Multiple Regression

Template for Analysis

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Purpose of Regression Analysis

The two primary purposes of regression analysis:

1. Understand the relations among the variables, especially as related to the target variable.
[via slope coefficients]

2. Predict the value of the target from the values of the predictor variables.
[from computing fitted values on new data, the predictions]

To accomplish these goals, estimate a model from the data to get the equation that provides the slope coefficients and the full equation (model) for computing predicted values.

Problem Definition

Three advertising campaigns across three different media — TV, Radio, and Newspaper — promoted a variety of products in 200 different markets.

Spending for each set of media in each market is recorded along with Sales

Purpose of the analysis:
How is spending in these markets related to Sales of the company's products?

lessR Input: `reg(Sales ~ TV + Radio + Newspaper)`

The Data

TV	Radio	Newspaper	Sales
230.1	37.8	69.2	22.1
44.5	39.3	45.1	10.4
17.2	45.9	69.3	9.3
151.5	41.3	58.5	18.5
180.8	10.8	58.4	12.9
8.7	48.9	75	7.2
57.5	32.8	23.5	11.8
120.2	19.6	11.6	13.2
8.6	2.1	1	4.8
199.8	2.6	21.2	10.6

plus 190 more rows

Sales: in 1000 units
   e.g., first row has recorded Sales of 22,100 units

Ad Budgets: in $1000 USD
   e.g., first row has ad budgets of $230,100 and $37,800, and $69,200, respectively

Correlation Matrix

Develop Intuition for the Model

Relevance: Do the predictor variables relate to the target (response) variable?

[Sec 5.2]

```
Correlation Matrix

            Sales     TV    Radio   Newspaper
  Sales     1.00    0.78    0.58       0.23
     TV     0.78    1.00    0.05       0.06
  Radio     0.58    0.05    1.00       0.35
Newspaper   0.23    0.06    0.35       1.00
```

TV ads correlates much with Sales, Radio ads also correlate well, though to a lesser extent

Newspaper ad spending does not correlate well with Sales

Uniqueness: Is collinearity a problem?

```
Correlation Matrix

               Sales     TV    Radio   Newspaper
     Sales   1.00    0.78     0.58        0.23
        TV   0.78    1.00     0.05        0.06
     Radio   0.58    0.05     1.00        0.35
 Newspaper   0.23    0.06     0.35        1.00
```

Newspaper ads has a slight correlation with Radio of 0.35

The two most relevant predictor variables, TV ads and Radio ads, do not correlate with each other, only 0.05

Conclude: Not much of a collinearity problem at least in terms of pairs of variables

Model Selection: Given the correlations, what is the most likely candidate for the final model?

```
Correlation Matrix

                Sales     TV    Radio   Newspaper
     Sales     1.00    0.78    0.58      0.23
        TV     0.78    1.00    0.05      0.06
     Radio     0.58    0.05    1.00      0.35
 Newspaper     0.23    0.06    0.35      1.00
```

The model will clearly include TV ads because of its high correlation with Sales

Given the low correlation between TV ad spending and Radio ad spending, and the moderately high correlation of Radio ads with Sales, Radio ads adds new, useful information to the model beyond TV ads, so it also will likely be included in the model

Newspaper ad spending is not relevant

Write the estimated regression model

[Sec 2.3]

```
Estimated Model for Sales

              Estimate   Std Err   t-value   p-value   Lower 95%   Upper 95%
(Intercept)      2.939     0.312     9.422     0.000       2.324       3.554
         TV      0.046     0.001    32.809     0.000       0.043       0.049
      Radio      0.189     0.009    21.893     0.000       0.172       0.206
  Newspaper     -0.001     0.006    -0.177     0.860      -0.013       0.011
```

Sales_fitted = 2.939 + 0.046(TV) + 0.189(Radio) - 0.001(Newspaper)

Show the fitted value and its calculation

Use the advertising campaign of TV=232, Radio=8.6, Newspaper=8.7

[Sec 1.2.2, 2.4.1]

	Estimate	Std Err	t-value	p-value	Lower 95%	Upper 95%
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	0.046	0.001	32.809	0.000	0.043	0.049
Radio	0.189	0.009	21.893	0.000	0.172	0.206
Newspaper	-0.001	0.006	-0.177	0.860	-0.013	0.011

