HW 1 — Due: 8 October 2025

As always, your answer will be graded on the quality of presentation as well as the correct answer. To get a good score: write your answer neatly, use complete sentences, and *justify your work*.

1. Problems 3,4,5 from [Ash10, Section 4.1].
2. Suppose that R, S are rings, that M is an S-module, and $\phi\colon R \to S$ is a homomorphism or rings. Define the operation "$\bullet$" by
$$\text{for all } r \in R \text{ and } m \in M : \qquad r \bullet m = \phi(r)m$$
Prove that $(M, \bullet)$ is an R-module.
3. Let V be the inner product space $\mathbb{R}^2$ (with the dot product). Let M be the sub-$\mathbb{Z}$-module of V generated by $\begin{bmatrix}143\\100\end{bmatrix}$ and $\begin{bmatrix}356\\249\end{bmatrix}$. Find a generating set for M consisting of vectors of length less than 6.

References

[Ash10] Robert B. Ash, *Abstract Algebra: The Basic Graduate Year*, 2010.