

Chapter 5

Hypothesis Test of the Mean

Section 5.4

Power Analysis

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- Power Analysis and a Meaningful Effect
- Managerial vs Statistical Decisions
- Type I and Type II Errors
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- The Concept of Power
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 - Needed Sample Size for Sufficient Power
- Practical Importance
- Comprehensive Strategy for Statistical Inference
- Application

5.4a

Managerial vs Statistical Decisions

Managerial Decision versus Statistical Decision

Purpose of statistical analysis is to guide managerial decisions

- ▶ **Managerial decision:** Implement, or not, a specific action
- ▶ **Key Concept:** A technical decision regarding statistical inference provides information about reality that guides a subsequent managerial decision
- ▶ Statistical inference, confidence interval or hypothesis test, often focuses on a specific value in terms of the null hypothesis
- ▶ **Statistical decision** regarding a mean: Does a difference exist from the hypothesized value?
- ▶ The statistical decision is to either reject or not reject this reference value of interest, $\mu = \mu_0$
- ▶ **Each outcome** of the statistical decision, reject or not, may lead to a different managerial decision

Statistical Decision Guides the Managerial Decision

The statistical analysis informs the decision maker

- ▶ Consider the goal to assess a potential improved process with a trial version, implemented on a small scale, which management is considering to implement company wide
- ▶ To assess, compare a sample mean from the new process against the current mean, which is the null hypothesis
- ▶ One outcome is that the statistical analysis detects a difference from the null in the desired direction
 - Statistical decision → Reject the null hypothesis, so ...
 - typical Managerial decision → Implement the change
- ▶ Another outcome is that the statistical analysis does not detect a difference from the null
 - Statistical decision → Do not reject the null, so ...
 - typical Managerial decision → Retain the current process

Slippage Between Statistical and Managerial Decisions

Issue: These two decisions are not so tightly coupled

- ▶ However, this relation between statistical and managerial decisions must be qualified because the statistical and managerial decisions are distinct concepts
- ▶ **Key Concept:** The managerial decision does not necessarily follow directly from the statistical decision
- ▶ The statistical decision guides the managerial decision, but the statistical decision does not rigidly imply what action the manager should take
- ▶ In terms of deciding to retain or change to a new process
 - Rejection of the null hypothesis does not necessarily imply that the new process should be implemented
 - Failure to reject the null hypothesis does not necessarily imply that the current process should be retained
- ▶ This potential slippage is explored next

5.4b

Type I and Type II Errors

Binary Decisions

Definition and Examples of Binary Decisions

Choose one alternative or the other

- ▶ **Binary decision:** Choose between two alternatives
- ▶ Professional and personal life presents a series of binary decisions, from the trivial to the profound
 - Cross the road now, do not cross now
 - Hire the applicant, do not hire the applicant
 - Invest in increased manufacturing capacity, do not invest
 - Marry the person, do not marry
 - Reject the null hypothesis, do not reject

Consequences of Binary Decisions

Four possible outcomes from one binary decision

- ▶ Each binary decision provides two ways to be right (opportunity) and two ways to be wrong (error)
- ▶ Each outcome implies different consequences
 - 😊 Correct: Cross road now and do not get run over
 - 😊 Correct: Do not cross road now and watch the car go by
 - 😢 Error: Do not cross now and no car goes by – consequence is to lose a little bit of time
 - 😢 Error: Cross now and get run over – consequence is death or severe injury
- ▶ These two types of errors have names, rather unimaginatively called **Type I** and **Type II** errors

Four Possible Outcomes for an Hypothesis Test

Four possible outcomes from one binary decision

- ▶ The manager's decision regards an unknown reality, which at some later time reveals itself
- ▶ **Statistical Decision Rule** for analysis of the mean: If m is in the rejection region, decide H_0 is false, otherwise the data is consistent with H_0
 - 😊 Correct: Decide the null is true, and it really is
 - 😊 Correct: Decide a false null is false, a real difference detected
 - 😢 Type I Error, False Positive: An unusually deviant event leads to decide the null is false, but it really is true
 - 😢 Type II Error, False Negative: Decide the null is true, but fail to detect a difference that actually exists

Four Possible Outcomes for an Hypothesis Test

		Decision	
		Do Not Reject H_0	Reject H_0
Reality	H_0 is true	CORRECT: "accept" a true null	TYPE I ERROR: reject a true null
	H_0 is false	TYPE II ERROR: accept a false null	CORRECT: reject a false null

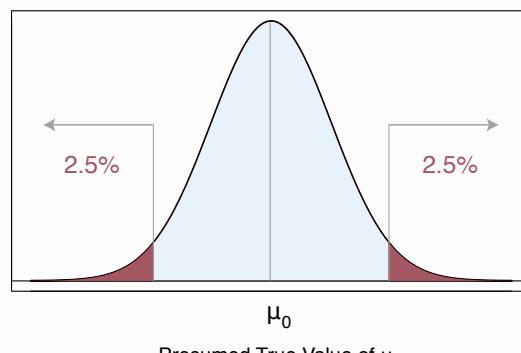
Type I Error

Null Hypothesis is True: Probability of a Type I Error

Consider the null hypothesis to be true

- Distribution of sample mean, m , centered over hypothesized value, μ_0 , assumed true *for purposes of the test*
- Presume the hypothesized value, μ_0 , *really is true*, not just for purposes of the test
- IF μ_0 is true, then ...
 - The applicable error is a Type I error, rejecting a true μ_0
 - A Type I Error occurs when m falls in the rejection region
 - What is the (conditional) probability that m falls in the rejection region given that the null hypothesis is true?