	TV	Radio	Newspaper	Sales	fitted	resid	rstdnt	dffits	cooks
13	23.800	35.100	65.900	9.200	10.577	-1.377	-0.829	-0.147	0.005
53	216.400	41.700	39.600	22.600	20.663	1.937	1.159	0.146	0.005
200	**232.100**	**8.600**	**8.700**	13.400	**15.173**	-1.773	-1.062	-0.144	0.005
130	59.600	12.000	43.100	9.700	7.884	1.816	1.087	0.144	0.005

Sales_fitted = 2.939 + 0.046(TV) + 0.189(Radio) - 0.001(Newspaper)

= 2.939 + 0.046(232) + 0.189(8.6) - 0.001(8.7)

= 15.173

Show the associated residual, its calculation, interpret

Use the advertising campaign of TV=232, Radio=8.6, Newspaper=8.7

[Sec 2.4.1]

```
    RESIDUALS AND INFLUENCE

              TV  Radio Newspaper  Sales fitted  resid rstdnt dffits cooks
    35   95.700  1.400     7.400   9.500  7.575  1.925  1.154  0.160 0.006
    10  199.800  2.600    21.200  10.600 12.551 -1.951 -1.169 -0.154 0.006
     1  230.100 37.800    69.200  22.100 20.524  1.576  0.947  0.152 0.006
    13   23.800 35.100    65.900   9.200 10.577 -1.377 -0.829 -0.147 0.005
    53  216.400 41.700    39.600  22.600 20.663  1.937  1.159  0.146 0.005
   200  232.100  8.600     8.700  13.400 15.173 -1.773 -1.062 -0.144 0.005
   130   59.600 12.000    43.100   9.700  7.884  1.816  1.087  0.144 0.005
    34  265.600 20.000     0.300  17.400 18.864 -1.464 -0.880 -0.143 0.005
```

e_{200} = Sales - Sales_fitted = 13.400 – 15.173 = –1.773

The actual Sales for Sample 200 fell 1,773 units short of its fitted value

Specify and interpret the sample slope coefficient

[Secs 2.1, 2.3, 3.3.2]   Apply to the 1st predictor variable

```
Estimated Model for Sales

             Estimate    Std Err   t-value   p-value    Lower 95%   Upper 95%
(Intercept)    2.939       0.312     9.422     0.000       2.324       3.554
         TV    0.046       0.001    32.809     0.000       0.043       0.049
      Radio    0.189       0.009    21.893     0.000       0.172       0.206
  Newspaper   -0.001       0.006    -0.177     0.860      -0.013       0.011
```

$b_{TV} = 0.046$

In this *particular* sample, on average, for each increase of $1000 of TV spending, Sales increase by 46 units-- *holding the values of the remaining variables constant*

Hypothesis Test

Is a Slope Coefficient Different from Zero?

Specify the null hypothesis and its alternative for the hypothesis test of the slope coefficient

[Sec 3.2.1]   Apply to the 1st predictor variable

```
Estimated Model for Sales

             Estimate    Std Err   t-value   p-value   Lower 95%   Upper 95%
(Intercept)     2.939      0.312     9.422     0.000       2.324       3.554
         TV     0.046      0.001    32.809     0.000       0.043       0.049
      Radio     0.189      0.009    21.893     0.000       0.172       0.206
  Newspaper    -0.001      0.006    -0.177     0.860      -0.013       0.011
```

Null Hypothesis: Partial slope coefficient, that is, the values of Radio and Newspaper held constant, of TV on Sales is 0. Or, in symbols, $\beta_{TV} = 0$

Alternative to the Null: Holding the values of Radio and Newspaper constant, there is a relationship between TV advertising and Sales, $\beta_{TV} \neq 0$

Specify the calculation of the *t*-statistic by applying the relevant numbers from this specific analysis

Apply to the 1st predictor variable

```
Estimated Model for Sales

              Estimate    Std Err   t-value   p-value    Lower 95%   Upper 95%
(Intercept)      2.939      0.312     9.422     0.000        2.324       3.554
         TV      0.046      0.001    32.809     0.000        0.043       0.049
      Radio      0.189      0.009    21.893     0.000        0.172       0.206
  Newspaper     -0.001      0.006    -0.177     0.860       -0.013       0.011
```

How close is the estimate to what is hypothesized?

