 to be detected with any reasonable probability in this analysis, as the power for this value is only 0.302**
- ▶ The next section shows how to **address this issue**

Needed Sample Size for Sufficient Power

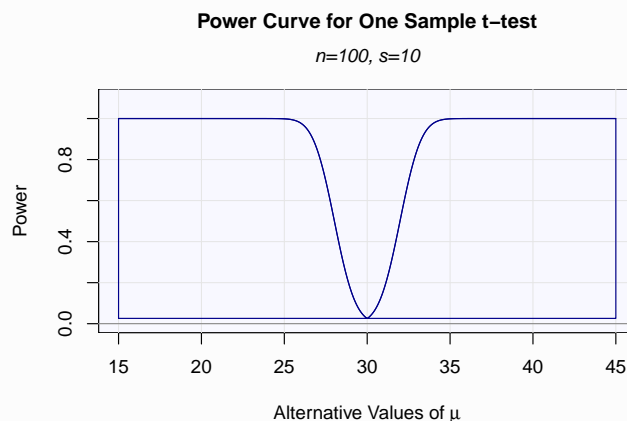
Effect of Sample Size on Power

The more data, the more ability to detect a real difference

- ▶ The **solution** to the problem of trying to detect a smaller change from the null hypothesized value is the usual solution in statistical analysis, **get more information, that is, more data**
- ▶ The **larger the sample size, n** , the smaller the standard error, s_m , so **the narrower the curves for the hypothesized and actual distributions of m**
- ▶ Narrower curves for the two distributions means **less overlap, which means fewer Type II Errors**
- ▶ **Key Concept:** As n gets larger, Power increases, β decreases
- ▶ To visualize, **compare** the Power Curve for $n = 100$, on the next figure, with the previous Power Curve for $n = 25$
- ▶ For the **larger sample size** of $n = 100$, obtain power of at least 0.8 for a real μ of 33 or larger, or for a real μ of 27 or smaller

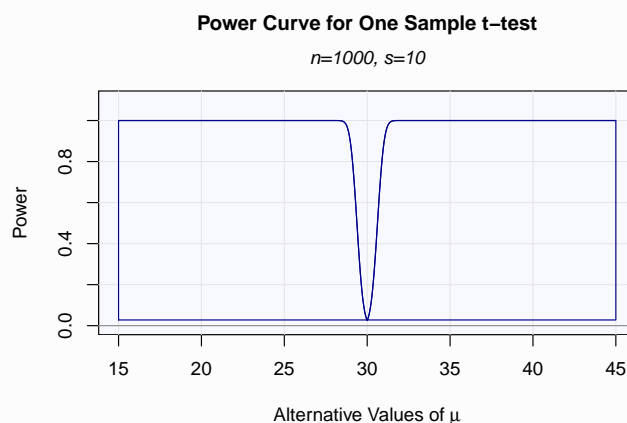
Power Curve for a Sample Size of 100

- ▶ Consider a **larger sample size**, $n = 100$, a power of 0.8 is obtained for a real $\mu \approx 33$ or $\mu \approx 27$, at $\mu_0 = 30$



Power Curve for a Sample Size of 1000

- ▶ Consider a **dramatically larger sample size**, $n = 1000$, a power of 0.8 is obtained for a real $\mu \approx 31$ or $\mu \approx 29$, at $\mu_0 = 30$

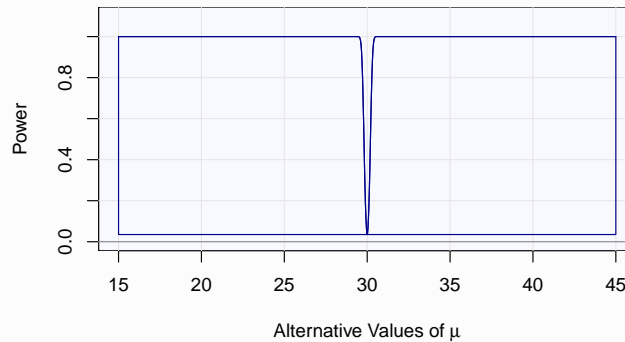


Power Curve for a Sample Size of 10000

- ▶ Consider a **huge sample size**, $n = 10000$, a power of 0.8 is obtained for a real $\mu \approx 29\frac{3}{4}$ or $\mu \approx 30\frac{1}{4}$, at $\mu_0 = 30$

Power Curve for One Sample t-test

$n=10000, s=10$



Obtain the Sample Size to Achieve the Needed Power

Set all but one, and then calculate the remaining value

- ▶ With the R `power.t.test` function, had previously set n , s and δ and then solved for power
- ▶ Now consider a related problem in which the **information entered into the `power.t.test` function is the desired power of 0.8 and also s and δ , and then solve for n**

```
> power.t.test(power=0.8, sd=10, delta=5,
               type="one.sample")
```

```
n = 33.36720
      :
```

- ▶ For the original sample with a size of $n = 25$, a **power of 0.67 is obtained** for a δ of 5
- ▶ For a δ of 5, to achieve a **power of 0.80**, a sample of size $n = 34$ is needed