Null Hypothesis is True: Probability of a Type I Error



- Probability of a Type I Error for any one sample is not computed, but instead set at the probability of the complete rejection region, α , usually $\alpha = .05$ as shown here

Type II Error

More of the What-If Game

Assume $\mu = \mu_0$, but what if this assumption is false?

- ▶ To understand and estimate the value of the unknown value of μ , **probe the consequences of assuming different values of μ**
- ▶ The basis of analyzing Type II error is that **the assumed null hypothesis, $\mu = \mu_0$ is wrong**
- ▶ **Alternative value** of the mean: To presume $\mu \neq \mu_0$ means that some alternative value of μ , μ_{ALT} , is true, $\mu = \mu_{ALT}$
- ▶ **Type II Error Analysis:**
 - First, **set up the hypothesis test** by assuming **WHAT IF $\mu = \mu_0$?**
 - Second, **add a second assumption** beyond the first, **WHAT IF** we first assume $\mu = \mu_0$, but then ask actually **WHAT IF**, instead, $\mu = \mu_{ALT}$?
- ▶ Will we properly **reject** our first assumption of $\mu = \mu_0$ if, instead, the **truth is that $\mu = \mu_{ALT}$?**

Concepts and Notation

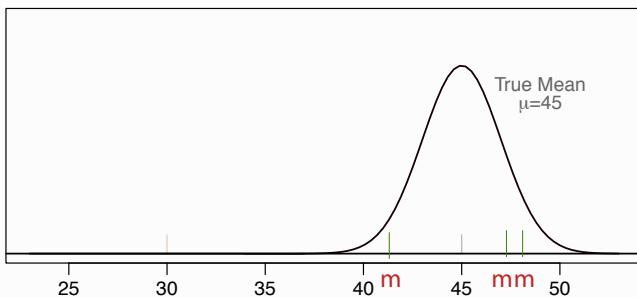
Many presumed values of μ when μ_0 is false

- ▶ Decision rule: **Fail to reject μ_0 if m is reasonably close to μ_0** , as assessed by the corresponding hypothesis test
- ▶ The **error here**, a Type II error, is to “accept” $\mu = \mu_0$ when some alternative value is actually the true mean, $\mu = \mu_{ALT}$
- ▶ The **probability of a Type II error** is defined in terms of one **corresponding specific value of μ_{ALT}** relative to the specified value of μ_0
- ▶ There are **many probabilities of a Type II error**, each probability calculated relative to a specific value of the presumed true mean, $\mu = \mu_{ALT}$
- ▶ There are **many possibilities for μ_{ALT}** , which each can be **systematically investigated**, as shown next, beginning with a specific example where $\mu_{ALT} = 45$

Probability of a Type II Error

Presume true mean is actually $\mu_{ALT} = 45$

- ▶ Hypothetical distribution of m is centered over the presumed true mean of 45, with most values between 40 and 50
- ▶ For example, if a sample were taken, values might be obtained such as $m=48.8$, $m=47.1$ or $m=41.5$



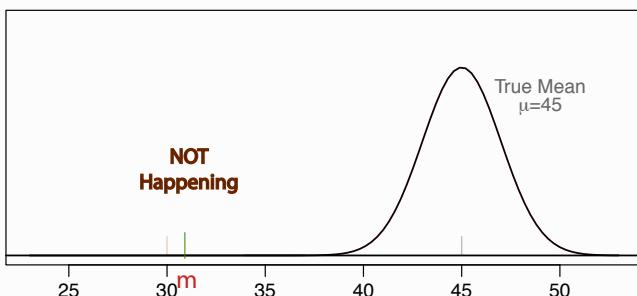
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Probability of a Type II Error

Consider an extremely deviant sample mean, $m = 31.0$

- ▶ A value of $m = 31.0$ would be highly unlikely to ever occur for this distribution with $\mu_{ALT} = 45$
- ▶ Although theoretically possible, a value of m this far below the mean of 45 would not happen in practice