The *t*-statistic shows how many standard errors the sample estimate is from the null, hypothesize value, here 0

$t = (b_{TV} - 0) / 0.001 = 0.046 / 0.001 = 32.809$

Adapt the definition of the *p*-value to this analysis

Apply to the 1st predictor variable

```
Estimated Model for Sales

            Estimate    Std Err   t-value   p-value    Lower 95%    Upper 95%
(Intercept)    2.939      0.312     9.422     0.000        2.324        3.554
        TV     0.046      0.001    32.809     0.000        0.043        0.049
     Radio     0.189      0.009    21.893     0.000        0.172        0.206
 Newspaper    -0.001      0.006    -0.177     0.860       -0.013        0.011
```

Assuming the null hypothesis of no relation is true, the probability of getting a sample slope coefficient that is almost 33 standard errors from 0 is extremely small, less than 0.000

The usual 95% cutoff for a t-statistic is around 2.00, closer to 1.96 for large samples, and larger for very small samples.

Specify the basis for the statistical decision for the hypothesis test and the resulting statistical conclusion

[Sec 3.2.1, 3.3.2.1]   Apply to the 1st predictor variable

```
Estimated Model for Sales

              Estimate    Std Err   t-value   p-value    Lower 95%   Upper 95%
(Intercept)      2.939      0.312     9.422     0.000        2.324       3.554
         TV      0.046      0.001    32.809     0.000        0.043       0.049
      Radio      0.189      0.009    21.893     0.000        0.172       0.206
  Newspaper     -0.001      0.006    -0.177     0.860       -0.013       0.011
```

Given the p-value=0.000 < alpha=0.05, reject the null hypothesis of no relation between TV ad spending and Sales for any given levels of Radio and Newspaper advertising

Hypothesis Test: Interpretation, as an executive summary you would report to management

[Sec 3.2.1, 3.3.2.1]   Apply to the 1st predictor variable

```
Estimated Model for Sales

            Estimate   Std Err   t-value   p-value   Lower 95%   Upper 95%
(Intercept)    2.939     0.312     9.422     0.000      2.324       3.554
         TV    0.046     0.001    32.809     0.000      0.043       0.049
      Radio    0.189     0.009    21.893     0.000      0.172       0.206
  Newspaper   -0.001     0.006    -0.177     0.860     -0.013       0.011
```

For any given level of Radio and Newspaper advertising, TV advertising is related to *increasing* Sales

Specify the null hypothesis and its alternative for the hypothesis test of the slope coefficient

[Sec 3.2.1]   Apply to the 3rd predictor variable

```
Estimated Model for Sales

              Estimate    Std Err   t-value   p-value    Lower 95%    Upper 95%
(Intercept)     2.939      0.312      9.422     0.000        2.324        3.554
         TV     0.046      0.001     32.809     0.000        0.043        0.049
      Radio     0.189      0.009     21.893     0.000        0.172        0.206
  Newspaper    -0.001      0.006     -0.177     0.860       -0.013        0.011
```

Null Hypothesis: Partial slope coefficient, that is, the values of Radio and TV held constant, of Newspaper ad spending on on unit Sales is 0. Or, in symbols, $\beta_{NP} = 0$

Alternative to the Null: Holding the values of Radio and TV ad spending constant, there is a relationship between Newspaper advertising and unit Sales, $\beta_{NP} \neq 0$

Specify the calculation of the *t*-statistic by applying the relevant numbers from this specific analysis

Apply to the 3rd predictor variable

```
Estimated Model for Sales

              Estimate    Std Err    t-value    p-value    Lower 95%    Upper 95%
(Intercept)     2.939      0.312      9.422      0.000        2.324        3.554
         TV     0.046      0.001     32.809      0.000        0.043        0.049
      Radio     0.189      0.009     21.893      0.000        0.172        0.206
  Newspaper    -0.001      0.006     -0.177      0.860       -0.013        0.011
```

How close is the estimate to what is hypothesized? The *t*-statistic is how many standard errors the sample estimate is from the null, hypothesize value, here 0

$t = (b_{NP} - 0) / 0.006 = -0.001 / 0.006 = -0.177$

Adapt the definition of the *p*-value to this analysis

Apply to the 3rd predictor variable

```
Estimated Model for Sales

              Estimate   Std Err   t-value   p-value   Lower 95%   Upper 95%
(Intercept)     2.939     0.312     9.422     0.000       2.324       3.554
         TV     0.046     0.001    32.809     0.000       0.043       0.049
      Radio     0.189     0.009    21.893     0.000       0.172       0.206
  Newspaper    -0.001     0.006    -0.177     0.860      -0.013       0.011
```