5.4e

Practical Importance

Practical Importance

Distinguish the meaningful from the trivial

- ▶ **Practical importance:** The importance of the extent or size of a deviation from the true value of the population mean, μ , from the null hypothesized value
- ▶ **Minimal mean difference (mmd):** Smallest deviation from the null hypothesized value that is of practical importance
- ▶ For a hypothesized value of 30, is a difference from
 - 30 to 32 sufficiently large to achieve practical importance?
 - 30 to 30.01 too small to be of practical importance?
- ▶ **Key Concept:** The size of mmd follows from the return on investment given implementation of the policy that resulted in the change of the observed magnitude
- ▶ Ex: Determine the size of the needed improvement following the re-engineering of a process to justify its cost

Managerial vs Statistical Decisions

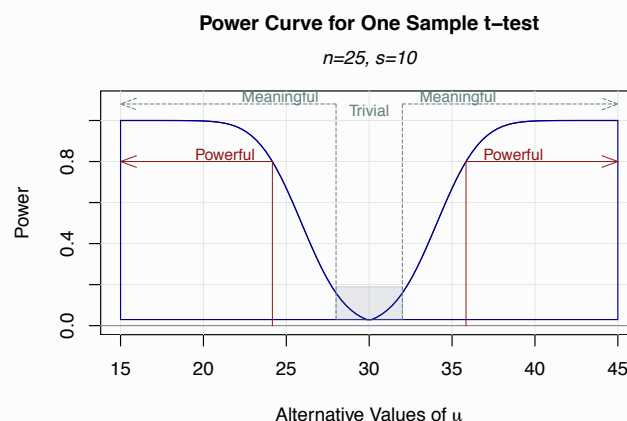
Two levels of decisions

- ▶ **Key Concept:** The ultimate purpose of statistical analysis for the manager is to facilitate decision making
- ▶ Distinguish between the ...
 - **Statistical decision:** A difference from the null detected?
 - **Managerial decision:** Follows from a meaningful difference
- ▶ The statistical decision may corroborate the managerial decision, but not necessarily
 - **Statistically significant but meaningless:** A trivial difference is rendered statistically significant by a large sample that yields an extremely powerful test
 - **Statistically insignificant but meaningful:** A perhaps small but meaningful difference is not detected by a statistical test with insufficient power → *the possibility examined next*

Power Curve with mmd=2

```
> ttestPower(n=25, s=10, mu=30, mmd=2)
```

mmd option: Range of meaningful changes from μ_0



Analysis of Power Curve with Meaningful Changes

What changes are detected and which are meaningful?

- ▶ Remember, the power analysis is only of interest if the null is not rejected, a result *not* statistically significant
- ▶ To include the minimal mean difference in the analysis of the power curve, add the `mmd` option to the `lessR ttestPower` function, which then annotates the power curve accordingly
- ▶ The resulting power curve contains two different annotations that delineate two sets of values of μ_{ALT} , those that are ...
 - **Powerful:** Values likely to be detected, with power > 0.8
 - **Meaningful:** Values sufficiently far from μ_0
- ▶ The purpose of both annotations is to compare meaningfulness with power

Analysis of Power Curve with Meaningful Changes

What changes are likely detected and which are meaningful?

- ▶ Text output of previous `ttestPower` analysis includes

```
Given n1, n2 and s, Power for mmd of 2 is 0.159
Warning: Meaningful differences from 32 to 35.84
         have Power < 0.8
```

- ▶ **Key Concept:** Failure to detect a difference from μ_0 does not imply there is no meaningful difference if some meaningful differences have low power
- ▶ If a result is *not* significant, then either
 - There is no difference from the null hypothesis
 - Or, a meaningful difference exists but was not detected
- ▶ To help distinguish between these alternatives, discover if if all meaningful values of μ_{ALT} would have been likely detected with the test

Follow-up Analysis to the Hypothesis Test

If the hypothesis test *fails* to reject the null, then ...

- ▶ Look at the meaningful alternatives, particularly the smallest
- ▶ Do a power analysis against these meaningful alternatives ...
 - If power is low, then maybe a real, meaningful effect exists despite the failure to achieve significance
 - If power is high, then *not* finding a difference from the null means “no meaningful effect likely exists”

If the hypothesis test rejects the null, then ...