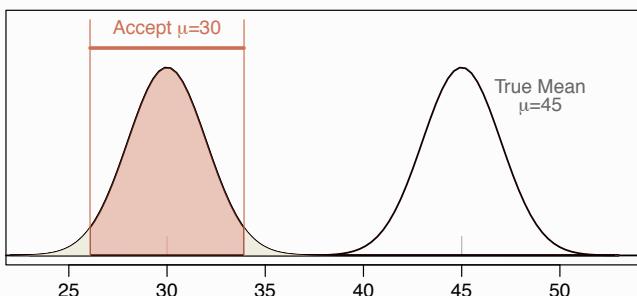
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Probability of a Type II Error

Hypothesized mean is $\mu_0 = 30$

- ▶ Now consider a hypothesis test with μ_0 set at 30, which, in this scenario is *false* as the true mean is presumed to be 45
- ▶ Around the value of the hypothesized mean of 30 is the "acceptance" region of the hypothesis test



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Probability of a Type II Error

Presume true mean is actually $\mu_{ALT} = 45$

- ▶ $\mu = 30$ is false in this scenario, so a Type II error occurs when m falls in the “acceptance” region, “accepting” the false null
- ▶ But m will virtually never fall in the “acceptance” region
- ▶ The false $\mu_0 = 30$ would almost certainly be correctly rejected for any one sample because the sample mean, m , would be much higher than the hypothesized mean, μ_0
- ▶ The value of the one observed m is not known until the data are collected, but whatever value it assumes, a difference from the null value will be properly detected in this situation

Basis of a Type II Error

Focus on a specific alternative to μ

- ▶ The hypothesis test “accepts” the null hypothesis when m falls in the “acceptance” region, close to μ_0
- ▶ If μ_{ALT} is the true mean, then the sample mean m is sampled from the distribution centered about μ_{ALT}
- ▶ **Key Concept:** If μ_{ALT} is the true mean, m can still fall within the “acceptance” region defined by the null hypothesis
- ▶ A Type II Error occurs when the null is false, but m is in the “acceptance” region:
The difference from the null value μ_0 to the presumed reality μ_{ALT} exists, but the hypothesis test does not detect the difference
- ▶ What is the probability that a value of m is in the “acceptance” region given the value of μ_{ALT} ?

5.4c The Concept of Power

Power Defined and Computed

Probability of the Correct Choice: Power

Focus on a specific alternative to the hypothesized value of μ

- ▶ Typically prefer to state the probability of making the *correct* choice, of rejecting the null hypothesis when it is false
- ▶ **Power** of an Hypothesis Test: Probability of *correctly* detecting a real difference from a false null hypothesis
- ▶ When the null is false, so that μ is different from μ_0 ,
 - Either make the right choice, reject the false null, or
 - Make the wrong choice, do not reject the false null
- ▶ **Notation:** Probability of a Type II Error is β
- ▶ The probabilities of making the correct or incorrect choice, here for $\mu \neq \mu_0$, sum to 1,

$$\text{Power} + \beta = 1$$

or

$$\text{Power} = 1 - \beta$$

Calculate Power and Probability of a Type II Error

Use R as an electronic table

- ▶ To use R to calculate Power is to use R as an electronic table of probabilities – no data file, no data analysis
- ▶ Provide a specific sample size, n , and standard deviation, s , such as from an initial data analysis
- ▶ To calculate Power and β , the actual values of the alternative and hypothesized values of μ per se are not of interest
- ▶ Rather, the focus is on the *difference* or “delta” between the hypothesized value of μ and the specific proposed alternative

$$\text{delta} = \mu_{ALT} - \mu_0$$

- ▶ For the same n and s , the power of detecting a real μ of 35 from a hypothesized value of 30 is *exactly the same* as detecting a real μ of 105 from a hypothesized value of 100

R Input: Power for a one-sample t -test

R function `power.t.test` calculates power

- ▶ Suppose for a specific data analysis:

$n = 25$, $s = 10$, $\mu_0 = 30$ and $\mu_{ALT} = 35$

- ▶ R input:

```
> power.t.test(n=25, sd=10, delta=5,  
    type="one.sample")
```

- ▶ Required values:

- `n`: sample size
- `sd`: standard deviation of data
- `delta`: change from μ_0 to the alternative value of μ

- ▶ Defaults:

- `alternative="two.sided"`, i.e., two-tailed test
- `type="two.sample"`, i.e., for a mean difference

R Output: Power for a one-sample t -test

One-sample t -test power analysis

```
> power.t.test(n=25, sd=10, delta=5,  
    type="one.sample")
```

One-sample t test power calculation

```
n = 25  
delta = 5  
sd = 10  
sig.level = 0.05  
power = 0.6697014  
alternative = two.sided
```

- ▶ For a sample of size 25 with a standard deviation of 10, and a hypothesized value of 30, the probability of correctly detecting a difference when $\mu = 35$ is 0.67, and so $\beta = 1 - 0.67 = 0.33$

