

As always, your answer will be graded on the quality of presentation as well as the correct answer. To get a good score: write your answer neatly, use complete sentences, and *justify your work*.

1. Let  $R$  be a commutative ring, and let  $A, B, C, M$  be  $R$ -modules. Write  $H$  for the functor  $\text{Hom}_R(\bullet, M)$ . Suppose that

$$0 \leftarrow A \xleftarrow{\alpha} B \xleftarrow{\beta} C \leftarrow 0 \quad (\star)$$

is a short exact sequence of  $R$ -modules. Prove: if  $(\star)$  is split, then

$$0 \rightarrow H(A) \xrightarrow{H(\alpha)} H(B) \xrightarrow{H(\beta)} H(C) \rightarrow 0$$

is a split short exact sequence.