- ▶ Conclude that a difference from the null value has been detected
- ▶ Obtain a confidence interval to estimate the likely value of the true population value μ to understand the magnitude of the effect

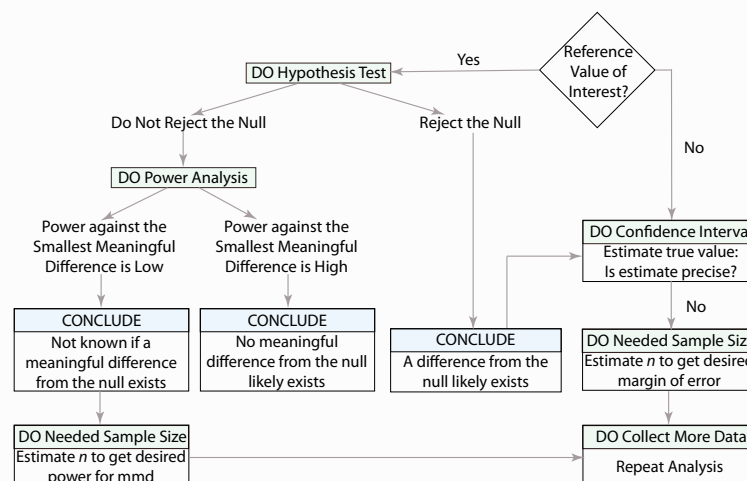
5.4f Comprehensive Strategy for Statistical Inference

Strategy for Managerial Decisions from Stat Inference

An integrated strategy

- ▶ Statistical inference is always in pursuit of unknown population values, such as the population mean, μ
- ▶ Two complementary forms of statistical inference: Confidence interval and hypothesis test, and related techniques such as power analysis
- ▶ Need an integrated strategy of statistical inference that provides a guide to form conclusions about μ or other population values of interest that involves
 - confidence interval
 - hypothesis test
 - power analysis
 - practical importance

Strategy for Managerial Decisions from Stat Inference



5.4g Application

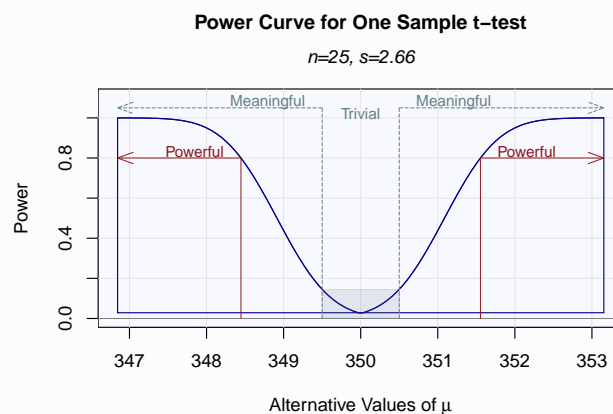
Follow-up Analysis from Hypothesis Test with No Rejection

Consider the previous application of filling cereal boxes

- ▶ A **previous application** provided a hypothesis test of the average weight of packaged cereal to be 350g
- ▶ Traditional Notation: $n = 25$ $m = 350.86$ $s = 2.66$
- ▶ **Statistical Decision:** $p\text{-value} = 0.119 > \alpha = .05$
so **do not reject the null hypothesized value of 350g**, the sample mean of 350.86 is close to 350
- ▶ **No difference in average weight of cereal from 350g detected**
- ▶ **Failing to reject the null hypothesis**, the question becomes:
What is the power against meaningful alternatives that may exist but were not detected?
- ▶ Suppose management decides that **any change from the target of 350g is not meaningful if it is 0.5g or less**
- ▶ That is, $mmd = 0.5$

Power Curve with $mmd = .5$

```
> ttestPower(n=25, s=2.66, mu=350, mmd=.5)
```



Analysis of Power Curve with Meaningful Changes

What changes are detected and which are meaningful?

- ▶ $m = 350.86$, but not so much larger than $\mu_0 = 350g$ to reject μ_0 as unreasonable
- ▶ Text output of previous `ttestPower` analysis includes

```
Given n1, n2 and s, Power for mmd of 0.5 is 0.145
Warning: Meaningful differences from 350.5 to 351.554
         have Power < 0.8
```

- ▶ So maybe a meaningful difference exists from the null value of 350g that this hypothesis test failed to detect
- ▶ If a more definitive conclusion is desired, then the only alternative is to gather more data, of which `ttestPower` provides the relevant information

```
Needed n to achieve power=0.8 for mmd of 0.5: n=225
```

Managerial Summary

What is important from this analysis for the manager?

- ▶ Deviations from the target weight of cereal in the cereal box of 350g in either direction need to be corrected
- ▶ Deviations less than 0.5g are deemed to small to provide any reliable or meaningful corrected action that includes shutting down the production line to make the adjustment
- ▶ So management wishes to detect any deviation of the average weight from the current production process that is smaller than 349.5g and larger than 350.5g
- ▶ With an initial sample size of $n = 25$, the null hypothesis of a population mean of 350g was not rejected

Managerial Conclusion

What is important from this analysis for the manager?

- ▶ The failure to reject lead to an analysis of power to determine if the detection of some meaningful differences from the null were unlikely
- ▶ The primary conclusions are that
 - Differences less than 351.55g have low power
 - The minimal value of interest larger than the target of 350g is 350.5g, which has a power of only 0.145
- ▶ To pursue this analysis further would require a considerable increase of sample size from $n = 25$ to $n = 225$ to achieve high power against against the value of $\mu_{ALT} = 350.5g$

Index Subtract 2 from each listed value to get the Slide #

► The End