

Final Exam

Math 344

- Write your name here: _____.
- Answer all questions.
- Proofs are graded for clarity, rigor, neatness, and style.
- Good luck!

1 Computations

1. (a) Using cycle notation, write down all elements of S_4 of order 2.
(b) Using cycle notation, write down all elements of S_4 of order 4.

2. Write down:

- (a) An infinite group G and a subgroup H such that G/H is finite.
- (b) An infinite group G and a subgroup H such that G/H is infinite.
- (c) A group G and a nonnormal subgroup H .
- (d) A noncommutative group G and a normal subgroup H with the property that G/H is commutative.

3. Let f be the following element of S_9 :

$$f: \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \rightarrow \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$1 \mapsto 6$$

$$2 \mapsto 4$$

$$3 \mapsto 1$$

$$4 \mapsto 9$$

$$5 \mapsto 3$$

$$6 \mapsto 7$$

$$7 \mapsto 8$$

$$8 \mapsto 2$$

$$9 \mapsto 5.$$

- (a) Write f in disjoint cycle form.
- (b) Write f^{-1} in disjoint cycle form.
- (c) Write f as a product of transpositions.
- (d) State the parity of f .
- (e) Write $(162) \circ (37) \circ f$ in disjoint cycle form.

2 Proofs

1. Suppose that G is a group and $|G| < 200$. Suppose further that G has subgroups of size 15 and 36. What is the size of G ? Prove your answer is correct.

2. Let $G = \mathbb{Z} \times \mathbb{Z}$ and $H = \{(m, n) \in G \mid 2m + 3n = 0\}$. In Exam One, we proved H is a subgroup of G . But now we know: the fact G is abelian tells us that H is a normal. Prove G/H is isomorphic to $\mathbb{Z}$.

3. Suppose that G_1, G_2 are groups, and $\phi: G_1 \rightarrow G_2$ is a homomorphism. Recall that $\phi(G_1) = \{\phi(g) \mid g \in G_1\}$, and that $\phi(G_1)$ is a subgroup of G_2 . Prove that if $|G_1| = 16$, then $|\phi(G_1)|$ is a divisor of 16.