Probability Distribution that Underlies a Type II Error

Technical Note

- ▶ The hypothesis test of the mean follows from the distribution of t_m based on the sample mean, assuming the null hypothesized value, μ_0 , is true
- ▶ The probability of a Type II Error follows from a non-central t -distribution, the distribution of the t -statistic, t_m , when μ_0 is assumed true but is actually false
- ▶ For pedagogical simplicity, the following illustrations are based on the sample mean, m , and the normal distribution of m instead of t_m and the non-central t -distribution
- ▶ The basic concepts remain the same, and numerical computations only suffer when the sample size, n , is small
- ▶ Fortunately, Type II Error calculations with R are based the correct non-central t -distribution, so actual applications of these probabilities from R are accurate for all values of n

Power: An Example Continued

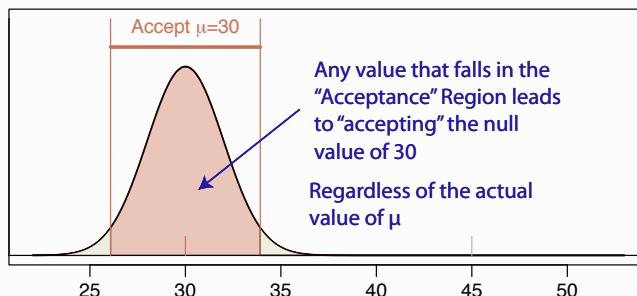
Illustrate Probability of Type II Error with Graphics

Consider a specific example

- ▶ Suppose the hypothesized value is $\mu_0 = 30$
- ▶ Collecting a sample of size $n = 25$ yielded a standard deviation of $s = 10$, for a standard error of the mean of $s_m = 2$
- ▶ Suppose the sample mean m was larger than 30, but close enough to $\mu_0 = 30$ that the null hypothesis was *not* rejected
- ▶ Although no difference from the null was detected, the analyst wonders if the test might have failed to detect a real difference with the true value of μ actually larger than 30
- ▶ What is the probability of detecting this difference if it actually exists?
- ▶ Power analysis provides the answer

Probability of a Type II Error

Null hypothesis is $\mu = 30$



- ▶ Here the null hypothesized value, μ_0 , is 30
- ▶ The hypothesis test sets up an "acceptance" region around 30
- ▶ Whatever the actual value of μ , a m that falls in the "acceptance" region leads to "accepting" the null

Probability of a Type II Error

Probability depends on the true value of μ

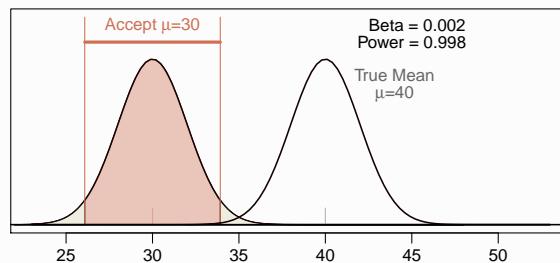
- ▶ So the computation of the probability of a Type II Error, β , depends on the true value of μ
- ▶ Presume the value of the null hypothesis, μ_0 , is false, so what is the alternative value of μ , μ_{ALT} to use for the probability calculation?
- ▶ We do not know the true value of μ , which is the whole point of statistical inference, to estimate μ
- ▶ In absence of the actual value of μ , explore what happens for a range of potential different alternative values of μ , from values far away from μ_0 to values that are close to μ_0
- ▶ What are Power and β ... IF the actual $\mu = \mu_{ALT}$ was equal to a value just one unit above or below μ_0 , or two units, or ...

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Probability of a Type II Error

Hypothesize 30; Presume true mean is actually $\mu_{ALT} = 40$



```
> power.t.test(n=25, sd=10, delta=10, type="one.sample")
```

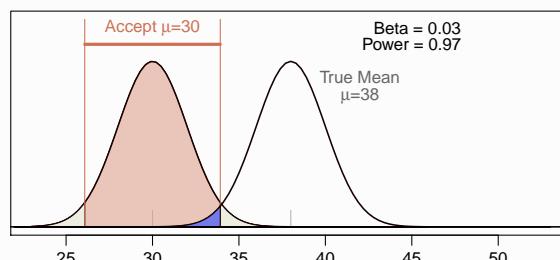
- ▶ For $\mu_{ALT} = 40$, m likely varies somewhere between 34 and 46
- ▶ Rarely a m may just cross the line into the "acceptance" region, resulting in a Type II Error

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Probability of a Type II Error

Hypothesize 30; Presume true mean is actually $\mu_{ALT} = 38$



```
> power.t.test(n=25, sd=10, delta=8, type="one.sample")
```

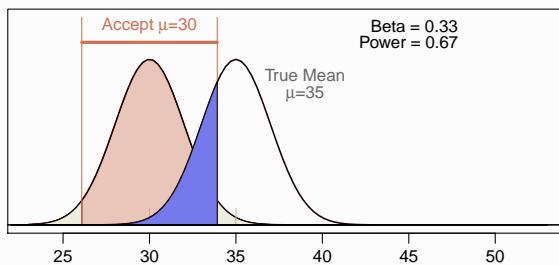
- ▶ For $\mu_{ALT} = 38$, m likely varies somewhere between 32 and 44
- ▶ Here m occasionally lands in the "acceptance" region, as a Type II Error has a probability of only 0.03 (blue area)