Assuming the null hypothesis of no relation is true, the probability of getting a sample slope coefficient that is only 0.177 or more standard errors from 0 is high, 0.860

This sample slope coefficient of -0.001 is consistent with the null hypothesized value of 0

Specify the basis for the statistical decision for the hypothesis test and the resulting statistical conclusion

[Sec 3.2.1, 3.3.2.1]   Apply to the 3rd predictor variable

```
Estimated Model for Sales

              Estimate   Std Err   t-value   p-value   Lower 95%   Upper 95%
(Intercept)      2.939     0.312     9.422     0.000       2.324       3.554
         TV      0.046     0.001    32.809     0.000       0.043       0.049
      Radio      0.189     0.009    21.893     0.000       0.172       0.206
  Newspaper     -0.001     0.006    -0.177     0.860      -0.013       0.011
```

Given the *p*-value=0.860 > alpha=0.05, so cannot reject the null hypothesis of no relation between Newspaper ad spending and Sales for any given levels of Radio and TV advertising

Hypothesis Test: Interpretation, as an executive summary you would report to management

Apply to the 3rd predictor variable

```
Estimated Model for Sales

              Estimate    Std Err    t-value    p-value    Lower 95%    Upper 95%
(Intercept)     2.939       0.312      9.422      0.000       2.324        3.554
         TV     0.046       0.001     32.809      0.000       0.043        0.049
      Radio     0.189       0.009     21.893      0.000       0.172        0.206
  Newspaper    -0.001       0.006     -0.177      0.860      -0.013        0.011
```

For any given level of Radio and TV advertising, no relationship between Newspaper ad spending and unit Sales is detected

Confidence Interval

What is the True Value of a Slope Coefficient?

Specify the value the confidence interval estimates

Apply to the 1st predictor variable

```
Estimated Model for Sales

              Estimate    Std Err   t-value   p-value    Lower 95%   Upper 95%
(Intercept)      2.939      0.312     9.422     0.000        2.324       3.554
         TV      0.046      0.001    32.809     0.000        0.043       0.049
      Radio      0.189      0.009    21.893     0.000        0.172       0.206
  Newspaper     -0.001      0.006    -0.177     0.860       -0.013       0.011
```

The confidence interval estimates the true (population) slope coefficient that relates TV advertising to Sales with given values of Radio and Newspaper advertising

Specify the computation of the margin of error by applying the relevant numbers of this analysis with 2 for the *t*-cutoff

[Secs 3.2.2, 3.3.2.2]   Apply to the 1st predictor variable

```
Estimated Model for Sales

               Estimate    Std Err   t-value   p-value    Lower 95%    Upper 95%
  (Intercept)     2.939      0.312     9.422     0.000        2.324        3.554
           TV     0.046      0.001    32.809     0.000        0.043        0.049
        Radio     0.189      0.009    21.893     0.000        0.172        0.206
    Newspaper    -0.001      0.006    -0.177     0.860       -0.013        0.011
```

The 95% margin of error is about plus or minus 2 standard errors, depending on the exact value of the *t*-cutoff value that cuts off the upper and lower 2.5% of the *t*-distribution over many hypothetical repeated samples

E = (2)(0.001)

Show the computations of the confidence interval illustrated with the specific numbers from this analysis

[Secs 3.2.2, 3.3.2.2]   Apply to the 1st predictor variable

```
Estimated Model for Sales

              Estimate    Std Err    t-value    p-value    Lower 95%    Upper 95%
(Intercept)      2.939      0.312      9.422      0.000        2.324        3.554
         TV      0.046      0.001     32.809      0.000        0.043        0.049
      Radio      0.189      0.009     21.893      0.000        0.172        0.206
   Newspaper    -0.001      0.006     -0.177      0.860       -0.013        0.011
```

The 95% confidence interval is plus and minus the margin of error about the sample value, here the value for b_{TV}

Lower Bound of CI: $b_{TV} - E = 0.046 - E = 0.043$
Upper Bound of CI: $b_{TV} + E = 0.046 + E = 0.049$

Confidence Interval: Interpretation, as an executive summary you would report to management

Apply to the 1st predictor variable

```
Estimated Model for Sales

              Estimate    Std Err   t-value   p-value    Lower 95%    Upper 95%
(Intercept)      2.939      0.312     9.422     0.000        2.324        3.554
         TV      0.046      0.001    32.809     0.000        0.043        0.049
      Radio      0.189      0.009    21.893     0.000        0.172        0.206
  Newspaper     -0.001      0.006    -0.177     0.860       -0.013        0.011
```

