

# HW 3

Math 643

Due: 13 May 2026

As always, your answer will be graded on the quality of presentation as well as the correct answer. To get a good score: write your answer neatly, use complete sentences, and *justify your work*.

1. Suppose that  $G$  is a group and  $A$  is a  $G$ -module. Let's transform  $\mathbb{Z}$  into a  $G$ -module by giving it the trivial  $G$ -action. Prove:

$A^G$  and  $\text{Hom}_{\mathbb{Z}G}(\mathbb{Z}, A)$  are isomorphic as abelian groups.

2. Let  $G$  be a group and  $0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$  a short exact sequence of  $G$ -modules. Suppose  $B$  has the property that: for all positive integers  $n$ , we know  $H^n(G, B) = 0$ . Prove: for all positive integers  $n$ ,

$H^n(G, C)$  and  $H^{n+1}(G, A)$  are isomorphic as abelian groups.