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Power Analysis: The Concept of Power 36

Probability of a Type II Error

Hypothesize 30; Presume true mean is actually $\mu_{ALT} = 35$

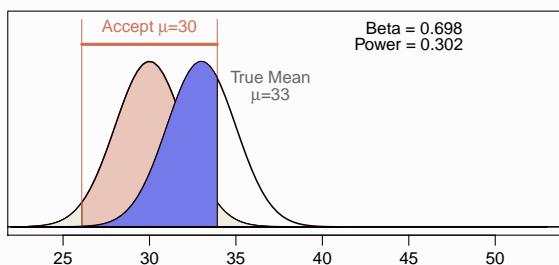


```
> power.t.test(n=25, sd=10, delta=5, type="one.sample")
```

- ▶ For $\mu_{ALT} = 35$, m likely varies somewhere between 29 and 41
- ▶ Here m lands in the “acceptance” region with reasonable frequency, as a Type II Error has probability of 0.330

Probability of a Type II Error

Hypothesize 30; Presume true mean is actually $\mu_{ALT} = 33$

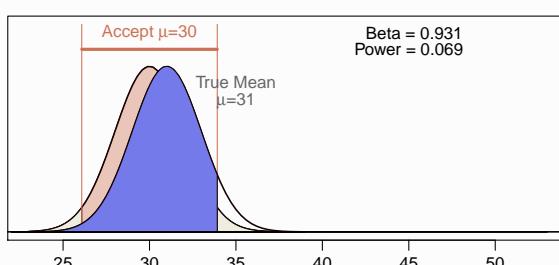


```
> power.t.test(n=25, sd=10, delta=3, type="one.sample")
```

- ▶ For $\mu_{ALT} = 33$, m likely varies somewhere between 27 and 39
- ▶ Now m lands in the “acceptance” region more often than not, as the probability of a Type II Error is 0.698

Probability of a Type II Error

Hypothesize 30; Presume true mean is actually $\mu = 31$



```
> power.t.test(n=25, sd=10, delta=1, type="one.sample")
```

- ▶ For $\mu_{ALT} = 31$, m likely varies somewhere between 25 and 37
- ▶ Now m lands in the “acceptance” region most of the time, with a probability of a Type II Error at 0.931

5.4d The Power Curve

Power Over a Range of Alternative Values of μ

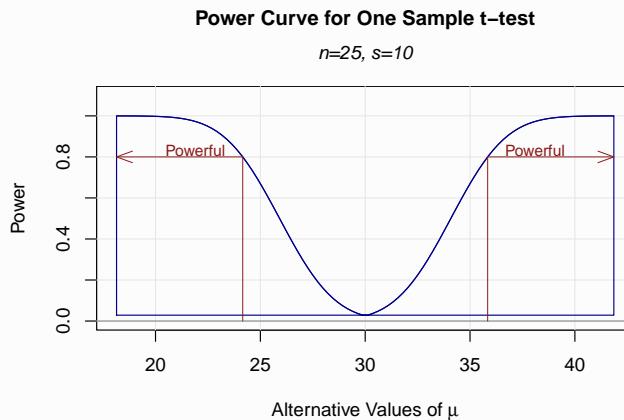
Simultaneously Consider Many Alternative Values of μ

Only one μ_0 , but many possible alternative values of μ

- ▶ Organize this information regarding power for multiple alternative values of μ into a single graph
- ▶ **Power curve:** Plot the power relative to a specified null value, μ_0 , across a range of alternative values of μ , μ_{ALT}
- ▶ By analyzing power over a range of possible values of μ , an interesting pattern emerges that can provide the analyst much information for the interpretation of a specific hypothesis test
 - Values of μ far from μ_0 are almost assuredly detected
 - Values of μ close to μ_0 are almost assuredly *not* detected
- ▶ Discover “far from” and “close to” for a specific analysis
- ▶ One tradition is to establish a minimum threshold of desirable power of 0.8 in general, which may be adjusted up or down for a specific situation

Power Curve for Previous Example with Sample Size of 25

- lessR `ttestPower` function provides the power curve
 - > `ttestPower(n=25, s=10, mu=30)`



Power Curve Interpretation

What ranges of μ have high and low power, respectively?