For any given level of Radio and Newspaper advertising, each additional $1000 of TV advertising relates to an average increase of unit Sales likely somewhere between 43 to 49

Demonstrate the consistency of the confidence interval and hypothesis test using the specific numbers for this analysis for both results

Apply to the 1st predictor variable

```
Estimated Model for Sales

              Estimate    Std Err   t-value   p-value    Lower 95%    Upper 95%
(Intercept)     2.939      0.312     9.422     0.000        2.324        3.554
         TV     0.046      0.001    32.809     0.000        0.043        0.049
      Radio     0.189      0.009    21.893     0.000        0.172        0.206
  Newspaper    -0.001      0.006    -0.177     0.860       -0.013        0.011
```

The *p*-value for the hypothesis test of a 0 partial population slope coefficient for TV is less than 0.05, so 0 was rejected as a plausible value

The confidence interval contains the range of plausible values for the slope coefficient, which does not contain 0, again rendering 0 as implausible

Are any predictor variables much less useful for predicting the response variable? Why or why not?

[Secs 5.2, 5.4]

```
Estimated Model for Sales

            Estimate   Std Err   t-value   p-value   Lower 95%   Upper 95%
(Intercept)    2.939     0.312     9.422     0.000       2.324       3.554
         TV    0.046     0.001    32.809     0.000       0.043       0.049
      Radio    0.189     0.009    21.893     0.000       0.172       0.206
  Newspaper   -0.001     0.006    -0.177     0.860      -0.013       0.011
```

The *p*-value for the hypothesis test of a 0 partial population slope coefficient for Newspaper ad spending is much greater than 0.05 so 0 cannot be rejected, and the confidence interval contains 0 and both - and + values, all rendered as plausible

No relationship is detected for Newspaper ad spending and Sales given a set of values for TV and Radio ad spending

Evaluate fit with the standard deviation of residuals

[Secs 2.4, 3.3.3]

```
Sales:  Mean = 14.022   SD = 5.217
        Min = 1.600   Max = 27.000


Model Fit

Standard deviation of residuals:  1.686 for 196 degrees of freedom
95% range of residual variation:  6.648    [2 * 1.972 * 1.686]

R-squared:  0.897    Adjusted R-squared:  0.896    PRESS R-squared:  0.891
```

The standard deviation of unit Sales is 5.217, yet the standard deviation of the residuals about the regression line reduces down to 1.686, a considerable reduction

The model fits at least reasonably well, though ultimately fit depends on the value judgement of the impact of the size of the 95% range of variation of the residuals, 6.6

Evaluate fit with R² and PRESS R², including their comparison

```
Sales:  Mean = 14.022   SD = 5.217
        Min = 1.600   Max = 27.000


Model Fit

Standard deviation of residuals:  1.686 for 196 degrees of freedom
95% range of residual variation:  6.648    [2 * 1.972 * 1.686]

R-squared:  0.897    Adjusted R-squared:  0.896    PRESS R-squared:  0.891
```

R-sq above 0.6 is usually considered agood fit to the data, a descriptive statistic

PRESS R-sq generalizes to new data and so usually is lower

Here PRESS R-sq is almost as large as R-sq, indicating no practical overfitting and excellent fit to new data as well

Any collinearity problems? Why or why not?

[Sec 5.3]

```
            Collinearity

                    Tolerance        VIF
            TV        0.995         1.005
         Radio        0.873         1.145
     Newspaper        0.873         1.145
```

Tolerance below 0.2 usually is considered to indicate linear dependence of the predictor variable on the others, which corresponds to VIF > 5

Clearly no collinearity with these predictor variables

Any potential outliers? Why or why not?

[Secs 6.2, 6.3]

```
RESIDUALS AND INFLUENCE

         TV  Radio Newspaper  Sales fitted  resid rstdnt dffits cooks
131   0.700 39.600     8.700  1.600 10.428 -8.828 -5.758 -1.127 0.273
  6   8.700 48.900    75.000  7.200 12.478 -5.278 -3.288 -0.734 0.128
 76  16.900 43.700    89.400  8.700 11.858 -3.158 -1.943 -0.478 0.056
 36 290.700  4.100     8.500 12.800 17.007 -4.207 -2.571 -0.459 0.051
179 276.700  2.300    23.700 11.800 16.011 -4.211 -2.570 -0.435 0.046
127   7.800 38.900    50.600  6.600 10.577 -3.977 -2.422 -0.403 0.040
166 234.500  3.400    84.800 11.900 14.224 -2.324 -1.433 -0.391 0.038
```

Two samples have a much larger Cook's Distance values and Studentized residuals than the others, and so are likely outliers

Both samples have negative residuals, which means that Sales was drastically over predicted

For example, for the first sample, unit Sales was 1600 and fitted Sales is 10,428

From this analysis, what model do you recommend? Why?

