

# HW 1

Due: 21 January 2026

As always, your answer will be graded on the quality of presentation as well as the correct answer. To get a good score: write your answer neatly, use complete sentences, and *justify your work*.

1. Since  $\mathbb{C}$  is commutative, it is
  - (a) an  $(\mathbb{R}, \mathbb{R})$  bimodule, using restriction of scalars twice and
  - (b) an  $(\mathbb{C}, \mathbb{C})$  bimodule, using restriction of scalars once.

By (a), we can construct the  $\mathbb{R}$ -module  $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C}$ , and by (b) we can construct the  $\mathbb{C}$ -module  $\mathbb{C} \otimes_{\mathbb{C}} \mathbb{C}$ . However, using restriction of scalars, we can consider  $\mathbb{C} \otimes_{\mathbb{C}} \mathbb{C}$  as an  $\mathbb{R}$ -module. Show that the  $\mathbb{R}$ -module  $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C}$  and the  $\mathbb{R}$ -module  $\mathbb{C} \otimes_{\mathbb{C}} \mathbb{C}$  are not isomorphic.

2. [Ash10, Exercise 8.7.2]
3. [Ash10, Exercise 8.7.7]

## References

[Ash10] Robert B. Ash, *Abstract Algebra: The Basic Graduate Year*, 2010.