- Small changes more difficult to detect, so always have low power against values close to the hypothesized value
- Minimum desired power is usually set at 0.8, here not obtained until real $\mu > 36$ or $\mu < 24$
- Probability of detecting a real change from 30 is less than 50% for values of μ all the way up to 34 and down to 26
- So what if the true mean is $\mu = 33$, which management decides is a sufficiently large increase from $\mu_0 = 30$ to indicate a meaningful, interesting difference from 30?
- If $\mu = 33$, then such a value is too close to the hypothesized value of 30 to be detected with any reasonable probability in this analysis, as the power for this value is only 0.302
- The next section shows how to address this issue

Needed Sample Size for Sufficient Power

Effect of Sample Size on Power

The more data, the more ability to detect a real difference

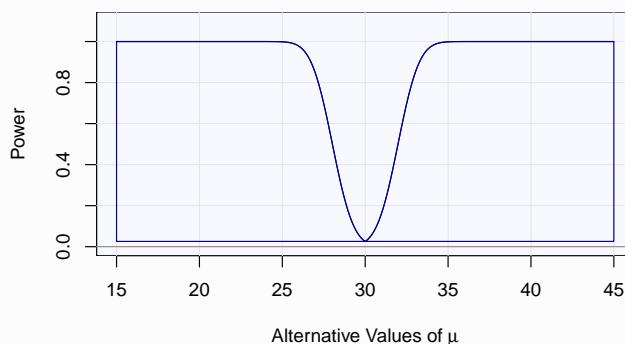
- ▶ The **solution** to the problem of trying to detect a smaller change from the null hypothesized value is the usual solution in statistical analysis, **get more information, that is, more data**
- ▶ **The larger the sample size, n , the smaller the standard error, s_m , so the narrower the curves for the hypothesized and actual distributions of m**
- ▶ Narrower curves for the two distributions means **less overlap, which means fewer Type II Errors**
- ▶ **Key Concept:** As n gets larger, Power increases, β decreases
- ▶ To visualize, **compare** the Power Curve for $n = 100$, on the next figure, with the previous Power Curve for $n = 25$
- ▶ For the **larger sample size** of $n = 100$, obtain power of at least 0.8 for a real μ of 33 or larger, or for a real μ of 27 or smaller

Power Curve for a Sample Size of 100

- ▶ Consider a **larger sample size, $n = 100$** , a power of 0.8 is obtained for a real $\mu \approx 33$ or $\mu \approx 27$, at $\mu_0 = 30$

Power Curve for One Sample t-test

$n=100, s=10$

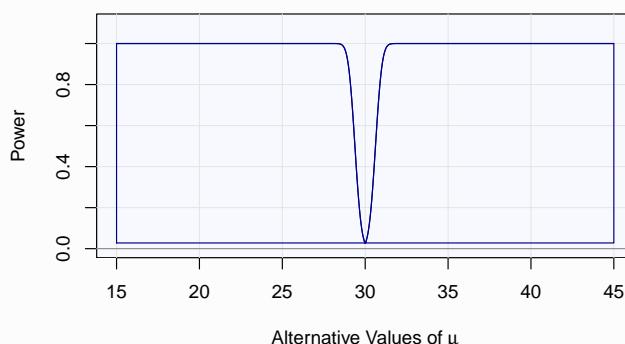


Power Curve for a Sample Size of 1000

- ▶ Consider a **dramatically larger sample size, $n = 1000$** , a power of 0.8 is obtained for a real $\mu \approx 31$ or $\mu \approx 29$, at $\mu_0 = 30$

Power Curve for One Sample t-test

$n=1000, s=10$

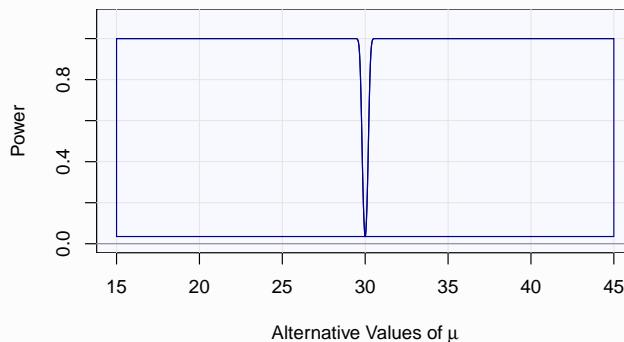


Power Curve for a Sample Size of 10000

- ▶ Consider a **huge sample size**, $n = 10000$, a power of 0.8 is obtained for a real $\mu \approx 29\frac{3}{4}$ or $\mu \approx 30\frac{1}{4}$, at $\mu_0 = 30$

Power Curve for One Sample t-test

$n=10000, s=10$



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Power Analysis: The Power Curve 49

Obtain the Sample Size to Achieve the Needed Power

Set all but one, and then calculate the remaining value

- ▶ With the R `power.t.test` function, had previously set n , s and δ and then solved for power
- ▶ Now consider a related problem in which the information entered into the `power.t.test` function is the desired power of 0.8 and also s and δ , and then solve for n

```
> power.t.test(power=0.8, sd=10, delta=5,  
type="one.sample")
```

n = 33.36720

:

- ▶ For the original sample with a size of $n = 25$, a power of 0.67 is obtained for a delta of 5
- ▶ For a delta of 5, to achieve a power of 0.80, a sample of size $n = 34$ is needed