[Sec 5.4]

```
        Best Subset Regression Models

    TV Radio Newspaper     R2adj     X's
     1     1         0     0.896      2
     1     1         1     0.896      3
     1     0         1     0.642      2
     1     0         0     0.610      1
     0     1         0     0.329      1
     0     1         1     0.326      2
     0     0         1     0.047      1
```

The most parsimonious model is the model with TV and Radio advertising as the predictor variables

The fit of R2-adjusted is as large as the full three variable model, 0.896, and noticeably improved over the just the single predictor model with TV as the predictor, 0.610

Prediction Intervals

How Much Error is Expected from Predictions of y?

Show the calculation of the 95% prediction interval approximated with the t-cutoff of 2

Use the advertising campaign of TV=232, Radio=8.6, Newspaper=8.7

[Sec 4.3.4.2]

```
    PREDICTION ERROR

            TV   Radio Newspaper  Sales   pred     sf pi.lwr pi.upr width
    97  197.600  3.500     5.900 11.700 12.636 1.701  9.281 15.991 6.710
    17   67.800 36.600   114.000 12.500 12.824 1.757  9.359 16.288 6.929
    58  136.200 19.200    16.600 13.200 12.775 1.691  9.439 16.110 6.672
...
   200  232.100  8.600     8.700 13.400 15.173 1.701 11.819 18.527 6.708
    46  175.100 22.500    31.500 14.900 15.162 1.690 11.828 18.495 6.667
   147  240.100  7.300     8.700 13.200 15.294 1.702 11.937 18.651 6.714
```

The 95% prediction interval is plus and minus the margin of error about the sample value, the standard error of forecast, s_f, multiplied by about 2

Lower Bound of PI: Sales_fitted – E = 15.173 – E = 11.819
Upper Bound of PI: Sales_fitted + E = 15.173 + E = 18.527

Interpret the prediction interval

Use the advertising campaign of TV=232, Radio=8.6, Newspaper=8.7

```
PREDICTION ERROR

            TV  Radio Newspaper  Sales    pred     sf pi.lwr pi.upr width
    97 197.600  3.500     5.900 11.700  12.636  1.701  9.281 15.991 6.710
    17  67.800 36.600   114.000 12.500  12.824  1.757  9.359 16.288 6.929
    58 136.200 19.200    16.600 13.200  12.775  1.691  9.439 16.110 6.672
...
   200 232.100  8.600     8.700 13.400  15.173  1.701 11.819 18.527 6.708
    46 175.100 22.500    31.500 14.900  15.162  1.690 11.828 18.495 6.667
   147 240.100  7.300     8.700 13.200  15.294  1.702 11.937 18.651 6.714
```

With 95% confidence, the actual value of Sales -- for a sample with TV ad spending of 232, Radio of 8.6, and Newspaper of 8.7 -- will lie between 11,819 and 18,527 units

Conclusion

Management Recommendation

What decision do you recommend to management?

	Estimate	Std Err	t-value	p-value	Lower 95%	Upper 95%
(Intercept)	2.939	0.312	9.422	0.000	2.324	3.554
TV	0.046	0.001	32.809	**0.000**	0.043	0.049
Radio	0.189	0.009	21.893	**0.000**	0.172	0.206
Newspaper	-0.001	0.006	-0.177	**0.860**	-0.013	0.011

TV	Radio	Newspaper	R2adj	X's
1	**1**	**0**	**0.896**	**2**
1	1	1	0.896	3
1	0	1	0.642	2
1	0	0	0.610	1
0	1	0	0.329	1
0	1	1	0.326	2
0	0	1	0.047	1

Controlling the values of TV and Radio ad spending, Newspaper ad spending contributes little to Sales

Maybe by itself Newspaper ad spending would be useful, but that hypothesis is not evaluated in this analysis. Instead, implement TV and Radio ads, dropping Newspaper ads

The End