HW 5 Due: 18 February 2026

As always, your answer will be graded on the quality of presentation as well as the correct answer. To get a good score: write your answer neatly, use complete sentences, and *justify your work*.

Computations

1. Write down all the orders of all the elements of D_5.
2. Write down all the orders of all the elements of $S_3 \times \mathbb{Z}_2$.
3. Let $f = (14563) \in S_9$. Write down all the orders of all the elements of $\langle f \rangle$.

Proofs

(I) Let G be a group and suppose $g, h \in G$ have finite order. Prove: if $gh = hg$, then $\operatorname{ord}(gh)$ is a divisor of $\operatorname{lcm}(\operatorname{ord}(g), \operatorname{ord}(h))$.

(II) Suppose that G, H are groups and $\phi\colon G \to H$ is an isomorphism. Prove: if G is cyclic, then H is cyclic.

(III) Let G be a group, and define

$$D = \{(g, g) \mid g \in G\}.$$

Assume that D is a subgroup of $G \times G$.[1] Prove that G is isomorphic to D.

(IV) Suppose that G is a finite cyclic group of order $n \in \mathbb{Z}_{\geq 1}$, with generator $g \in G$. Let $j \in \mathbb{Z}_{>0}$. Prove: if there exist $a, b \in \mathbb{Z}$ with $an + bj = 1$, then $G = \langle g^j \rangle$.

[1] This is always true and in principle, we could prove it. However, I'm not requiring a proof. At this time.