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Power Analysis: The Power Curve 50

5.4e Practical Importance

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Power Analysis: Practical Importance 51

Practical Importance

Distinguish the meaningful from the trivial

- ▶ **Practical importance:** The importance of the extent or size of a deviation from the true value of the population mean, μ , from the null hypothesized value
- ▶ **Minimal mean difference (mmd):** Smallest deviation from the null hypothesized value that is of practical importance
- ▶ For a hypothesized value of 30, is a difference from
 - 30 to 32 sufficiently large to achieve practical importance?
 - 30 to 30.01 too small to be of practical importance?
- ▶ **Key Concept:** The size of mmd follows from the return on investment given implementation of the policy that resulted in the change of the observed magnitude
- ▶ Ex: Determine the size of the needed improvement following the re-engineering of a process to justify its cost

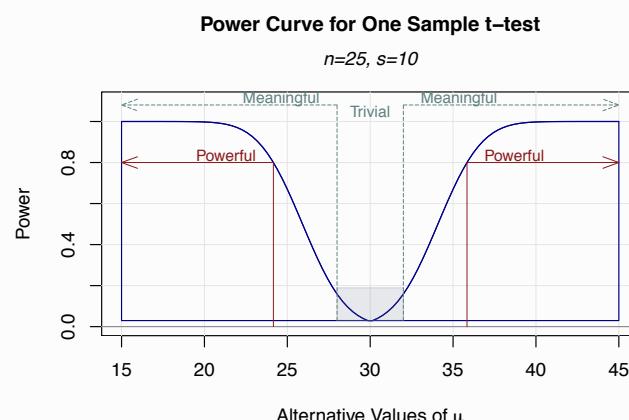
Managerial vs Statistical Decisions

Two levels of decisions

- ▶ **Key Concept:** The ultimate purpose of statistical analysis for the manager is to facilitate decision making
- ▶ Distinguish between the ...
 - Statistical decision: A difference from the null detected?
 - Managerial decision: Follows from a meaningful difference
- ▶ The statistical decision may corroborate the managerial decision, but not necessarily
 - **Statistically significant but meaningless:** A trivial difference is rendered statistically significant by a large sample that yields an extremely powerful test
 - **Statistically insignificant but meaningful:** A perhaps small but meaningful difference is not detected by a statistical test with insufficient power → *the possibility examined next*

Power Curve with mmd=2

```
> ttestPower(n=25, s=10, mu=30, mmd=2)  
mmd option: Range of meaningful changes from  $\mu_0$ 
```



Analysis of Power Curve with Meaningful Changes

What changes are detected and which are meaningful?

- ▶ Remember, the power analysis is only of interest if the null is not rejected, a result *not* statistically significant
- ▶ To include the minimal mean difference in the analysis of the power curve, add the `mmd` option to the `lessR ttestPower` function, which then annotates the power curve accordingly
- ▶ The resulting power curve contains two different annotations that delineate two sets of values of μ_{ALT} , those that are ...
 - Powerful: Values likely to be detected, with $\text{power} > 0.8$
 - Meaningful: Values sufficiently far from μ_0
- ▶ The purpose of both annotations is to compare meaningfulness with power

Analysis of Power Curve with Meaningful Changes

What changes are likely detected and which are meaningful?

- ▶ Text output of previous `ttestPower` analysis includes

```
Given n1, n2 and s, Power for mmd of 2 is 0.159
Warning: Meaningful differences from 32 to 35.84
          have Power < 0.8
```

- ▶ **Key Concept:** Failure to detect a difference from μ_0 does not imply there is no meaningful difference if some meaningful differences have low power
- ▶ If a result is *not* significant, then either
 - There is no difference from the null hypothesis
 - Or, a meaningful difference exists but was not detected
- ▶ To help distinguish between these alternatives, discover if if all meaningful values of μ_{ALT} would have been likely detected with the test

Follow-up Analysis to the Hypothesis Test

If the hypothesis test *fails* to reject the null, then ...

- ▶ Look at the meaningful alternatives, particularly the smallest
- ▶ Do a power analysis against these meaningful alternatives ...
 - If power is low, then maybe a real, meaningful effect exists despite the failure to achieve significance
 - If power is high, then *not* finding a difference from the null means "no meaningful effect likely exists"

If the hypothesis test *rejects* the null, then ...

