Chapter 5
Hypotheses Test of the Mean

Section 5.3
Confidence Intervals Related to Hypothesis Tests

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- Confidence Intervals Related to Hypothesis Tests
  - Relation to Confidence Interval

5.3
Confidence Interval Related to the Hypothesis Test

Range of Sampling Variation and Statistical Inference
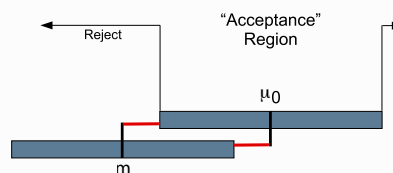
Confidence intervals and hypothesis tests

- ▶ Statistical inference of the mean follows from the range of sampling variation of m, in terms of the corresponding t-distribution
- ▶ Usually use the 95% range of variation, defined by $t_{.025} \approx 2$
- ▶ Center this range of sampling variation over ...
  - ○ m: yields the 95% confidence interval
  - ○ μ_0: yields "acceptance" region of $\alpha = 5\%$ hypothesis test
- ▶ Not much of a surprise to know that correctly implemented the two forms of statistical inference, a confidence interval and hypothesis test, always yield consistent results
- ▶ To relate the two forms of statistical inference, begin with the definition of the confidence interval of the mean:

Range of plausible values for the population mean

Consistency of Confidence Interval and Hypothesis Test

Reject null hypothesized value, considered implausible
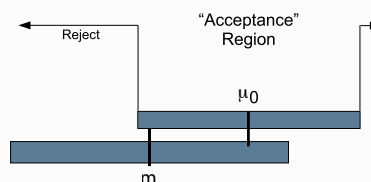
- ▶ Consider a confidence interval centered about m

- ▶ The hypothesized value μ_0 is *outside* of the confidence interval
  - ○ μ_0 is implausible according to the confidence interval
  - ○ m necessarily lies in the rejection region of the hypothesis test, so μ_0 is considered implausible

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Consistency of Confidence Interval and Hypothesis Test

Do not reject null hypothesized value

- ▶ Consider a confidence interval centered about m

- ▶ The hypothesized value μ_0 is *inside* the confidence interval
  - ○ μ_0 is plausible according to the confidence interval
  - ○ m necessarily lies in the "acceptance" region of the corresponding hypothesis test, and so renders μ_0 plausible

Generality of the Confidence Interval

One confidence interval vs many, many hypothesis tests

- ▶ All values in the confidence interval are plausible values of the population mean of Y, μ
- ▶ All values outside the confidence interval are implausible, with such values rejected by the corresponding hypothesis test
- ▶ One confidence interval provides the same qualitative result as many, many hypothesis tests
- ▶ The primary advantage of a hypothesis test over a confidence interval is to provide a p-value for a specific μ_0, instead of just concluding that the value is, or is not, in the confidence interval

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Confidence Interval Compared to the Hypothesis Test

Results always in agreement for any specified μ_0

- ▶ **Significant Result**: Reject the null hypothesis
  - ○ The hypothesized value of μ_0 is rejected because the p-value $< .05$, which renders μ_0 implausible
  - ○ The value of μ_0 is *outside*, below or above, the 95% confidence interval, so the range of plausible values for the true average, μ, are either all above or all below μ_0
  - ○ Conclude: Value of true average mean of Y, μ, is not μ_0
- ▶ **Not significant Result**: Null hypothesis *not* rejected
  - ○ The hypothesized value of μ_0 is not rejected because the p-value $> .05$, which renders μ_0 a plausible value
  - ○ The value of μ_0 is *inside* the confidence interval, which renders μ_0 a plausible value
  - ○ Conclude: No difference of the value of μ detected from μ_0

- ▶ The End