- ▶ Conclude that a difference from the null value has been detected
- ▶ Obtain a confidence interval to estimate the likely value of the true population value μ to understand the magnitude of the effect

5.4f

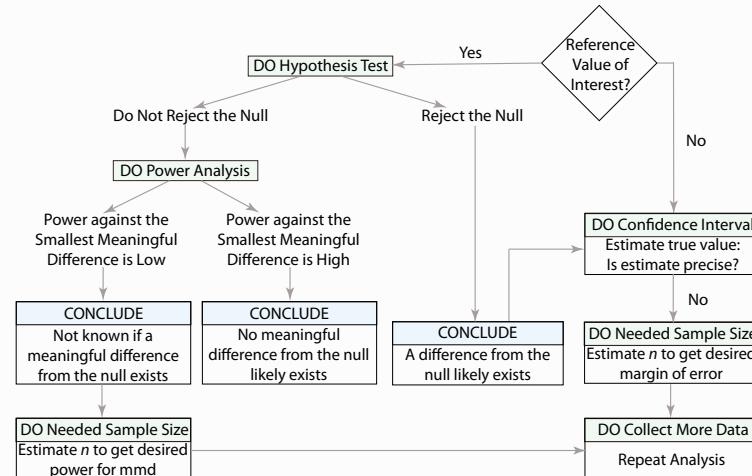
Comprehensive Strategy for Statistical Inference

Strategy for Managerial Decisions from Stat Inference

An integrated strategy

- ▶ Statistical inference is always in pursuit of unknown population values, such as the population mean, μ
- ▶ Two complementary forms of statistical inference: **Confidence interval** and **hypothesis test**, and related techniques such as power analysis
- ▶ Need an **integrated strategy of statistical inference** that provides a guide to form conclusions about μ or other population values of interest that involves
 - confidence interval
 - hypothesis test
 - power analysis
 - practical importance

Strategy for Managerial Decisions from Stat Inference



5.4g Application

Follow-up Analysis from Hypothesis Test with No Rejection

Consider the previous application of filling cereal boxes

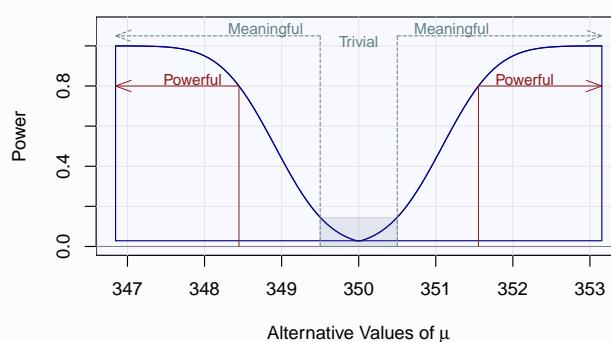
- ▶ A previous application provided a hypothesis test of the average weight of packaged cereal to be 350g
- ▶ Traditional Notation: $n = 25$ $m = 350.86$ $s = 2.66$
- ▶ Statistical Decision: $p\text{-value} = 0.119 > \alpha = .05$ so do not reject the null hypothesized value of 350g, the sample mean of 350.86 is close to 350
- ▶ No difference in average weight of cereal from 350g detected
- ▶ Failing to reject the null hypothesis, the question becomes: What is the power against meaningful alternatives that may exist but were not detected?
- ▶ Suppose management decides that any change from the target of 350g is not meaningful if it is 0.5g or less
- ▶ That is, $mmd=0.5$

Power Curve with $mmd=.5$

```
> ttestPower(n=25, s=2.66, mu=350, mmd=.5)
```

Power Curve for One Sample t-test

$n=25, s=2.66$



Analysis of Power Curve with Meaningful Changes

What changes are detected and which are meaningful?

- ▶ $m = 350.86$, but not so much larger than $\mu_0 = 350g$ to reject μ_0 as unreasonable
- ▶ Text output of previous `ttestPower` analysis includes

```
Given n1, n2 and s, Power for mmd of 0.5 is 0.145
Warning: Meaningful differences from 350.5 to 351.554
          have Power < 0.8
```

- ▶ So maybe a meaningful difference exists from the null value of 350g that this hypothesis test failed to detect
- ▶ If a more definitive conclusion is desired, then the only alternative is to gather more data, of which `ttestPower` provides the relevant information

```
Needed n to achieve power=0.8 for mmd of 0.5: n=225
```

Managerial Summary

What is important from this analysis for the manager?

- ▶ Deviations from the target weight of cereal in the cereal box of 350g in either direction need to be corrected
- ▶ Deviations less than 0.5g are deemed to small to provide any reliable or meaningful corrected action that includes shutting down the production line to make the adjustment
- ▶ So management wishes to detect any deviation of the average weight from the current production process that is smaller than 349.5g and larger than 350.5g
- ▶ With an initial sample size of $n = 25$, the null hypothesis of a population mean of 350g was not rejected

Managerial Conclusion

What is important from this analysis for the manager?

- ▶ The failure to reject lead to an analysis of power to determine if the detection of some meaningful differences from the null were unlikely
- ▶ The primary conclusions are that
 - Differences less than 351.55g have low power
 - The minimal value of interest larger than the target of 350g is 350.5g, which has a power of only 0.145
- ▶ To pursue this analysis further would require a considerable increase of sample size from $n = 25$ to $n = 225$ to achieve high power against the value of $\mu_{ALT} = 350.5g$

Index Subtract 2 from each listed value to get the Slide #

► The End